STATIC OLIGOPOLY MODELS

- COURNOT MODEL (1838)
- BERTRAND MODEL OF PRICE COMPETITION (1883)
- STACKELBERG MODEL

COURNOT (1838)

ASSUMPTIONS

1. CONSUMERS ARE PRICE TAKERS
2. HOMOGENEOUS PRODUCTS
3. NO ENTRY

CHOICE : LEVEL OF OUTPUT / QUANTITY / SALE VOLUMES

SETUP

1. 2 IDENTICAL FIRMS W/ NO ENTRY
2. ONE-SHOT GAME
3. HOMOGENEOUS GOODS WHERE $q_1 + q_2 = Q$.
4. $P = A - BQ$: MARKET DEMAND FUNCTION
   $P = A - B(q_1 + q_2)$
   $P = A - Bq_1 - Bq_2$.
5. EACH FIRM HAS A CONSTANT AND EQUAL $MC = c$.
6. EACH FIRM CHOOSES ITS OWN OUTPUT SIMULTANEOUSLY.

COURNOT-NASH EQUILIBRIUM : FOR q_1^c AND q_2^c TO BE A NASH EQUILIBRIUM QUANTITY 2 FOLLOWING CONDITIONS MUST BE TRUE :

$$\pi_1(q_1^c, q_2^c) \geq \pi_1(q_1, q_2^c) \text{ FOR ANY } q_1 \quad (1)$$

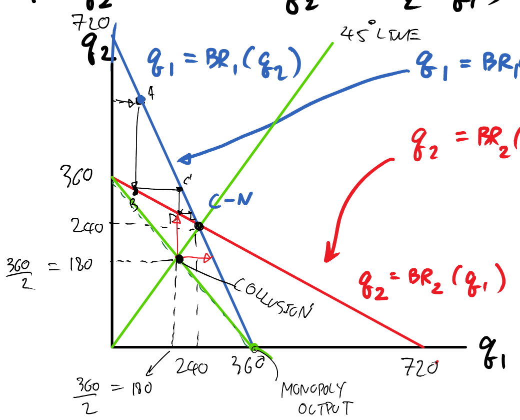
$$\pi_2(q_1^c, q_2^c) \geq \pi_2(q_1^c, q_2) \text{ FOR ANY } q_2 \quad (2)$$

C-N EQUILIBRIUM CAN BE DERIVED BY USING "BEST-RESPONSE FUNCTIONS".

SOCIAL STABLE CONVENTION.

$\boxed{Q = 1000 - 1000P}$

$$q_1^c = BR_1(q_2^c) \quad \& \quad q_2^c = BR_2(q_1^c)$$

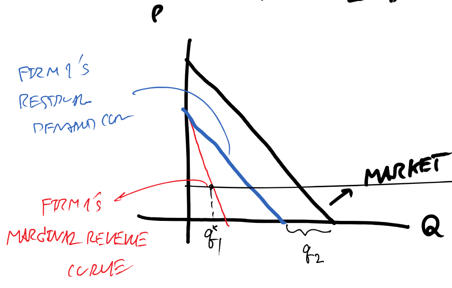


CONSIDER A MARKET DEMAND CURVE : $P(Q) = A - BQ$

WHERE $Q = q_1 + q_2$

COST OF PRODUCTION : $C = c \cdot q_i$ WHERE $i = 1, 2$

- FIRM 1'S RESIDUAL DEMAND CURVE : $P(q_1, q_2) = \underbrace{(A - Bq_2)}_{\text{INTERCEPT}} - Bq_1$



- FIRM 1'S MR CURVE : $MR(q_1, q_2) = (A - Bq_2) - 2Bq_1$.

- PROFIT MAXIMIZING OUTPUT FOR FIRM 1 : $A - Bq_2 - 2Bq_1 = c$

$$\boxed{q_1 = \frac{A - Bq_2 - c}{2B}}$$

BY SYMMETRY, $\boxed{q_2 = \frac{A - Bq_1 - c}{2B}}$

2-EQUATION & 2-UNKNOWN, WE CAN SOLVE FOR q_1^c, q_2^c

$$\boxed{q_1^c = q_2^c = \frac{A - c}{3B}}$$ C-N EQUILIBRIUM QUANTITIES

$$\boxed{Q = q_1^c + q_2^c = 2 \cdot \left(\frac{A - c}{3B}\right).}$$

$$P = A - BQ = A - \cancel{B} \cdot 2\left(\frac{A - c}{3\cancel{B}}\right)$$

$$\boxed{P^c = \frac{A + 2c}{3}}$$

$$\pi_i = P \cdot q_i - c \cdot q_i$$

$$\boxed{\pi_i^c = \frac{(A - c)^2}{9B}}$$

PROPERTIES OF C-N EQUILIBRIUM

RECALL THAT $TR = P(Q) \cdot Q$

$$\boxed{MR = \frac{\partial TR}{\partial Q} = P(Q) + Q \cdot \frac{\partial P(Q)}{\partial Q}}$$ OR $P(Q) + Q \frac{dP(Q)}{dQ}$

