Lecture 15-16
Dynamic Game with Incomplete Information

Read ch.4 and two handouts

- Review dynamic game and SPNE
- Predatory pricing (in handout)
- Limit Pricing or Entry deterrence (Milgrom-Roberts model) in handouts
- Perfect Bayesian equilibrium
- Job-market signaling (Spence) in textbook

TOPICS

- Remind dynamic game with complete information and Subgame Perfect Nash Equilibrium
- Applications
  - Predatory Pricing
  - Limit Pricing
- Dynamic Game with Incomplete Information
- Perfect Bayesian or Weak Sequential equilibrium concept

Dynamic game

- In a static game or game with simultaneous moves, players take their opponents' strategies as given and thus do not consider the possibility of influencing them. We employ a NE concept.
- But in games in which a player chooses some actions after observing some of his opponents' actions (or in dynamic game), this NE concept can lead to some absurd Nash equilibria.
- Thus we need some refinement of NE for a class of dynamic games.

Dynamic game

- We use the game tree to describe the dynamic game.
- A subgame is a smaller part of the whole game starting from any node and continuing to the end of the entire game. Thus, a subgame can be analyzed as a game in its own right.
- To describe what players know when they move, we identify the information set. A single node or singleton is exactly one information set.
- But one information set can contain many nodes, using dashed line to connect them.
- At each information set has only one player to move and the same set of choices.

