EE461
Lecture 3: Neoclassical Growth Models

January 28th, 2012

NEOCLASSICAL GROWTH MODEL: SOLOW MODEL

Solow Model: Background

- Growth model postulated by Robert Solow (b.1924) in 1956
  - "A Contribution to the Theory of Economic Growth". Quarterly Journal of Economics 70 (1): 65–94. 1956
- A major paradigm widely used in policy making, and a benchmark against which most recent growth theories are compared
- Focuses on the determinants of economic growth and the standard of living in the long run

Some questions to think along the way…

- Why are some countries rich while others are not?
- Why do some countries grow fast while others do not?
- How to explain sustained economic growth?

>>> After this lecture, we should be able to answer all these 3 questions **under Solow's assumptions**

The Solow Model: Supply Side

- Aggregate production of the economy generates one aggregate output and requires two input factors, capital stock and labor

$$Y^s = F(K,L)$$

- **Capital stock (K)** can vary
- **Labor (L)** can vary

The Solow Model: Supply Side

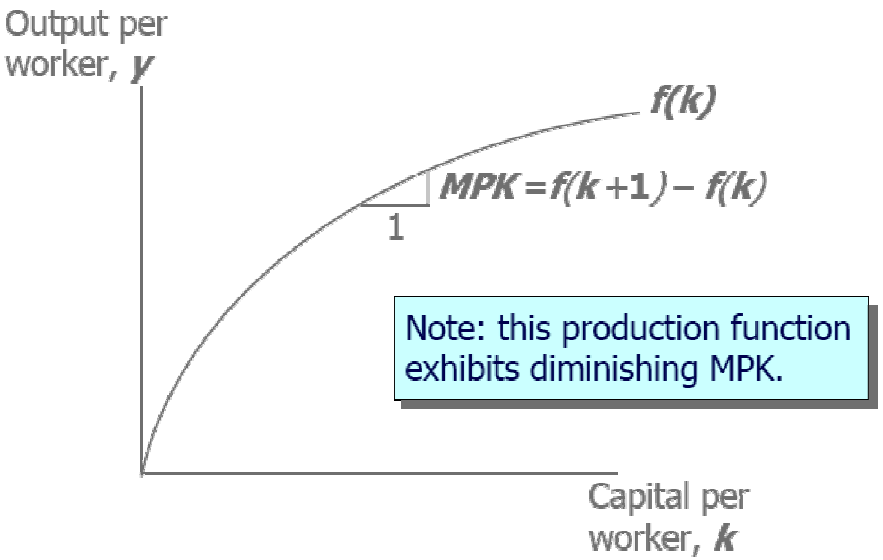
- Assume constant returns to scale (CRS) for this production function

$$zY = F(zK, zL) \text{ for any } z > 0$$

- Define: $y = Y/L$ = output per worker
  $k = K/L$ = capital per worker
- If we let $z = 1/L$:

$$Y/L = F(K/L, L/L)$$
$$y = F(k, 1)$$

Production Function



Example: Cobb-Douglas Production Function

Suppose $Y^s = K^{0.3}L^{0.7}$

1. Is this function CRS?

2. How can we transform this function into per-worker economy?

Example: Cobb-Douglas Production Function

•As long as the production function is CRS, we can virtually eliminate L in our per-worker setting

•From now on, we basically look at per-worker units of factors: this makes sense as we are interested more in individuals' living standards (e.g., per-capita income, or YOUR income) than in the size of an economy (e.g., GDP)

The Solow Model: Demand Side

- National Income Identity:

$$Y^d = C + I$$

For simplicity, assume no government for the time being

- In "per worker" terms:

$$y^d = c + i$$

  where $c = C/L$ and $i = I/L$

- s = saving rate, the fraction of income that is saved (s is an exogenous parameter)