GIVEN THIS KNOWLEDGE, $MR(q_1, q_2) = P(q_1, q_2) + \frac{d P(q_1, q_2)}{dQ} \cdot q_1 = MC(q_1)$

LET'S DIVIDE BOTH SIDES BY $P(q_1^c, q_2^c)$:

$$\frac{P(q_i^c, q_j^c) - MC(q_i)}{P(q_i^c, q_j^c)} = -\frac{d P(q_i^c, q_j^c)}{dQ} \cdot q_i \cdot \frac{Q^c}{Q^c} \cdot \frac{1}{P(q_i, q_j)}$$

$$\frac{P - MC}{P} = -\frac{1}{\varepsilon} \cdot \frac{q_i}{Q^c}$$

$$\boxed{\frac{P - MC}{P} = -\frac{1}{\varepsilon} \cdot s_i}$$

$$\boxed{\frac{P - MC^c}{P} = \frac{1}{|\varepsilon|} \cdot s_i}$$

WHERE $|\varepsilon|$ = PRICE ELASTICITY OF DEMAND

s_i = MARKET SHARE OF FIRM i, $\dfrac{q_i^c}{Q^c}$

FACT #1 COURNOT DUOPOLISTS EXERCISE SOME DEGREE OF MARKET POWER.

FACT #2 THE HIGHER THE $|\varepsilon|$, THE LOWER THE DEGREE OF MARKUP

FACT #3 THE HIGHER THE s_i, THE HIGHER THE DEGREE OF MARKUP FOR FIRM i

FACT #4 COURNOT MARKUP < MONOPOLY MARKUP (WHY?)

[SINCE s_i IS LESS THAN ONE]

FACT #5 $\uparrow$ # OF COMPETITORS $\to$ $\downarrow$ EACH FIRM'S MARKET SHARE $\to$ $\downarrow$ MARKET POWER

IN OTHER WORDS, $\uparrow$ BARRIERS TO ENTRY $\to$ $\downarrow$ # OF COMPETITORS $\to$ $\uparrow \left(\frac{P - MC}{P}\right)$

SUPPOSE WE HAVE AN OLIGOPOLY W/ N FIRMS IN THE INDUSTRY ...

THEN, IN EQUILIBRIUM,
$$\frac{P(q_i^c, q_{-i}^c) - MC_i(q_i^c)}{P(q_i^c, q_{-i}^c)} = \frac{s_i}{|\varepsilon|}$$

LET'S MULTIPLY BOTH SIDES BY s_i AND SUM BOTH SIDES OVER ALL N FIRMS:

$$\sum_{i=1}^{N} s_i \left(\frac{P^c - MC(q_i^c)}{P^c} \right) = \frac{\sum_{i=1}^{N} s_i^2}{|\varepsilon|}$$

WHERE $\sum_{i=1}^{N} s_i^2$ = HERFINDAHL-HIRCHMAN INDEX (HHI)

P^c = COURNOT EQUILIBRIUM PRICE

MEASURES DEGREE OF MARKET CONCENTRATION WHICH

CAN VARY FROM 0 (PERFECT COMPETITION) TO 1 (MONOPOLY)

<u>CASE 1</u>

A	B	C	D	E	F
15%	15%	15%	15%	15%	15%

$\underbrace{15\%\ 15\%\ 15\%\ 15\%\ 15\%\ 15\%}_{\text{6 LARGEST FIRM}}$ + 10% CAPTURED BY 10 EQUALLY SIZED FIRMS

0.01 × 10

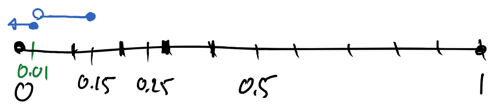
$$HHI = (0.15)^2 + (0.15)^2 + (0.15)^2 + (0.15)^2 + (0.15)^2 + (0.15)^2$$
$$+ \underbrace{(0.01)^2 + \ldots + (0.01)^2}_{\text{10 FIRMS}}$$
$$= 6(0.15)^2 + 10(0.01)^2$$
$$= 0.136$$