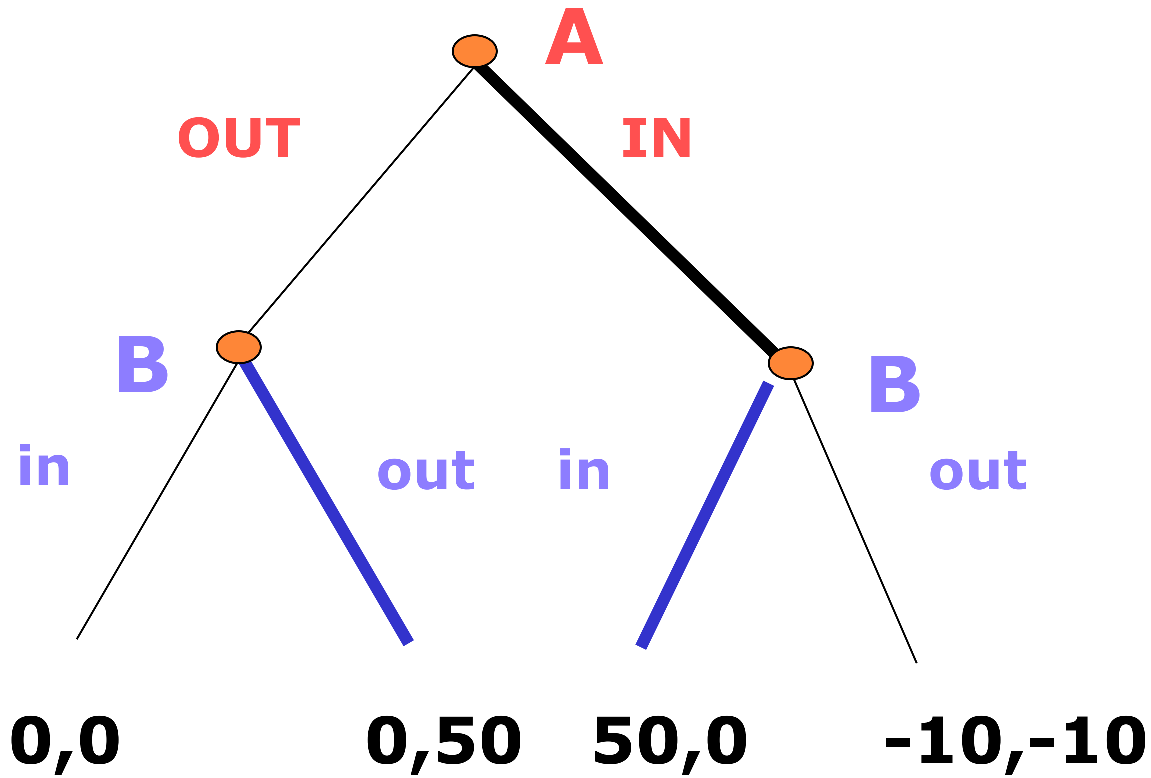
DYNAMIC GAME

- To make sure that we have NE in all subgames, we use a <u>solving backward</u> technique.
- This new solution or equilibrium concept is called "Subgame Perfect Nash Equilibrium".
- In dynamic game, strategy is a decision rule or a complete <u>contingent</u> plan of actions for all possible events.
- Thus, SPNE strategies give us the complete contingent actions of all players, regardless of players have a chance to play or not. But if they were, those equilibrium actions must be credible or self-enforcing.

Dynamic game

- For any extensive form game, there is a unique normal form. But the reverse is not true.
- We create the normal form game to help identifying NE's. This will tell us which NE's is possibly an empty move.
- In a simultaneous game, a strategy is a simple <u>non-contingent</u> choice of action.
- Thus, when we form the normal game from the dynamic game, we must present all possible action combinations at all information sets in the game.

Ex1. Entry game



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SPNE: A: plays IN; B plays OUT if A stays OUT, and B plays IN if A plays IN.

B

A	(out,out)	(in, in)	(out, in)	(in, out)
out	0,0	$\boxed{0,50}$	0,0	0,50
in	$\boxed{50,0}$	-10,-10	-10,-10	$\boxed{50,0}$

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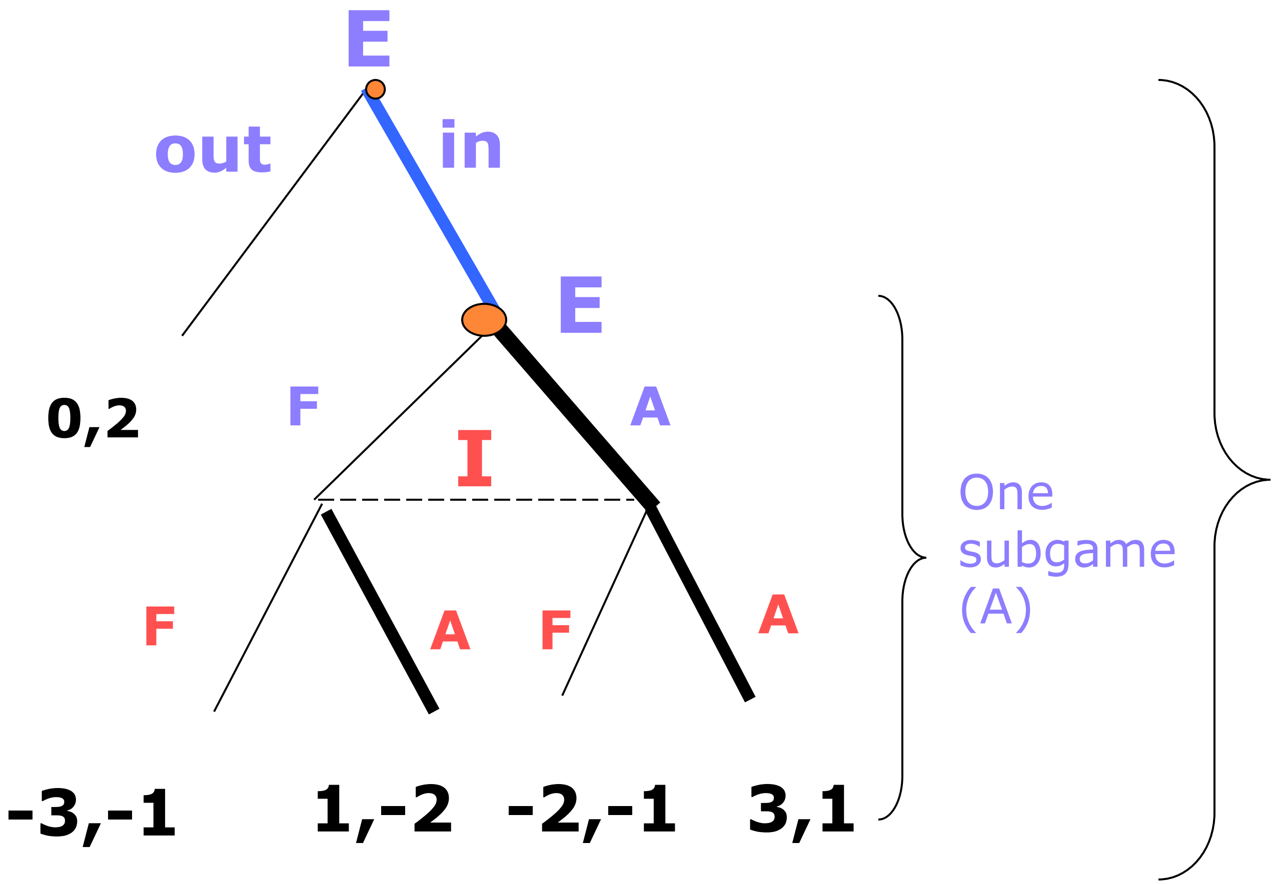
In this example, we found 3 NE's: {out, (in, in) }
{in, (out, out)} and {in, (in,out)}

