Tuesday, September 6, 2016 10:12 AM

Unbiased $\boxed{E(\hat{\beta}) = \beta}$ → Finite Sample Property

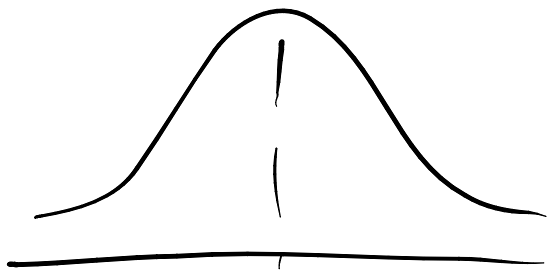
Consistent

$$\text{plim}\, \hat{\beta} = \beta$$

$$\lim_{n \to \infty} \text{Prob}(\hat{\beta} - \beta = \varepsilon) = 0$$

$\varepsilon \neq 0$, $\varepsilon \to 0$

→ Asymptotic Property $n \to \infty$



$$Y_i = \beta_1 + \beta_2 X_i + u_i \sim N(0, \sigma^2)$$

Linear combination of normal distributed variable will also be normal distributed.

$$\hat{\beta}_2 = \frac{\sum_1 x_i y_i}{\sum_1 x_i^2} = \frac{\sum_1 (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}$$

$$\hat{\beta}_2 \sim N(\beta_2, \sigma^2_{\hat{\beta}_2})$$

$$Z = \frac{\hat{\beta}_2 - \beta_2}{\sqrt{\sigma^2_{\hat{\beta}_2}}}$$

σ^2 $\sum_1 (X_i - \mu)^2$ s^2 $\sum_1 (X_i - \bar{X})^2$

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N} \qquad s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$$Y_i = \beta_1 + \beta_2 \underline{\underline{X_{2i}}} + \beta_3 X_{3i} + u_i$$

Individual Test

$H_0: \beta_2 = 0$

$H_a: \beta_2 \neq 0$

$$t = \frac{\hat{\beta}_2 - \overset{=0}{\beta_2}}{se_{\hat{\beta}_2}}$$

$H_0: \beta_3 = 0$

$H_a: \beta_3 \neq 0$

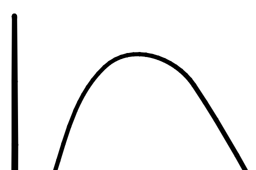
$$t = \frac{\hat{\beta}_3 - \overset{=0}{\beta_3}}{se_{\hat{\beta}_3}}$$

