ANSWER KEY HOMEWORK 1
CHAPTER 6: RISK AVERSION AND CAPITAL ALLOCATION TO RISKY ASSETS

PROBLEM SETS

2. (b) A higher borrowing rate is a consequence of the risk of the borrowers' default. In perfect markets with no additional cost of default, this increment would equal the value of the borrower's option to default, and the Sharpe measure, with appropriate treatment of the default option, would be the same. However, in reality there are costs to default so that this part of the increment lowers the Sharpe ratio. Also, notice that answer (c) is not correct because doubling the expected return with a fixed risk-free rate will more than double the risk premium and the Sharpe ratio.

4. a. The expected cash flow is: (0.5 × $70,000) + (0.5 × 200,000) = $135,000

With a risk premium of 8% over the risk-free rate of 6%, the required rate of return is 14%. Therefore, the present value of the portfolio is:

$135,000/1.14 = $118,421

b. If the portfolio is purchased for $118,421, and provides an expected cash inflow of $135,000, then the expected rate of return [E(r)] is as follows:

$118,421 × [1 + E(r)] = $135,000

Therefore, E(r) = 14%. The portfolio price is set to equate the expected rate of return with the required rate of return

c. If the risk premium over T-bills is now 12%, then the required return is:

6% + 12% = 18%

The present value of the portfolio is now:

$135,000/1.18 = $114,407

d.For a given expected cash flow, portfolios that command greater risk premia must sell at lower prices. The extra discount from expected value is a penalty for risk.

6. Points on the curve are derived by solving for E(r) in the following equation:

$$U = 0.05 = E(r) - 0.5A\sigma^2 = E(r) - 1.5\sigma^2$$

The values of E(r), given the values of σ^2, are therefore:

σ	σ^2	E(r)
0.00	0.0000	0.05000
0.05	0.0025	0.05375
0.10	0.0100	0.06500
0.15	0.0225	0.08375
0.20	0.0400	0.11000
0.25	0.0625	0.14375

The bold line in the graph on the next page (labeled Q6, for Question 6) depicts the indifference curve.

10. The portfolio expected return and variance are computed as follows:

(1) W_{Bills}	(2) r_{Bills}	(3) W_{Index}	(4) r_{Index}	$r_{Portfolio}$ (1)×(2)+(3)×(4)	$\sigma_{Portfolio}$ (3) × 20%	$\sigma^2_{Portfolio}$
0.0	5%	1.0	13.0%	13.0% = 0.130	20% = 0.20	0.0400
0.2	5%	0.8	13.0%	11.4% = 0.114	16% = 0.16	0.0256
0.4	5%	0.6	13.0%	9.8% = 0.098	12% = 0.12	0.0144
0.6	5%	0.4	13.0%	8.2% = 0.082	8% = 0.08	0.0064
0.8	5%	0.2	13.0%	6.6% = 0.066	4% = 0.04	0.0016
1.0	5%	0.0	13.0%	5.0% = 0.050	0% = 0.00	0.0000

11. Computing utility from $U = E(r) - 0.5 \times A\sigma^2 = E(r) - \sigma^2$, we arrive at the values in the column labeled U(A = 2) in the following table:

W_{Bills}	W_{Index}	$r_{Portfolio}$	$\sigma_{Portfolio}$	$\sigma^2_{Portfolio}$	U(A = 2)	U(A = 3)
0.0	1.0	0.130	0.20	0.0400	0.0900	.0700
0.2	0.8	0.114	0.16	0.0256	0.0884	.0756
0.4	0.6	0.098	0.12	0.0144	0.0836	.0764
0.6	0.4	0.082	0.08	0.0064	0.0756	.0724
0.8	0.2	0.066	0.04	0.0016	0.0644	.0636
1.0	0.0	0.050	0.00	0.0000	0.0500	.0500

The column labeled U(A = 2) implies that investors with A = 2 prefer a portfolio that is invested 100% in the market index to any of the other portfolios in the table.

12. The column labeled U(A = 3) in the table above is computed from:

$$U = E(r) - 0.5A\sigma^2 = E(r) - 1.5\sigma^2$$

The more risk averse investors prefer the portfolio that is invested 40% in the market, rather than the 100% market weight preferred by investors with A = 2.