Ex. 2



Dynamic game: subgame (A)

		Incumbent	
		Fight	**Accommodate**
Entrant	**Fight**	**-3, -1**	**1, -2**
	Accommodate	**-2, -1**	**3, 1**

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DYNAMIC GAME: WHOLE GAME

What is NE's and SPNE?

		Incumbent	
		Fight	Accommodate
entrant	Out, Fight	$\boxed{0,2}$	0,2
	Out, ACC	$\boxed{0,2}$	0,2
	IN, Fight	-3,-1	1,-2
	IN, ACC	-2,-1	$\boxed{3,1}$

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Entry Deterrence (or Milgrom Roberts model of limit pricing)

- Interaction between an incumbent and a potential entrant.
- We examine situations where a monopolist has the incentive to try and prevent other firms from entering
- If successful, the incumbent maintains its monopoly position.
- Assume product is close substitutes so entry will have adverse effect on monopolist's profits.
- We first discuss the limit pricing theory: how MP deters entry by increasing its output (lowering the price), hoping that the potential entrant will believe that its level of sales will be small and stay out of the market.

Entry Deterrence

- The limit price is the highest price at which entry is deterred.
- Using this pricing policy, the MP sacrifices some of its current profits to ensure higher profits in the future.
- Is this policy sound?
- Probably not, why entrant should believe that the incumbent will continue to produce the same level after the entry.
- The MP can threat, but we know this is not credible. So, it's not rational to issue such threat.
- Same to say this is not SPNE.

Entry Deterrence

- You can draw the extensive form game of this story with 2 periods.
- Solving backward, in the second period, suppose if the entry happens, both firms compete in Cournot fashion. Thus NE in in this second period is both select Cournot NE output levels.
- Entrant will enter if they make profits under Cournot competition, if not he will stay out.
- Importantly, the entry decision is not related to what the incumbent did in the first period.
- The incumbent realizes that, so he will ignore any limit price strategy. Thus, he will set price to maximize his current period profits in the first period.

Entry Deterrence

- There are two possible SPNEs: one is where the incumbent remains a monopolist in both periods, and the entrant stays out. The second equilibrium is where the incumbent sets the MP price in the first period and faces the duopolistic competition in the second period.
- There is no role for entry deterrence or limit pricing.
- So far we assume limited numbers of interaction, and there is no possibility of pre-commitment.
- What happens if we allow more interaction between the incumbent and the potential entrants.

