



B.E. International Program

Faculty of Economics, Thammasat University



EE 320 Introductory Mathematical Economics (Semester 2/2013)

In-Class Exercise 5 – Answer

The joint cost function for producing two products is

$$C(q_1, q_2) = 6q_1^2 - q_1q_2 + 10q_2^2 + 30,$$

and the firm has a production quota of $q_1 + q_2 = 34$.

- (a) Find the values q_1 and q_2 that minimize costs. Show the sufficient conditions for [cost minimization](#).
- (b) Estimate the effect on costs if the production quota is reduced by 1 unit.

Ans.

$$(a) \mathcal{L} = 6q_1^2 - q_1q_2 + 10q_2^2 + 30 + \lambda[34 - q_1 - q_2]$$

FONC:

$$\mathcal{L}_1 = 12q_1 - q_2 - \lambda = 0$$

$$\mathcal{L}_2 = -q_1 + 20q_2 - \lambda = 0$$

$$\mathcal{L}_\lambda = 34 - q_1 - q_2 = 0$$

$$\Rightarrow q_1^* = 21 \text{ and } q_2^* = 13$$

$$\Rightarrow C(q_1^*, q_2^*) = C(21, 13) = 4093 \text{ and } \lambda = 239.$$

SOSC:

$$|\bar{H}| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 12 & -1 \\ 1 & -1 & 20 \end{vmatrix} = -34 < 0$$

(b) If the production quota is reduced by 1 unit, then the cost will decrease by: $\Delta C(q_1^*, q_2^*) \approx \lambda \cdot \Delta \bar{q} = (239)(-1) = -239$. Thus, the new minimum cost will be equal to $4093 - 239 = \$3854$.