Practice problem set 2

Matrix algebra and linear economic model

EE320 Semester 2/2015

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Question 1 (Easy) Basic operations on matrix

1.1

$$\text{Find } A^{-1}B \quad \text{when} \quad \text{A} = \begin{bmatrix} 4 & 2 & 5 \\ 3 & 1 & 8 \\ 9 & 7 & 6 \end{bmatrix} \text{B} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

1.2

$$\text{If A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \text{B} = \begin{bmatrix} -2 & 3 \\ 4 & 0 \end{bmatrix}, \text{C} = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} \text{ Find } (ABC)^T.$$

1.3

$$\text{If A} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \text{ Find } A^{-1}B.$$

1.4

$$\text{Let A} = \begin{bmatrix} 4 & 2 & 5 \\ 3 & 1 & 8 \\ 9 & 7 & 6 \end{bmatrix} \text{ Find determinant of A}$$

1.5

$$\text{If A} = \begin{bmatrix} 3 & -1 \\ -4 & 0 \\ 2 & 1 \end{bmatrix} \text{B} = \begin{bmatrix} 2 & 1 \\ -1 & -2 \\ 1 & 1 \end{bmatrix} \text{ Find } [A^T B]^{-1}.$$

Question 2 (Moderate)

Consider a simple macroeconomic model.

$C = a + bY_d; \quad 0 < b < 1$

$I = I_a + iY; \quad 0 < i < 1$

$G = G_0$

$T = T_0 + tY; \quad 0 < t < 1$

$R = R_0$

$Y_d = Y - T + R$

where R is the government transfer and G is the government purchase. All the remainings are defined as usual.

a. Determine *all* the endogenous and exogenous variables in the model.
b. State the condition that characterizes the equilibrium of this model.
c. Simplify the model into a 3-variable system of equations that only includes on Y, C and I.
d. Rewrite the system of equations in 2.3 in the form of matrix.
e. Solve for the solution of Y, C and I. Use the Cramer's rule method.
f. Compare the multipliers of G and R. Which one has a bigger impact? Why?

Question 3 (moderate)

Assume that we have three markets in the economy. Each market can be characterized by demand and supply equations as given below.

$$\text{Demand for goods A} : q_A^d = 3 - P_A + P_B \qquad \text{Supply for goods A} : q_A^s = P_A - 2$$
$$\text{Demand for goods B} : q_B^d = 3 - 2P_B + P_C \qquad \text{Supply for goods B} : q_B^s = P_B - 1$$
$$\text{Demand for goods C} : q_C^d = 6 + 2P_A - P_C \qquad \text{Supply for goods C} : q_C^s = 2P_C - 2$$

a. State the equilibrium conditions for this multi-market economy, and write the model in the form of matrix.
b. Solve for the equilibrium solution by using the *Cramer's rule*.

Question 4 (Hard)

Consider a simple global economy model. There are but two countries, namely A and B. Model equations for each country are given below.

$$\begin{aligned} \text{Country A}: C_A &= 150 + 0.4Y_A \\ M_A &= 0.6Y_A \\ I_A &= 250 \end{aligned}$$

$$\begin{aligned} \text{Country B}: C_B &= 100 + 0.5Y_B \\ M_B &= 0.4Y_B \\ I_B &= 200 \end{aligned}$$

where Y is national income, C is consumption, M is import, and I is investment.

a. Let "X" be a new variable representing export of a country. That is, when I write X_A, this is referred to the value of export of country A. Based on the information given above, can we find the export function of both country A and country B?
b. State all the endogenous and exogenous variables.
c. State the equilibrium conditions for the global economy model.
d. Simplify the model into a 2-variable system of equations where only Y_A *and* Y_B are included. Then rewrite the simplified model in the matrix form.
e. Solve for the equilibrium income using the *inverse matrix method*.
f. Under the equilibrium, how much is the *net export* in both countries?

Question 5 (Moderate)

Given the demand and supply functions: Qd = 100 - 3P and Qs = 80 + 2P.

a. Write the equilibrium condition for this market, and translate the system of equations into matrix notation.
b. Use matrix inversion to solve for the equilibrium quantity and equilibrium price.
c. Suppose that the government subsidizes the consumption of this good by giving the consumer $5 per unit of the goods consumed. Use Cramer's rule

to solve for (i) the equilibrium price paid by the consumer, (ii) the price received by the producer, and (iii) the amount of money the government needs for this subsidization.

Question 6 (moderate)

Examine for what values of the constants *a* and *b* the system of equations

$ax + y = 3$
$x + z = 2$
$y + az + bu = 6$
$y + u = 1$

has a unique solution in the unknowns *x, y, z,* and *u*. Find the unique solution (expressed in terms of *a* and *b*).

Question 7 (Easy)

Given the IS equation $0.3Y + 100r - 252 = 0$ and the LM equation $0.25Y - 200r - 176 = 0$. Use matrix inversion to solve for the equilibrium of national income and rate of interest.

Question 8 (Easy)

Consider the following macroeconomic model:

$$Y = C + I_0 + X_0 - M$$
$$C = C_0 + bY$$
$$M = M_0 + mY$$

where Y is income, C is consumption, M is import. $I_0, X_0, C_0,$ and M_0, are autonomous investment, export, consumption and import, respectively. b and m are coefficients (both are between 0 and 1)

a. Write the above system of equations in matrix form.
b. Apply the Cramer's rule to solve for the solution
c. Calculate the multiplier of autonomous export (X_0).

Question 9 (Midterm 1/2015)

Consider the following IS-LM model:

Commodity market:

$Y = C + I + G_0,$ $(G_0 > 0)$

$C = a + bY_d,$ $(0 < b < 1)$

$Y_d = Y - T,$

$T = tY,$ $(0 < t < 1)$

$I = I_0 - kr + iY,$ $(I_0 > 0, k > 0, 0 < i < 1)$

Money market:

$M_s = M_0$

$M_D = mY - hr,$ $(m > 0, h > 0)$

1.1. ***(5 points)*** Write a matrix form of the IS-LM equations with Y and r as the endogenous variables.

1.2. ***(5 points)*** State the condition for the existence of the equilibrium national income and interest rate.

1.3. ***(10 points)*** Suppose that $a = 700, G_0 = 350,$ and $I_0 = 800.$ Solve for the equilibrium level of national income and interest rate by using Cramer's rule. (No point will be given if you do not use Cramer's rule!)

1.4. ***(5 points)*** Determine the rate of change of equilibrium interest rate with respect to money supply $\left(\frac{dr^*}{dM_0}\right)$, assuming that everything else remains constant.