Credible Entry Deterrence:
- Predatory Pricing

- Can the incumbent deter entry by engaging in a price war if and when any firms enters the market.
- After entry has occurred, the Incumbent chooses to undercut price or takes predatory pricing policy, making both worse off.
- Could this predatory pricing be rational behavior?
- In finite repeated game, this is not possible.
- Possible if there are infinite markets and there is possible entrants to all markets.
- Consider a Chain-store Game

Predatory Pricing

- Set up:
- A monopoly incumbent operates in an infinite number of separate but identical markets, and faces a potential entrant in each one.
- The entrants make their decision to enter each market sequentially.
- If entry occurs on any one market, the incumbent has two choices: engage in predatory pricing and both firms make a loss, or accommodate entry and both firms earn positive profits.
- If an entrant does not enter, his payoff =0; and the incumbent earns monopoly profits, M.

The Chain-store Game

Entrant

Incumbent

	Enter	Stay out
Fight	FI, FE	M, 0
Accom	AI, AE	M, 0

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Assume that M > AI > 0 > FI
And AE > 0 > FE

Predatory Pricing

- For the incumbent, monopoly position is preferred to Accommodate and Accommodation is preferred to Fight
- Entrant will enter if he expects to be Accommodated from Incumbent.
- Assume that all previous outcomes are known by all players.
- If the entrant finds Accommodate in the first market, he will believe that Accommodate will occur in all other markets.

PREDATORY PRICING

Incumbent's PDV of profits from $ACCOM$

$= AI + \delta AI + \delta^2 AI + \ldots$

$= AI \,/\, (1-\delta)$

and from Fight

$= FI + \delta M + \delta^2 M + \ldots = FI + \delta M \,/\, (1-\delta).$

So, he will fight if

$FI + \delta M \,/\, (1-\delta) > AI \,/\, (1-\delta).$

If δ is big enough or close to one (patient), then he will fight and deter entry in every market.

Predatory Pricing

- Conditions for predatory pricing
- Profits from Fight in the first period + subsequent profits from monopoly must be greater or equal to Profits from Accommodate in all markets
- Noting that this policy works because entrant knows that Incumbent has an aggressive behavior or enough incentives to fight such entry.
- Not because seeing Incumbent's behavior before entry periods.

-PRE-COMMITMENT

- When the incumbent cannot credibly pre-commit its future behavior, entry deterrence cannot occur in the finite and complete-information models.
- Some credible commitment is to undertake some actions in the current periods that affects the nature of competition in subsequent periods and it must be irreversible.
- For example, strategic investment in capital allowing for lowering marginal costs in the future period if seeing the entrant. Increased expenditure in R&D. Advertising can be a form of pre-commitment if its effects are durable. Entrant may have to match this cost to compete effectively, and thus could be deterred.

-Incomplete Information: Limit Pricing

- Entry deterrence might be credible, even if the game is only finitely repeated, is if there is incomplete information about the types of incumbent.
- For example, Limit pricing is a rational behavior if there is uncertainty about incumbent's cost.
- This is a game with incomplete information
- In the old way, we explain this Limit Pricing practice by claiming that entrant believes that Incumbent will keep undercutting price (or producing high output) if entrant enters.

Limit Pricing

- We know that this is an empty threat.
- So, it is not reasonable belief for Entrant.
- Set up (one entrant, one incumbent, 2 periods)
- Incumbent moves first, then Entrant sees the pricing policy of the incumbent, then decides whether or not to enter in the second period, if enter occurs, both plays duopoly game, and game ends.
- Incumbents have two types: low and high costs. And it is known only by the incumbent.

LIMIT PRICING

- Entrant will enter if expecting to see the high-cost incumbent.
- Both incumbents can pretend to be low-cost types, and choose low price.
- Entrant will believe that the incumbent is a real low-cost incumbent if the pricing signal is credible.
- The signal is credible if the pricing set by the low-cost type is low enough so that the high-cost type does not want to pretend to be.