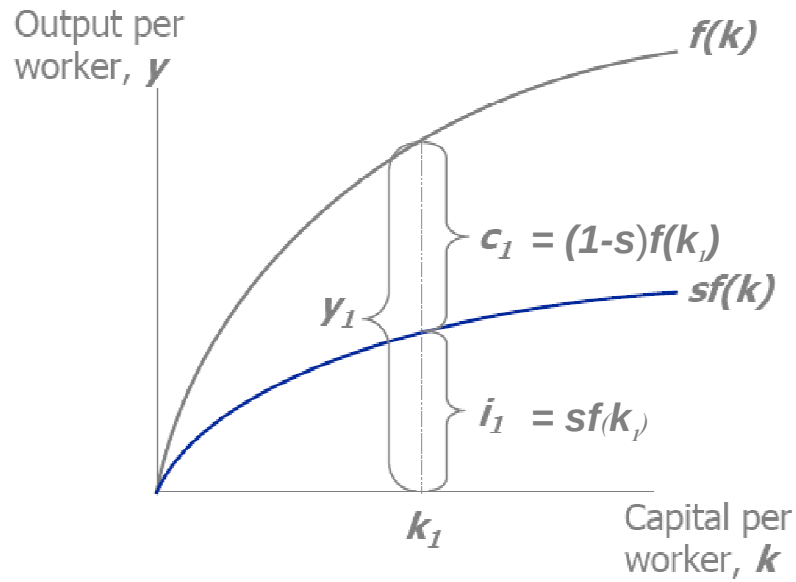
Consumption, Saving, and Investment

- Consumption function:
- Saving (per worker)

- National income identity is
  Rearrange to get:

- Using the results above, in equilibrium condition:

Consumption, Saving, and Investment



Capital Accumulation

- Depreciation is the fraction of the capital stock that wears out each period

•Basic Idea:

Change in capital stock =

$$k_t - k_{t-1} = \Delta k =$$

Capital Accumulation

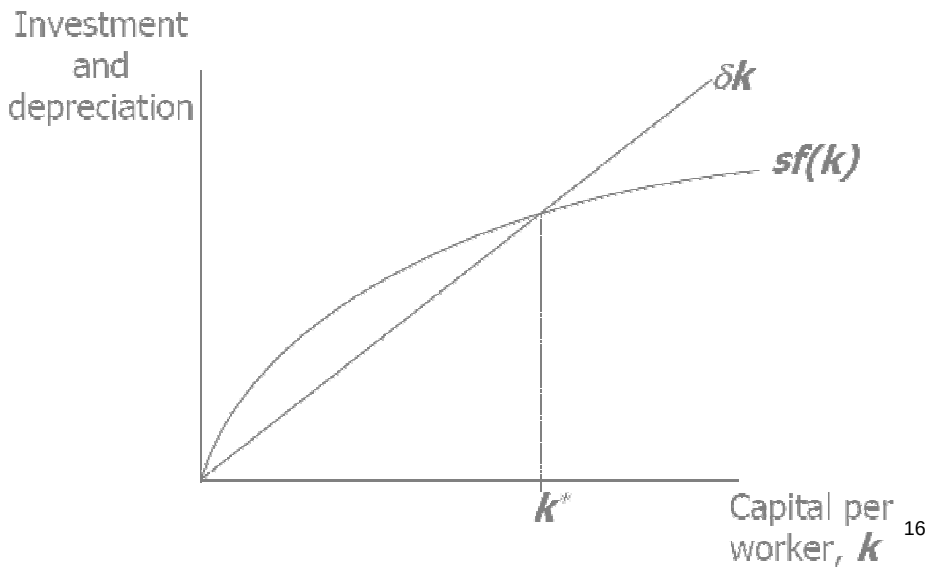
- Since $i = s \cdot f(k)$, this becomes

- The equation of motion for k determines behavior of capital over time...
- which, in turn, determines other endogenous variables, as they all depend on k