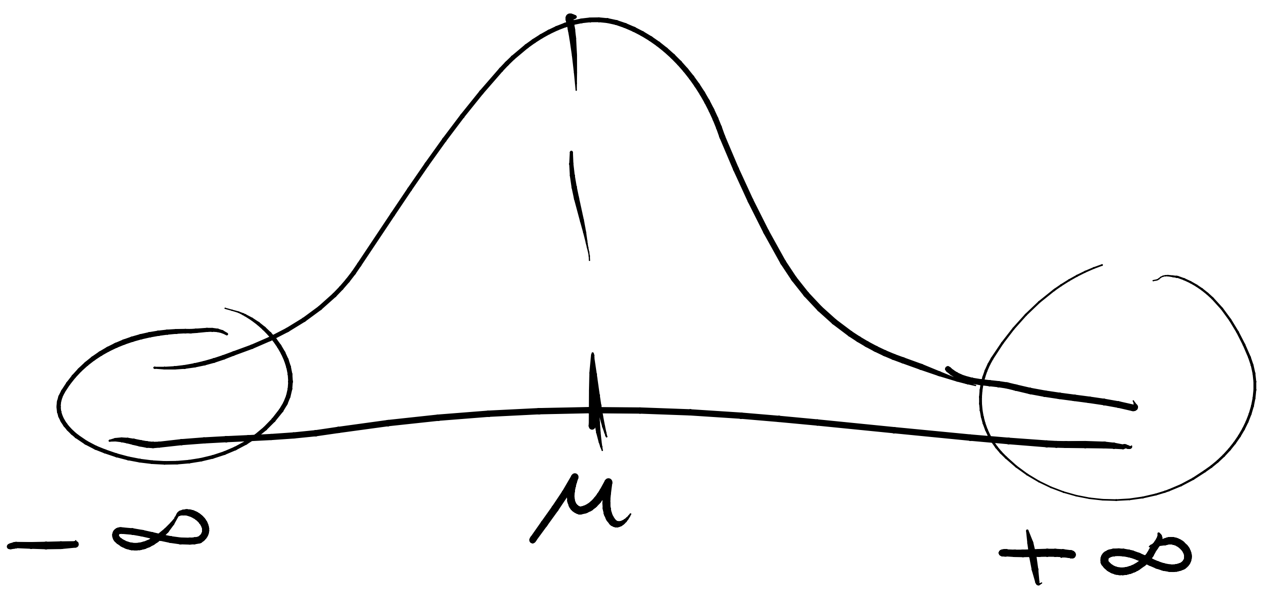
Overall Test

$H_0: \beta_2 = \beta_3 = 0$

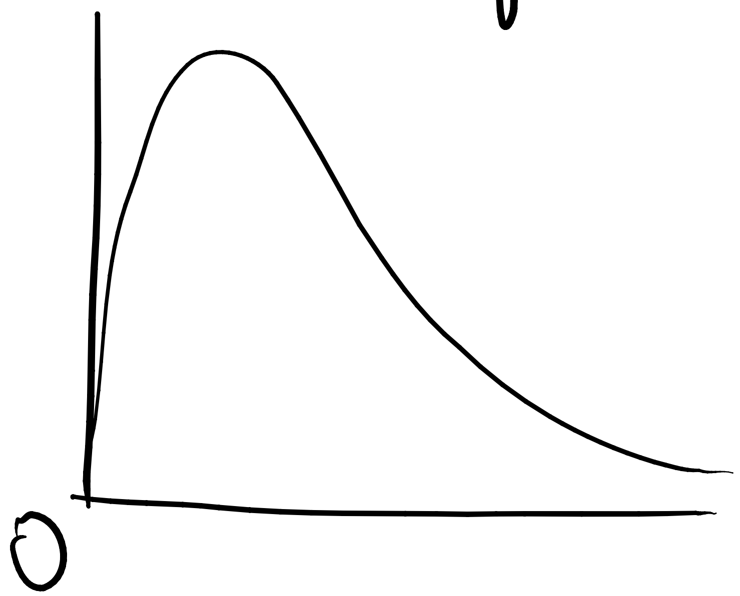
1 $\beta_2 = 0$ and 2 $\beta_3 = 0$

Chi-square.





Chi-square.