Limit Pricing

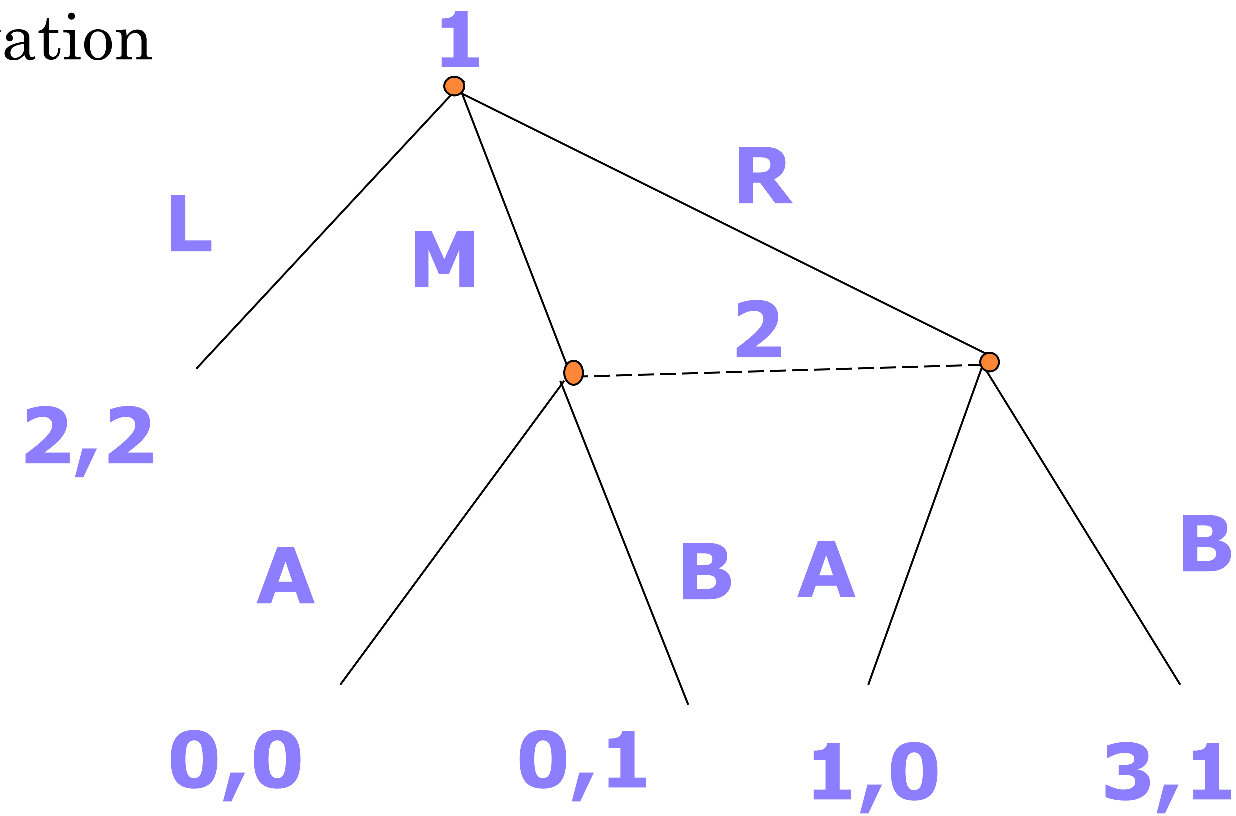
- **Case 1: complete information about types**
- If Incumbent is low-cost, he will set the monopoly price in both periods, and the entrant will not enter in the second period.
- If Incumbent is high-cost, he will set the monopoly price in the first period, the entrant will enter and both choose the NE of the second period duopoly game.
- Entrant will enter with probability of seeing high type; will stay out with probability of seeing low type.