13. Expected return = (0.7 × 18%) + (0.3 × 8%) = 15%

Standard deviation = 0.7 × 28% = 19.6%

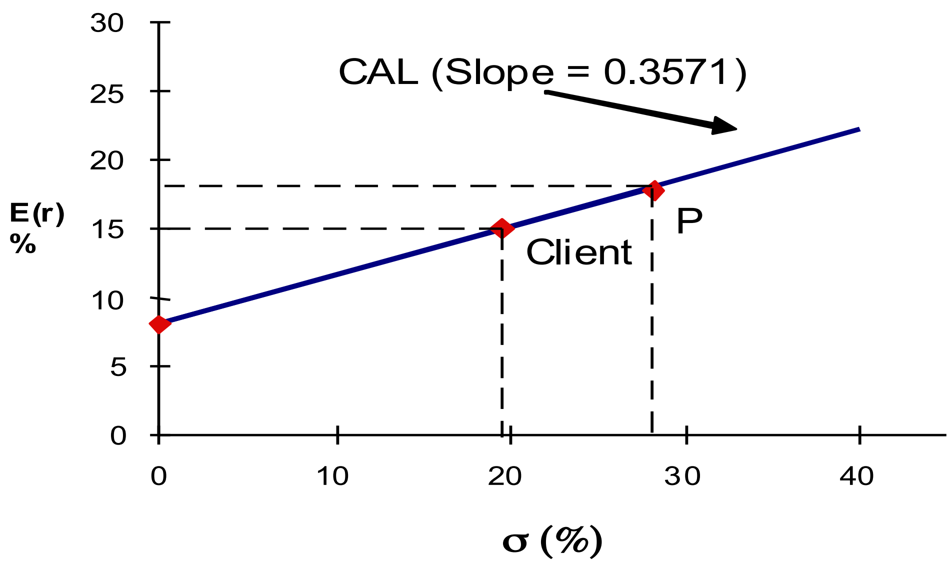
14. Investment proportions:

	30.0% in T-bills
0.7 × 25% =	17.5% in Stock A
0.7 × 32% =	22.4% in Stock B
0.7 × 43% =	30.1% in Stock C

15. Your reward-to-volatility ratio: $S = \dfrac{.18 - .08}{.28} = 0.3571$

Client's reward-to-volatility ratio: $S = \dfrac{.15 - .08}{.196} = 0.3571$

16.



17. a. $E(r_C) = r_f + y \times [E(r_P) - r_f] = 8 + y \times (18 - 8)$

If the expected return for the portfolio is 16%, then:

$$16\% = 8\% + 10\% \times y \Rightarrow y = \frac{.16 - .08}{.10} = 0.8$$

Therefore, in order to have a portfolio with expected rate of return equal to 16%, the client must invest 80% of total funds in the risky portfolio and 20% in T-bills b.

Client's investment proportions:	20.0% in T-bills
$0.8 \times 25\% =$	20.0% in Stock A
$0.8 \times 32\% =$	25.6% in Stock B
$0.8 \times 43\% =$	34.4% in Stock C

c. $\sigma_C = 0.8 \times \sigma_P = 0.8 \times 28\% = 22.4\%$

18. a. $\sigma_C = y \times 28\%$

If your client prefers a standard deviation of at most 18%, then:

$y = 18/28 = 0.6429 = 64.29\%$ invested in the risky portfolio

b. $E(r_C) = .08 + .1 \times y = .08 + (0.6429 \times .1) = 14.429\%$

19. a. $y^* = \dfrac{E(r_P) - r_f}{A\sigma_P^2} = \dfrac{0.18 - 0.08}{3.5 \times 0.28^2} = \dfrac{0.10}{0.2744} = 0.3644$

Therefore, the client's optimal proportions are: 36.44% invested in the risky portfolio and 63.56% invested in T-bills.

b. $E(r_C) = 8 + 10 \times y^* = 8 + (0.3644 \times 10) = 11.644\%$

$\sigma_C = 0.3644 \times 28 = 10.203\%$

21. a. $E(r_C) = 8\% = 5\% + y \times (11\% - 5\%) \Rightarrow y = \dfrac{.08 - .05}{.11 - .05} = 0.5$

b. $\sigma_C = y \times \sigma_P = 0.50 \times 15\% = 7.5\%$

c. The first client is more risk averse, allowing a smaller standard deviation.