The steady state

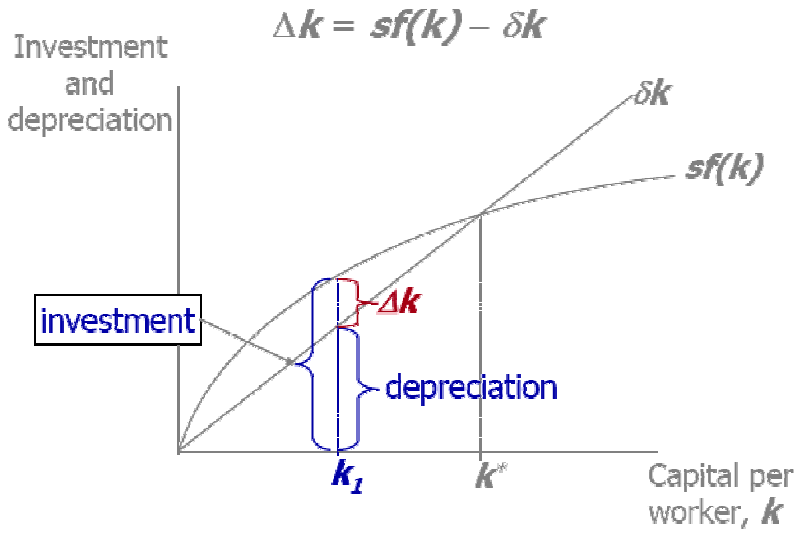
- If investment is just enough to cover depreciation, or $sf(k) = \delta k$
- then capital per worker will remain constant: $\Delta k = 0$.
- This constant value, denoted k^*, is called

The steady state

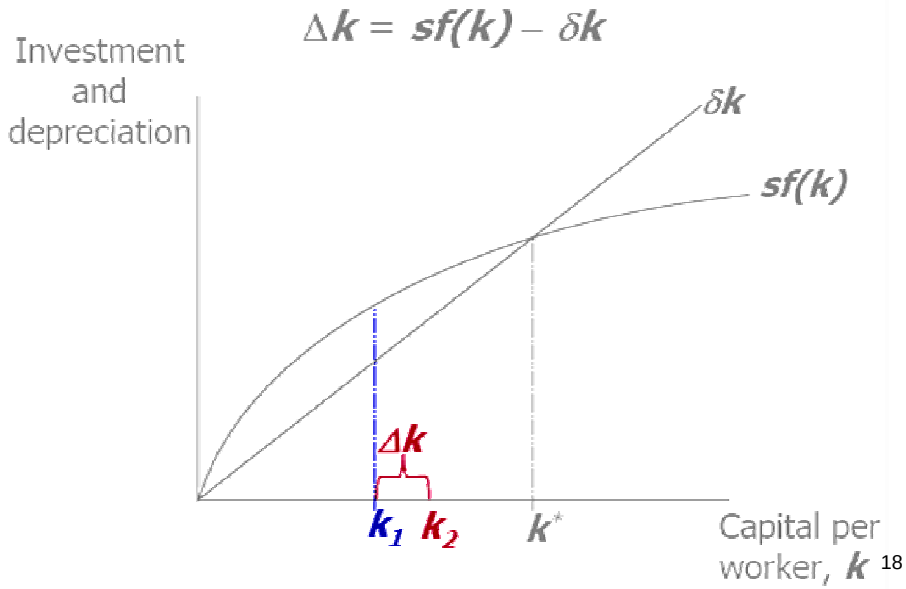


Moving toward the steady state (1)