Limit Pricing

- **Case 2: Incomplete information**
- We need a new equilibrium concept to answer this.
- This is known as signaling games by Milgrom and Roberts.
- Equilibrium can be either a “pooling equilibrium” or “ a separating equilibrium”.
- In Pooling Equil., types are not revealed by their actions because they do the same thing (both low cost and high cost incumbent).
- In Separating Eguil, type is revealed because their actions are different

Dynamic game with incomplete information: Weak Sequential Equilibrium

- motivation



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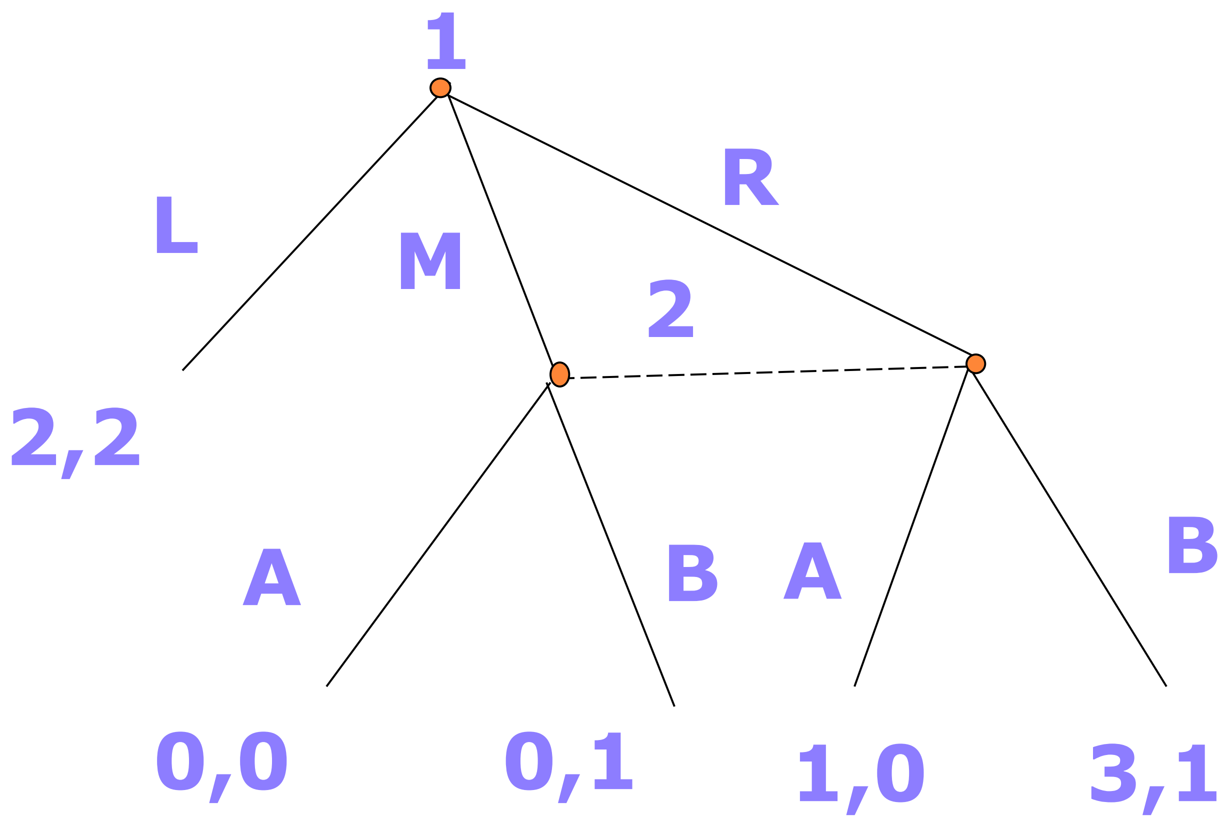
DYNAMIC GAME WITH INCOMPLETE INFORMATION: WEAK SEQUENTIAL EQUILIBRIUM

- When one subgame is the whole game, SPNE won't work. We have NE = SPNE.
- Thus, we cannot get rid of some incredible move in NE.
- When players face with uncertainty about the move of other players, or types of players when making his decision, the natural way is to make a guess about the probability of the unknown.
- In other words, players will form some beliefs about the unknown and then choose the best response given his belief. "Given his belief, he acts his best".