<u>CASE 2</u>

THE LARGEST FIRM PRODUCES 80% + THE NEXT FIVE LARGEST FIRMS PRODUCE 2% EACH + THE REST OF 10 FIRMS PRODUCE 1% EACH

$$HHI = (0.80)^2 + 5(0.02)^2 + 10(0.01)^2$$
$$= 0.643$$

<u>GUIDELINE</u>



$HHI < 0.01 \Rightarrow$ HIGHLY COMPETITIVE INDUSTRY

$0.01 < HHI < 0.15 \Rightarrow$ UNCONCENTRATED INDUSTRY

$0.15 < HHI < 0.25 \Rightarrow$ MODERATE CONCENTRATION.

$HHI > 0.25 \Rightarrow$ HIGHLY CONCENTRATED INDUSTRY

$\boxed{\text{NOTE}}$ IF WE HAVE N FIRMS w/ EQUAL MARKET SHARE...

EX $N = 4$, $s_i = 0.25$

$$HHI = (0.25)^2 + (0.25)^2 + (0.25)^2 + (0.25)^2$$
$$= 4(0.25)^2$$
$$= 0.25$$

SO IN THIS CASE $\boxed{\dfrac{1}{HHI} = \text{NUMBER OF FIRMS IN THE INDUSTRY}}$

(i.e., $\frac{1}{0.25} = 4$)

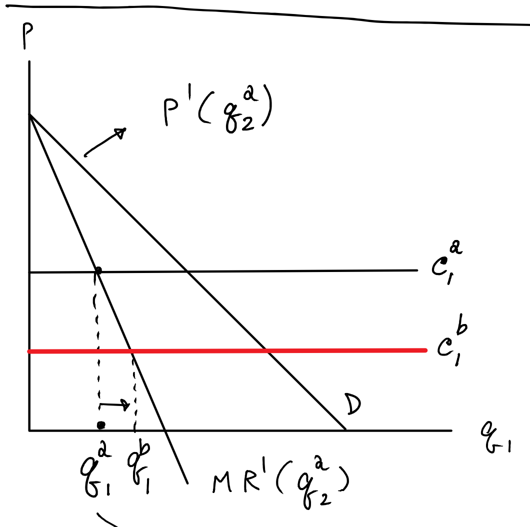
ONLY USE WHEN ALL HAVE SAME MKT SHARE.

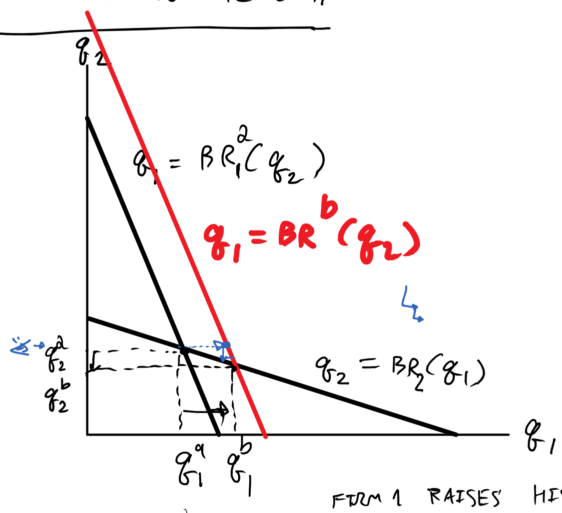
COMPARATIVE STATIC ANALYSIS

Q: HOW DO THE CHANGES IN THE EXOGENOUS PARAMETERS OF THE MODEL AFFECT THE COURNOT EQUILIBRIUM?

Ex: WHAT HAPPENS IF MC_i CHANGES?
WHAT HAPPENS IF MR_i CHANGES?
WHAT HAPPENS IF # OF FIRMS CHANGES?

EFFECT OF A DECREASE IN FIRM i's MARGINAL COST





FIRM 1 RAISES HIS OUTPUT
FIRM 2 LOWERS HIS OUTPUT

$$\boxed{\frac{P^c - MC^i}{P^c} = \frac{1}{|\varepsilon| \cdot N}}$$

WHERE $\delta_i = \frac{1}{N}$

IF ALL FIRMS HAVE THE SAME MARKET SHARE

① WHEN $N \uparrow$, $\frac{P^c - MC^i}{P^c} \downarrow$

② IN LIMIT, IF $N \to \infty$, $P \to MC^i$