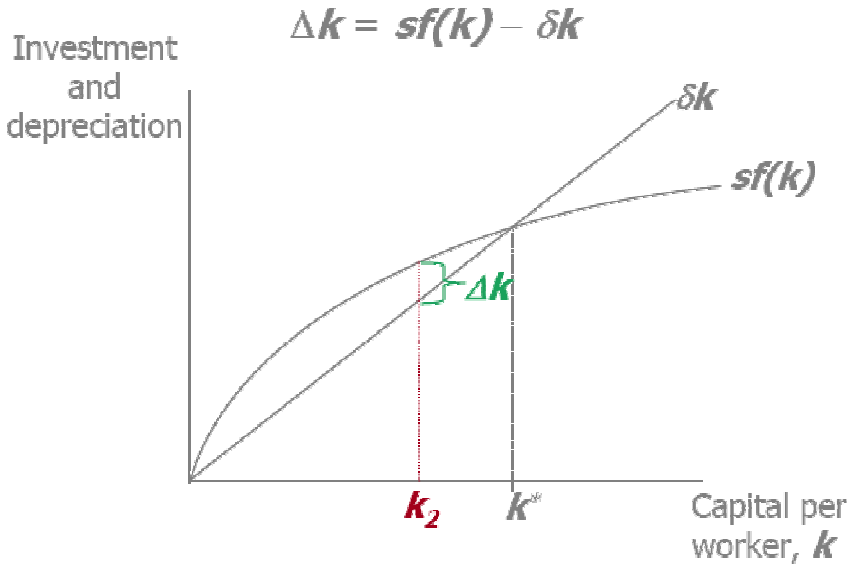
- What if k < k* ?



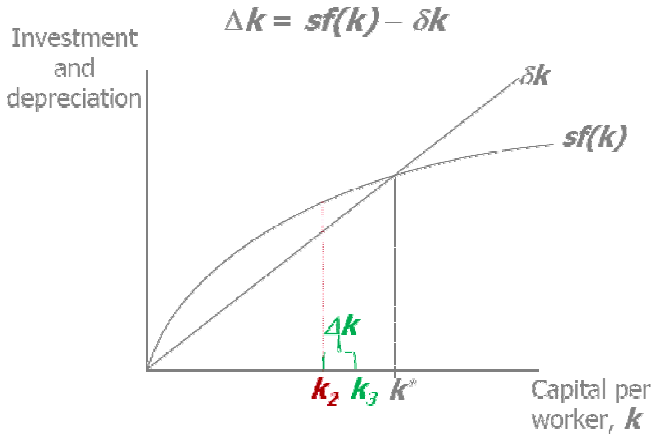
Moving toward the steady state (2)



Moving toward the steady state (3)

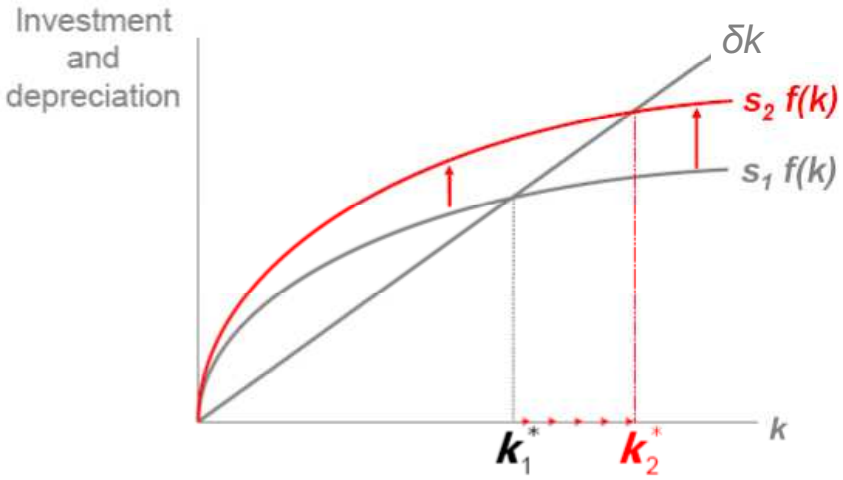


Moving toward the steady state (4)