Dynamic game with incomplete information

- Next question is how we make a guess or form our belief in the first place.
- Similarly, we can infer about beliefs from given or fixed strategies.
- Suppose we propose one equilibrium behavior strategies, we could then check if these beliefs are consistent with the proposed strategies. If so, it is a good belief or consistent belief.
- How do we check this consistency in our belief system?

DYNAMIC GAME WITH INCOMPLETE INFORMATION



Two pure NE's: (L, A) and (R, B).
Notice that (L, A) is not reasonable because it is supported by an empty threat.
Why? Because A is a dominated strategy for player 2.
To refine our solution concept, we introduce the notion of beliefs about what the player is uncertain.

DYNAMIC GAME WITH INCOMPLETE INFORMATION

- Assume: finite N a number of players, and finite X, a set of decision nodes.
- Let h(x) be information set containing x.
- n(h) is the player who plays at h.
- $\boldsymbol{\sigma}_n$ (. / h(x)) is behavior strategy of n at node x.
- Prob (x / $\boldsymbol{\sigma}$) is prob. that node x is reached under strategy $\boldsymbol{\sigma}$.
- $\boldsymbol{\mu}$ **(h) = (.., $\boldsymbol{\mu}$ (x) ,....)** beliefs at h;
- $\boldsymbol{\mu}$ **(x) belief of player n(h) on x conditional on h(x) being reached.**
- $\boldsymbol{\sigma}$ is a set of strategy for all players.
- $\boldsymbol{\mu}$ beliefs at all information sets
- $\pi_{n(h)}(\sigma / h, \mu(h))$ expected payoff of n(h) given that h is reached and he has belief $\boldsymbol{\mu}$ **(h)**

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Illustration of notation

Weak Sequential Equilibrium

- Needs two components
- 1. Sequential Rationality (S): Optimal action given your belief. Given belief, you play optimally from that point onwards.

- An assessment ($\boldsymbol{\sigma}, \boldsymbol{\mu}$) is sequentially rational if, for any information set h and alternative strategy $\sigma'_{n(h)}$,

$$\pi_{n(h)}(\sigma / h, \mu(h)) \geq \pi_{n(h)}(\sigma'_{n(h)}, \sigma_{-n(h)} / h, \mu(h)) .$$

That is, for any h, each player follows $\boldsymbol{\sigma}_n$ from that point onwards, given belief μ and her opponent's strategies $\boldsymbol{\sigma}_{-n}$.

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Weak Sequential Equilibrium

- 2. Weak Consistency (WC). Your belief needs to be consistent with the equilibrium strategy being played (mean you make a correct guess about your opponent's strategy choices).
- 2.1 If the information set h(x) is reached with a positive probability, we apply Bayes' rules to form our belief at that set.

$$\mu(x) = \frac{\text{Prob}(x/\sigma)}{\text{Prob}(h(x)/\sigma)}.$$

Belief at x = Prob (x | equil strategies) / Prob (h(x) | equil strategies)

