Chapter 2: Function and relation

2.1 Function and relation in economics

- Relation: mapping between two sets of variables.
  - One-to-one
  - Many-to-one
  - One-to-many
- Function is two special classes of relation
  - One-to-one
  - Many-to-one

Domain and Range

To properly define any mathematical functions, the function must be associated with domain/range. See example 2.A

Example 2.A : $y = \sqrt{x} - 3$ → Binary Relationship one x on the right

Domain for x $R; \geq 0$; $R^{+} \cup \{0\}$; $x \geq 0$

$D_f = [0, \infty)$

Range for y $y \geq -3$; $R_f = [-3, \infty)$

→ more than one x on the right ⇒ multivariate relationship

Generic function *and* Economic function:

Having applied math into economic function, we need to be very careful about the restricted sets of domains and ranges that makes the function *economically meaningful*. See example 2.B below!

Example 2.B: Does the domain set for x continue to be the same as before if we now are using X to represent price, and y to represent quantity? Discuss, why or why not!

math $y = \sqrt{x} - 3$; $D = x \geq 0$

$R \Rightarrow [-3, \infty)$

$x\uparrow \rightarrow y\uparrow$ → supply relationship

Economic: y: quantity

x: price

⇒ positive relationship b/w Price & quantity.

(i) ⇒ $x \geq 0$; make the function meaningful in mathematics

Price → Restrict Domain that make the f: economically meaningful

$x \geq 9 \Rightarrow$ Price ≥ 9

→ q; y ≥ 0

f^{n}: function

Eq^{m}: Equilibrium

2.2 Types of function commonly used in Economics

2.2.1) Linear function

A linear function takes the form:

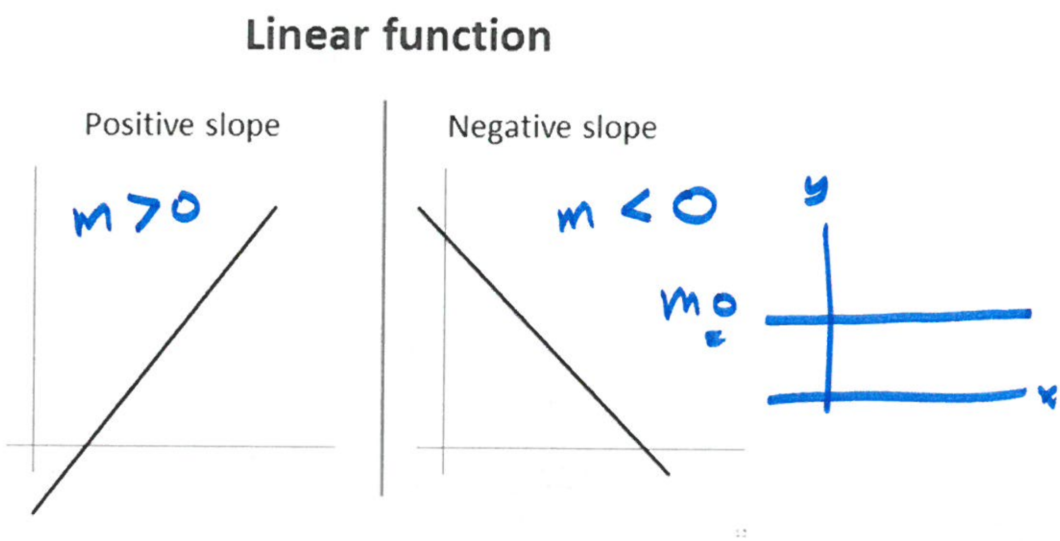
$$y = a + mx$$

$$a = intercept\ \ on\ y - axis\ (x = 0,\ y = a)$$

$$m = slope$$

What is the slope?

- A change in "y" with respect to a change in "x".
- Standard notation refers to $\frac{\Delta y}{\Delta x}$ as the slope of a function.
- Generally, slope can be varied with respect to changing values of X.
- But for the linear function, it's not. $\frac{\Delta y}{\Delta x} = m$ ➔ constant!
- Graphical illustration of a linear function depends on the value of slope, i.e. m.



Note: With the constant slope, suppose we know a line goes through two points (A and B), we can derive the equation representing that line, using the two-step procedure.