- The whole process continues until k reaches k*

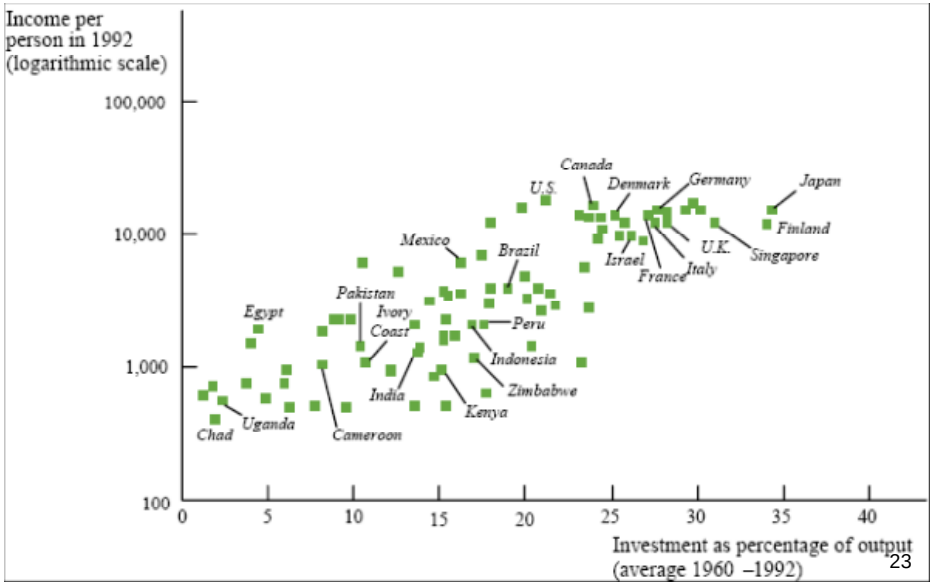
An increase in the saving rate



So what happens now?

- Higher ***s* =>** higher ***k****.
- And since ***y*** = ***f(k)*** ,
  higher ***k**** **=>** higher ***y**** .
- Thus, the Solow model predicts that...

International Evidence on Investment Rates and Income per Person



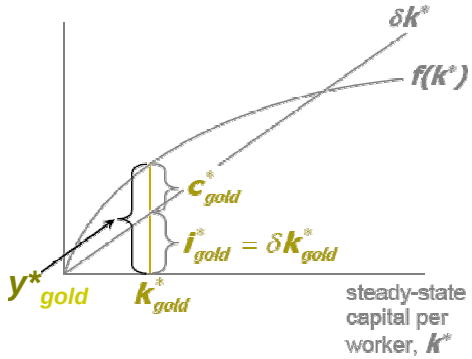
The Golden Rule

- Different values of s lead to different k*
  - Which k* and the corresponding s are the 'best' pair?
- Economic well-being depends on.....
- So the "best" steady state has the highest possible value of consumption per person: ***c**** = ***(1-s)f(k*)***
- We know that an increase in ***s***
  1. Leads to higher ***k**** and ***y****
  2. Reduces consumption's share of income ***(1-s)***

- So, how do we find the ***s*** and ***k**** that maximize ***c**** ?

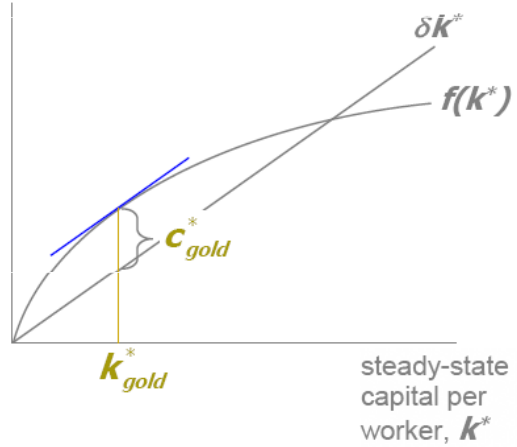
The Golden Rule

- To find it, first express ***c**** in terms of ***k****



The Golden Rule

$c^* = f(k^*) - \delta k^*$ is biggest where the slope of the production func. equals



How to get there?

- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires policymakers to adjust
- This adjustment leads to a new steady state with higher consumption.

