Chapter 5

Sampling Distributions

Sampling Distribution of Sample Means

Sampling Distribution of Sample Means

A *sampling distribution of sample means* is a distribution obtained by using the means computed from random samples of a specific size taken from a population.

Properties of the distribution of sample means:

1. $\mu_{\overline{X}} = \mu$

2. $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}} \quad \Rightarrow \quad \sigma^2_{\overline{X}} = \dfrac{\sigma^2}{n}$

Sampling Distribution of Sample Means(Cont.)

Consider the following example. Given the population data 2, 4, 6, and 8. Find the population mean and population variance.

Sampling Distribution of Sample Means (Cont.)

Next, choose a sample of size 2 with replacement and the mean of each sample is found, the distribution is as shown.

Sample	Mean	Sample	Mean	Sample	Mean	Sample	Mean
2,2		2,4		2,6		2,8	
4,2		4,4		4,6		4,8	
6,2		6,4		6,6		6,8	
8,2		8,4		8,6		8,8	

Sampling Distribution of Sample Means (Cont.)

The sampling distribution or the probability distribution of a sample means is as follows.

$\bar{X}$	2	3	4	5	6	7	8
$p(\bar{X})$	1/16	2/16	3/16	4/16	3/16	2/16	1/16

The Central Limit Theorem

<u>The Central Limit Theorem</u>:

In a random sampling from an arbitrary population with mean μ and standard deviation σ, when n is large (30 or more), the distribution of the sample means, $\overline{X}$, is approximately normal with mean μ and standard deviation $\sigma/\sqrt{n}$. That is

$$z = \frac{\overline{X} - \mu_{\overline{X}}}{\sigma_{\overline{X}}} = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

The Central Limit Theorem (Cont.)

Example 1: The mean and standard deviation of the strength of a packaging material are 55 and 7 lbs., respectively. If 40 specimens of this material are tested,

(a) What is the probability that the sample mean strength will be

(i) at least 57 lbs.

(ii) between 54 lbs. and 56 lbs.?

(b) Find the interval centered at 55 lbs., where the average strength will lie with probability 0.95.

The Central Limit Theorem (Cont.)

Example 2: Find the 1st quartile, Q_1, for the sampling distribution of the sample means when sample of size 144 are randomly selected from population with mean 50 and standard deviation 6.

Sampling Distribution of Sample Proportions

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Sampling distribution of proportions is a distribution of the proportions of "success" in all possible samples that could be taken in a given situation, where samples are simple random samples of fixed size n.

Properties of the distribution of sample proportions:

$1.\ \mu_{\hat{p}} = p$

$2.\ \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{pq}{n}} \Rightarrow \sigma_{\hat{p}}^2 = \frac{pq}{n}$

The Central Limit Theorem

The Central Limit Theorem:

In a random sampling from an arbitrary population, when n is large (that is np ≥ 5 and nq ≥ 5), the distribution of the sample proportions, $\hat{p}$, is approximately normally distributed. That is

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

The Central Limit Theorem (Cont.)

Example 3: Suppose that 53% of a voting population favours Mr.A in an election. Let's assume that only 50 people turn out to vote. If we can consider voters to be a random sample of the electorat, what is the probability that Mr.A loses (get less than 50% of the vote)?

The Central Limit Theorem (Cont.)

Example 4: 60% of patients survive COVID-19. Assume a random sample of 75 is selected from a people known to have contracted this disease. What is the probability that over 70% of patients survive?

The Central Limit Theorem (Cont.)

Example 5: Refer to Example 4. Assume that 10 patients were selected. What is the probability that more than 8 patients survive?