- Steps 1: Deriving slope
- Steps 2: Solve for "a" by using slope and one of the two points given.

See **example 2.C** below

Example 2.C: Suppose a research has a priori belief that the relationship between "q" and "p" is linear. From the survey, the research only knows that when p is equal to 1, q is equal to 4. Meanwhile, when p is equal to 5, q is then equal to 0. Find the equation of the line that represents the p-q relationship? What does the relationship represent?

~~q~~ ~~q~~, (P, q) ; p : Vertical axis

q: horizontal axis

$A(q^A, P^A) = A(4, 1)$

$B(q^B, P^B) = B(0, 5)$

$P = a + mq$

$\frac{\Delta q}{\Delta p} \cdot \frac{1}{\Delta P/\Delta q} = -1$

$m = \frac{\Delta P}{\Delta q} = \frac{1-5}{4-0} = \frac{-4}{4} = -1$

p-q $\Rightarrow$ $\boxed{m = -1}$

$P = a - q$

B $\Rightarrow P = a - (0) = 5$

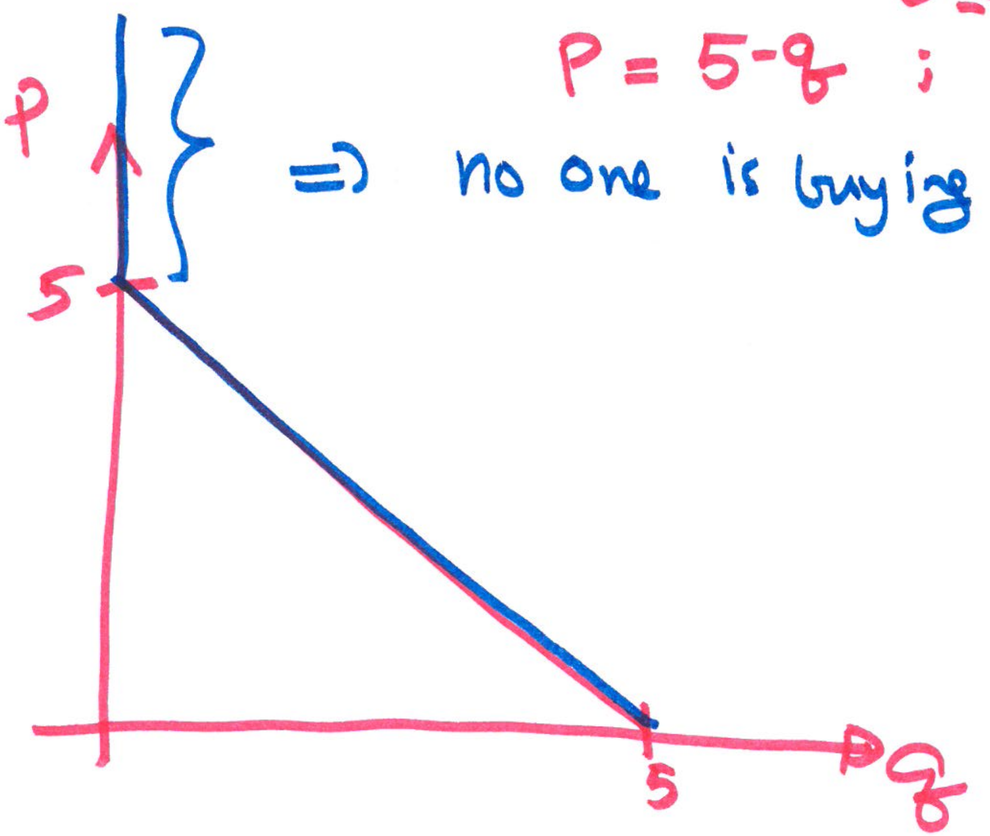
$\Rightarrow a = 5$; $P = 5 - q$

~~Demand f~~

~~De~~ f^n that represent demand curve

$P = 5 - q.$; $q = 0$; $P \geqslant 5$

$P = 5 - q$; $0 \leq P < 5$



$\Rightarrow$ no one is buying $\rightarrow q = 0$

~~multivariate~~ multivariate f^2

Example 2.D : Suppose $Q = 20 - P - Y$ as a market demand equation, where Q is quantity, P is price and Y is income.