Weak Sequential Equilibrium

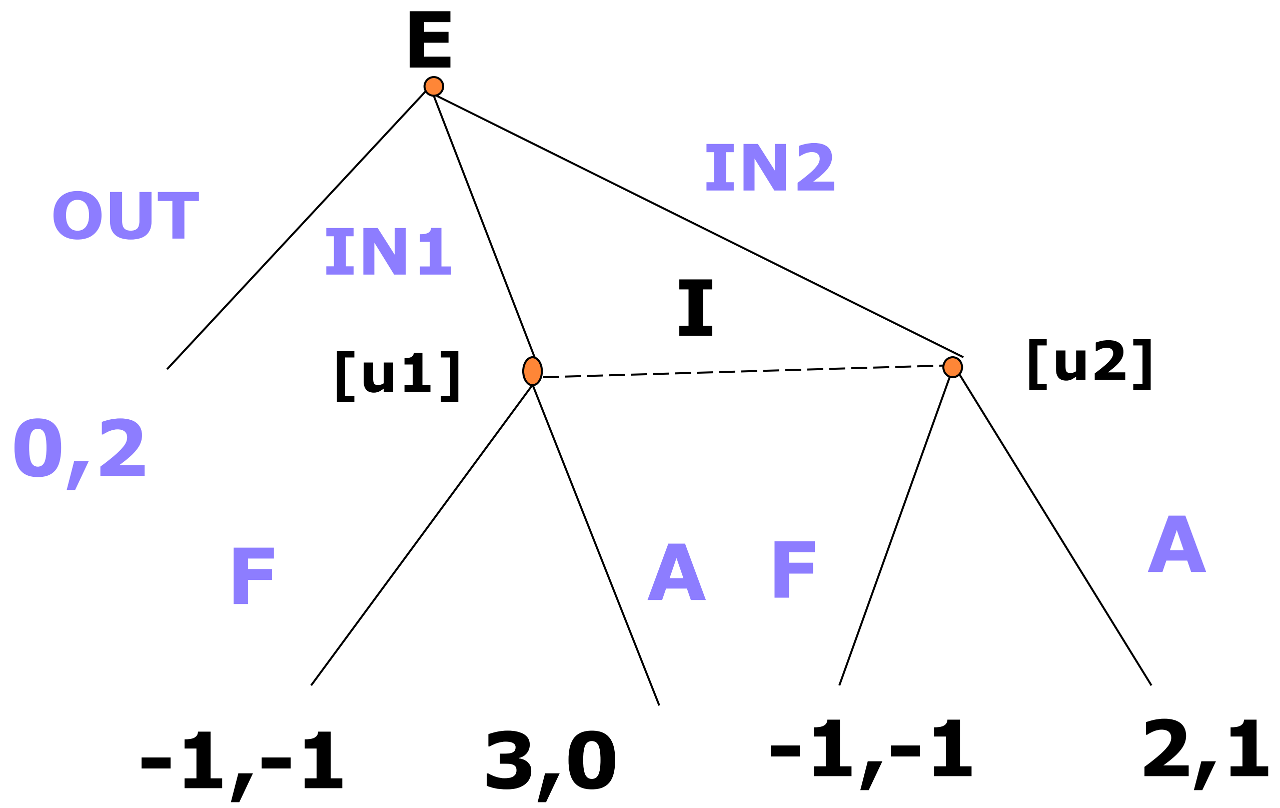
2.2 If h(x) is not reached according to strategies $\boldsymbol{\sigma}$, then we can assign any belief $\mu(x)$ between [0,1]. In other words, we place no restriction on beliefs off the equilibrium path, since such information sets are not reached with positive probability with equilibrium strategies that being played.

" An assessment ($\boldsymbol{\sigma}, \boldsymbol{\mu}$) Equilibrium strategies and beliefs is a weak sequential equilibrium if it satisfies both (S) and (WC)."

Weak Sequential Equilibrium

- We call this as a weak concept because we put no restriction at all on beliefs-off the equilibrium paths.
- Note that WSE may not be structurally consistent and not be SPNE.
- If we want to add restriction on out-of-equilibrium beliefs, we induce WSE in every subgame. That leads to a strong equilibrium concept where we need a strong consistency property.
- In our previous example, WSE is (player 1 plays R with prob=1, player 2 plays B with prob=1, and belief on the right node is =1).

Ex 2. WSE



Ex 2. WSE

- Two pure NE's: { (OUT, F if entry occurs),
  (IN1, A if entry occurs) }
- Are both NE's reasonable?
- First, we know that A is a dominant strategy for the incumbent.
- WSE requires the belief at [u1] and [u2] to support A.

Ex 2. WSE

- To support (IN1, A if entry occurs), we need to specify the value for u1 and u2.
- How about [u1]= 1 and [u2]=0.
- We know that we must apply the Bayes' rule here.
- Given these beliefs, both players choose their optimal actions, satisfying (S).
- But how about (OUT, F if entry occurs), we know that we can't support F with any belief.