When population grows…

- So far we have fixed the number of labor (population)
- What will happen if we let population grow?
- Assume that the population--and labor force grow at rate ***n***:
- Population growth spreads the capital stock more thinly to larger population, so it has a negative effect on capital stock accumulation

Break-even investment

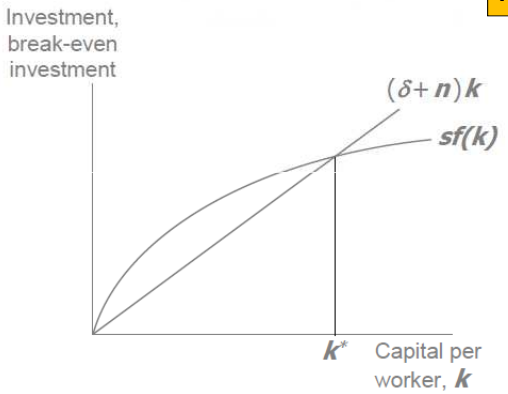
- $(\delta + n)k$ = break-even investment
  - the amount of investment necessary to keep ***k*** constant
- Break-even investment includes:
  - δk to replace capital as it wears out
  - nk to equip new workers with capital

The equation of motion has become…

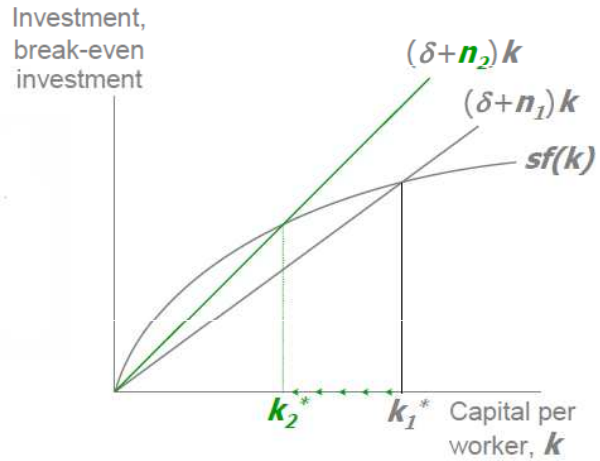
$$\Delta k = sf(k) - (\delta + n)k$$

Actual investment

Break-even investment



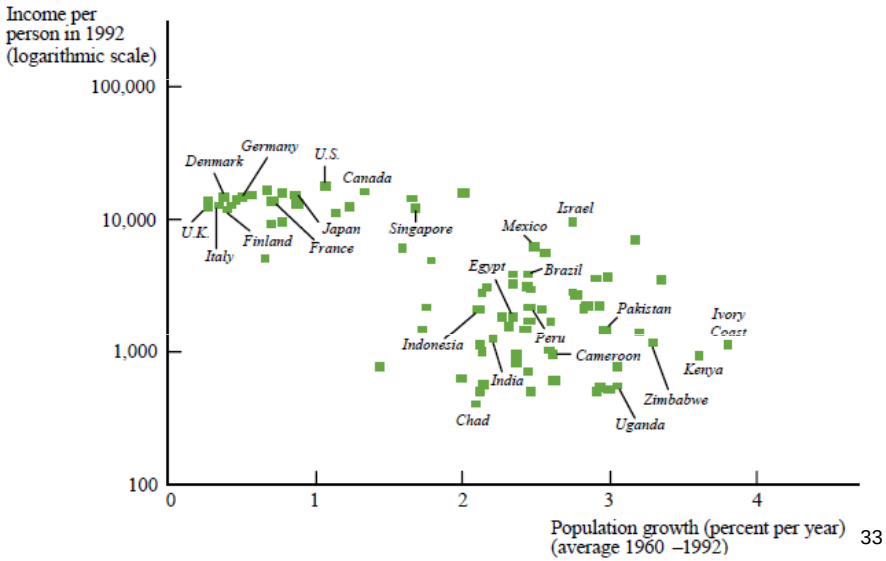
The impact of population growth



Prediction

- Higher ***n => lower k*.***
- And since ***y = f(k) ,*** lower ***k* => lower y* .***
- Thus, the Solow model predicts that