$\mu_1 = 3$

$\mu_2 = 100$

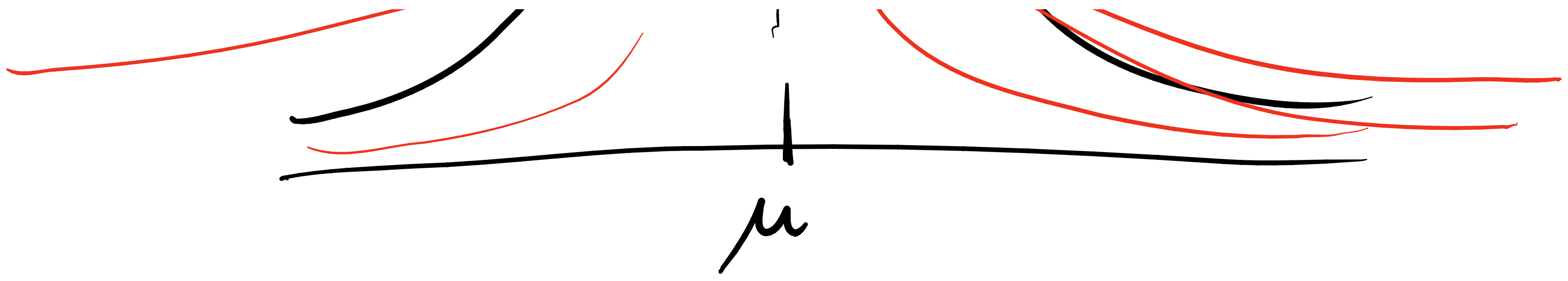
$y_i = x_i - \mu$

$X \sim N(\mu, \sigma^2)$

$Y \sim N(0, \sigma^2)$

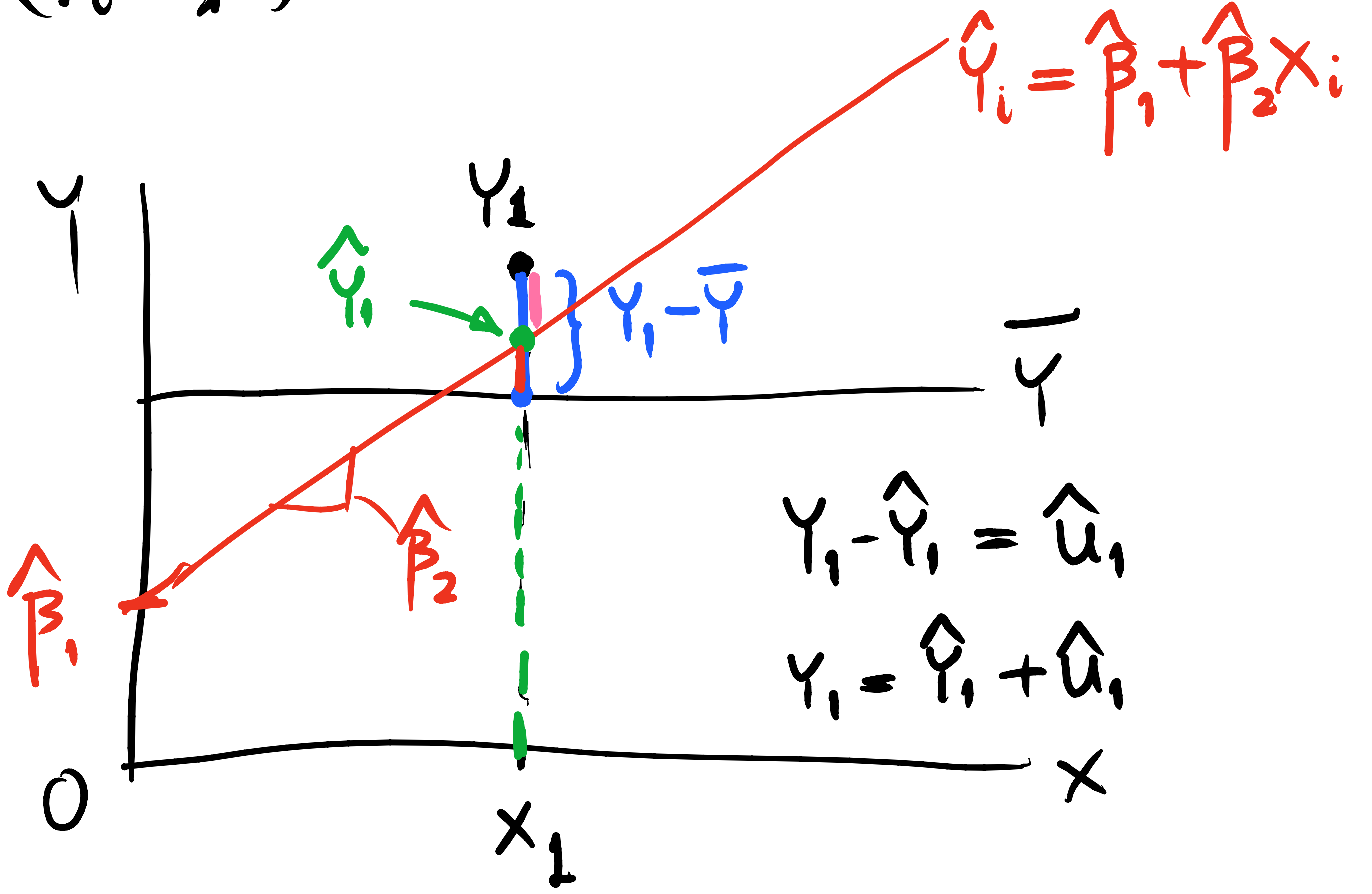
$Z_i = \left(\frac{x_i - \mu}{\sigma}\right)$

$Z \sim N(0, 1)$



$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

$$\sum_{i=1}^{n} (Y_i - \bar{Y})^2 = \text{Total Variation in } Y \qquad df = n-1$$



$$Y_1 - \hat{Y}_1 = \hat{u}_1$$

$$Y_1 = \hat{Y}_1 + \hat{u}_1$$

$$Y_i = \hat{Y}_i + \hat{u}_i$$

$$\Sigma(Y_i-\bar{Y})^2=\Sigma(\hat{Y}_i-\bar{Y})^2+\Sigma(\hat{u}_i-\cancel{\bar{u}})^2$$

$$\sum_{i=1}^{n}(Y_i-\bar{Y})^2=\Sigma(\hat{Y}_i-\bar{Y})^2+\Sigma\hat{u}_i^2$$

n $\quad 1$ $\quad k-1$ $\quad n-k$

$Y_i-\hat{Y}_i$

Total Sum of Square = Explained SS + Residual SS

TSS = ESS + RSS

$$\underset{n-1}{TSS}=\underset{k-1}{ESS}+\underset{n-k}{RSS}$$

$$F_{(k-1,n-k)}=\frac{(ESS/k-1)}{(RSS/n-k)}$$

13/9/2016

Multiple Linear Regression

$$Y_i=\beta+\beta X_{2i}+\beta X_{3i}+$$

Simple Linear Regression Model.