- Find the domain set of demand.

(i) $P \geqslant 0$

(ii) $y \geqslant 0$

$Q \geqslant 0$

(iii) $20 - P - y \geqslant 0 \rightarrow 20 \geqslant P + y \; ; \; P + y \leq 20$

Restriction to the domain set P & y 'coordinate'
↳ $Q \geqslant 0$ (not written) >cor

- Plot the demand curve for Y = 5, and compare with the demand curve for Y = 10.

$Q = 20 - P - y$

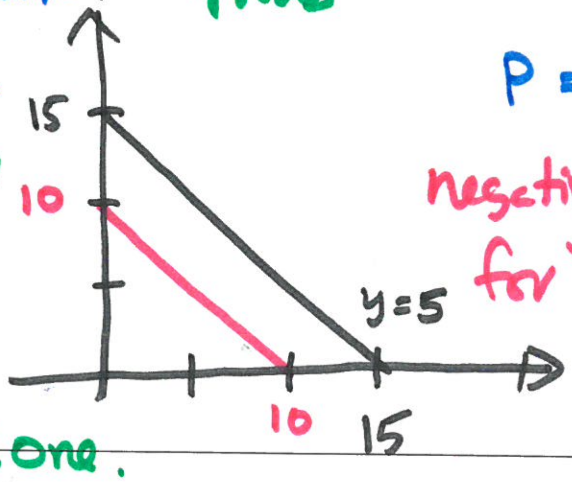
$y = 5$ → "increase" → $y = 10$

Inferior goods

$Q = 15 - P$ $\qquad$ $Q = 10 - P$

⇒ P-equal form

$P = 15 - Q$ $\qquad$ $P = 10 - Q$



"curve moving" inward from the black one to the red one.

negative coefficient for "y" = −1

$\frac{\Delta Q}{\Delta y}$; P is fixed

(1)

$$q = \begin{cases} 0 & ; \quad \underline{p \geq 0,\ y \geq 0} \ \text{; otherwise.} \\ 20 - p - y & ; \quad p \geq 0;\ y \geq 0;\ p + y \leq 20 \end{cases}$$

$$Q = 20 - P - y$$

$$\Delta Q = \Delta(20 - P - y)$$

$$= \cancel{\Delta(20)}^{\,0} - \Delta P - \Delta y$$

<u>Assume</u> that p is fixed $\Rightarrow \Delta P = 0$

$$\Delta Q = -\Delta y \Rightarrow \frac{\Delta Q}{\Delta y} = -1$$

} negatively related ⇒ negative slope

Inferior goods

Example 2.E: Suppose $Q_x = 10 - P_x + bP_y$ where

Q_x is quantity demanded for x.

P_x is price of good x.

P_y is price of good y.

What could be summarized as the relationship between good x and good y under which "b" is positive, zero, and negative?

$$Q_x = 10 - P_x + b \cdot P_y$$

$$\Delta Q_x = \Delta(10 - P_x + b P_y)$$

$$= \cancel{\Delta(10)}^{0} - \Delta P_x + b \cdot \Delta P_y$$

Suppose $\Delta P_y = 0 \Rightarrow \dfrac{\Delta Q_x}{\Delta P_x} = -1$

Suppose $\Delta P_x = 0 \Rightarrow \dfrac{\Delta Q_x}{\Delta P_y} = b$

$\boxed{b > 0}$ $\underline{P_y \uparrow}$; $q_y \downarrow$ $Q_x \uparrow$

sub product

$b < 0$ $P_y \uparrow$; $q_y \downarrow$ $Q_x \downarrow$

Complement product

$b = 0 \rightarrow$ neither substitute nor complement

Conclusion

Given $Q_x = k - aP_x + bP_y$, we can summarize that if

- b = positive ➔ an increase in P_y ➔ Q_x increases ➔ *substitute product*
    - I-phone vs Samsung
- b = negative➔ an increase in P_y ➔ Q_x decrease ➔ *complement product*
    - Coffee and tea
- b = 0 ➔ an increase in P_y ➔ Q_x stay the same ➔ *neither substitute nor complement*
    - toilet paper and Ferrari