International Evidence on Population Growth and Income per Person



The Golden Rule with Population Growth

- To find the Golden Rule capital stock, we again express ***c* in terms of k*:***

$$\begin{aligned} c^* &= y^* - i^* \\ &= f(k^*) - i^* \\ &= f(k^*) - \Delta k^* - (\delta + n)k^* \\ &= f(k^*) - (\delta + n)k^* \end{aligned}$$

- ***c* is maximized when*** $MPK = \delta + n$ or equivalently, $MPK - \delta = n$
- The marginal product of capital per labor net of depreciation equals the population growth rate

Solow Model with Technological Progress

- So far Technology is held constant. Possible?
- Examples of technological progress
  - 1970: 50,000 computers in the world
    2000: 51% of U.S. households have 1 or more computers
  - The real price of computer power has fallen an average of 30% per year over the past three decades.
  - The average car built in 1996 contained more computer processing power than the first lunar landing craft in 1969.
  - Modems are 22 times faster today than two decades ago.
  - 1981: 213 computers connected to the Internet
    2000: 60 million computers connected to the Internet

Introducing a New Variable

- A new variable: ***E = labor efficiency***
- Assume:
  - Technological progress is **labor-augmenting:** it increases labor efficiency at the exogenous rate ***g:***

$$g = \frac{\Delta E}{E}$$

More Assumptions

- We now write the production function as:

$$Y = F(K, L \times E)$$

where ***L x E*** = *the number of effective workers*

- Notation:

y = ***Y/LE*** = output per effective worker

k = ***K/LE*** = capital per effective worker

On the Supply Side

- Production function per effective worker:

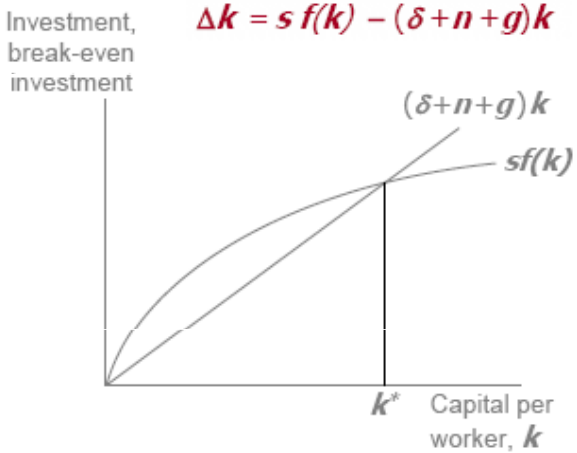
$$y = f(k)$$

- Saving and investment per effective worker:

$$s\,y = s\,f(k)$$

- Break-even investment = $(\delta + n + g)k$
  - the amount of investment necessary to keep ***k*** constant.
- Consists of:
  - δk to replace depreciating capital
  - nk to provide capital for new workers
  - gk to provide capital for the new "effective" workers created by technological progress

At the Steady State



Variable	Symbol	S-S growth rate
Capital per effective worker	k = K/(LE)	0
Output per effective worker	y = Y/(LE)	0
Output per worker	(Y/L) = yxE	g
Total output	Y = yxExL	n + g

The Golden Rule

- To find the Golden Rule capital stock, express ***c**** in terms of ***k****:

$$c^* = y^* - i^* = f(k^*) - (\delta + n + g)\,k^*$$

- ***c**** is maximized when...

Convergence in Solow Model

- Solow model predicts that, other things equal, "poor" countries (with lower ***y*** and ***k***) should grow faster than "rich" ones.

- In real world, many poor countries do NOT grow faster than rich ones. Does this mean the Solow model fails?

Convergence in Solow Model

- No, because "other things" aren't equal.
  - In samples of countries with similar savings & pop. growth rates, income gaps shrink about 2%/year.
  - In larger samples, if one controls for differences in saving, population growth, and human capital, incomes converge by about 2%/year.
- What the Solow model *really* predicts is **conditional convergence** - countries converge to their own steady states, which are determined by…
- And this prediction does have a bite in the real world.