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

OLS $\rightarrow \hat{\beta}_1, \hat{\beta}_2$

Evaluate

1. Sign & Meaning of Coefficients
2. Overall Test - F-test
3. R^2
4. Individual Test - t-test

Multiple Linear Regression

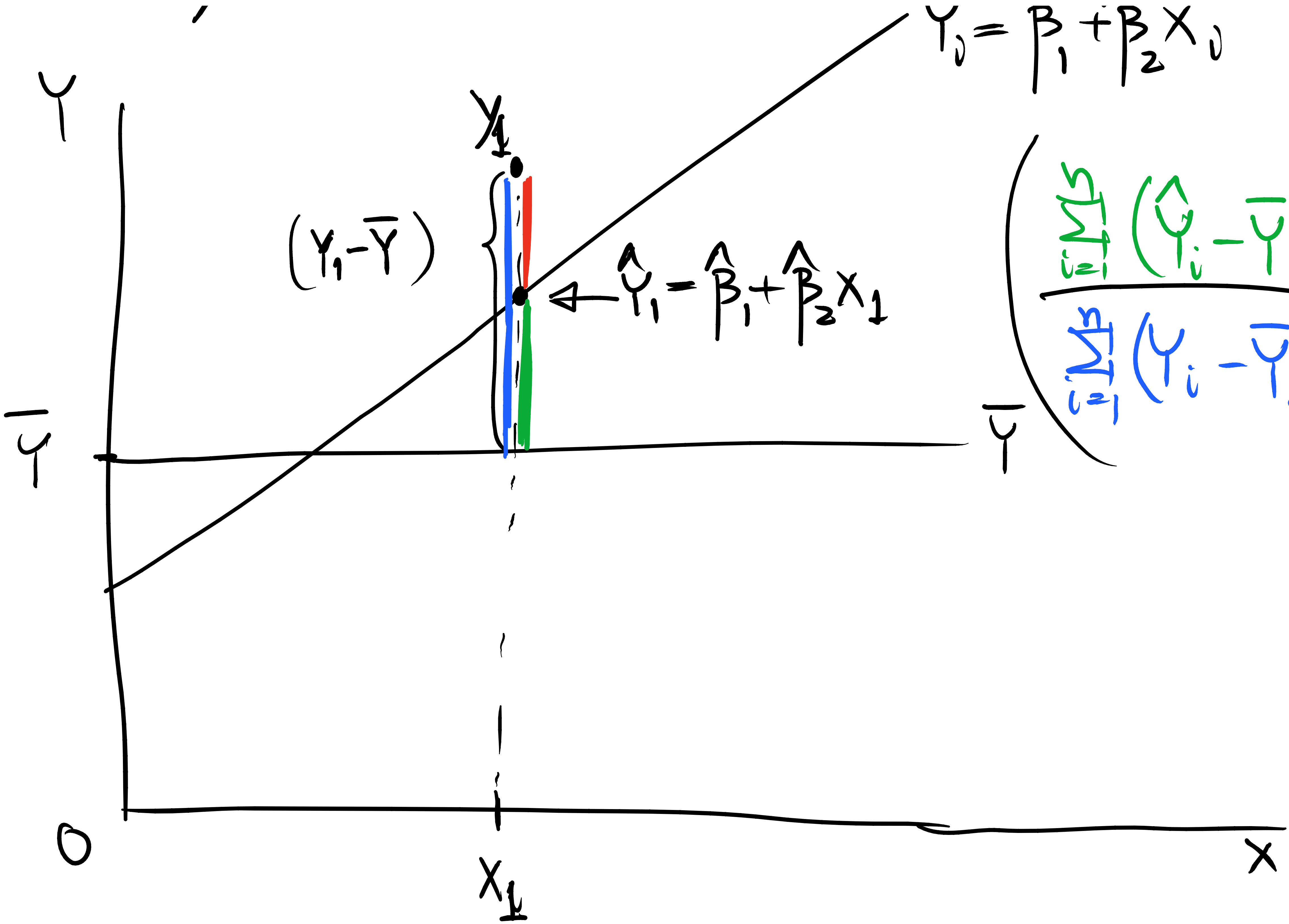
$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + u_i$$

Covariance

$$\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})$$

$\Delta x \rightarrow \Delta y$

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$



$$\left(\frac{\sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^{n} (Y_i - \bar{Y})^2} \right) = R^2$$