Solow Model: Conclusion

- Saving rate determines short-run growth rates, but not long-run.
- Long Run:
  - Although population growth makes income grow, it cannot make income *per capita* grow.
  - _____makes both income and income per capita grow.
- Conditional convergence is fine, but what about absolute convergence? We want to poor to catch up to the rich, not just to each other.
- More importantly, where do technological progress come from?

NEOCLASSICAL GROWTH MODEL: ENDOGENOUS GROWTH MODEL

Endogenous vs. Solow

- Solow model:
  - Sustained growth in living standards is due to technological progress
  - The rate of tech progress is exogenous
- Endogenous growth theory:
  - A set of models in which the growth rate of productivity and living standards is endogenous

The (very) Basic Model

- Production function: $Y = AK$

  where A is the amount of output for each unit of capital (A is exogenous & constant)
- Key difference between this model & Solow:

- Investment: $I = sY$
- Depreciation: δK
- Equation of motion for total capital:

$$\Delta K = sY - \delta K$$

Defining growth rate

- $\Delta K = sY - \delta K$
- Divide through by K and use $Y = AK$, get:

- If $sA > \delta$, then income will grow forever, and investment is the "engine of growth."
- Here, the permanent growth rate depends on s. In Solow model, it does not.
- **Food for thought:** *Should permanent (long-run) growth rate of an economy depends on its saving rate?*

Diminishing Returns of Capital?

- Yes, if "capital" is narrowly defined (plant & equipment).
- Perhaps not, with a broad definition of "capital" (physical & human capital, knowledge).
- Some economists believe that knowledge exhibits increasing returns (recall O-Ring Theory anyone?)

NEOCLASSICAL GROWTH MODEL: THE 2-SECTOR MODEL

The simplified version

- Two sectors:
  - manufacturing firms produce
  - research universities produce

- ***u*** = fraction of labor in research (***u*** is exogenous)
- Manufacturing production function:

$$Y = F(K, (1-u)EL)$$

- Research production function:

$$\Delta E = g(u)E$$

- Capital accumulation:

$$\Delta K = sY - \delta K$$

In the steady state

- In the steady state, manufacturing output per worker and the standard of living grow at rate
- Key variables:
  - ***s***: affects the level of income, but
  - ***u***: affects level and growth rate of income and income per capita in long run.
- Question: Would an increase in ***u*** be unambiguously good for the economy?

3 facts about R&D in the real world

1. Much research is done by ***firms seeking profits***.
2. Firms profit from research because
   - new inventions can be patented, creating a stream of monopoly profits until the patent expires
   - there is an advantage to being the first firm on the market with a new product
3. Innovation produces externalities that reduce the cost of subsequent innovation.

Much of the new endogenous growth theory attempts to incorporate these facts into models to better understand tech progress.

Is the private sector doing enough R&D?

- The existence of positive externalities in the creation of knowledge suggests that the private sector is not doing enough R&D. *(And we are talking about the US!)*
- But, there is much duplication of R&D effort among competing firms.
- Estimates: The social return to R&D is at least 40% per year.

Thus, many believe gov't should encourage R&D

SO WHAT NOW?

What have we learnt so far?

- No unified growth models.
  - Nowadays, growth models get more complex, but many conclusions remain similar.
  - R&D is the source for growth, as suggested by Solow and subsequent works.
  - New invention destroys existing invention in the "creative destruction" process. These are Schumpeterian models which we won't discuss.
- Solow model is still being widely discussed despite its age, but it does not predict absolute convergence. (Nor does any model?)

What have we learnt so far?

- All growth models focus on income per capita, but hardly any model discusses income distribution.
  - Interesting question is about dynamics of inequality (something we don't have time to discuss in this class)
- Economic growth V.S. Poverty
  - We tend to assume that economic growth reduces poverty, which IT DOES.
  - But can we assume that people under (extreme) poverty behave exactly like others (e.g. same saving rate and such). Probably the single biggest attack on growth models.