How can we measure the sensitivity of a demand to the change in Price?

Ex. Price of Pepsi 10฿, you buy 5/week.
20฿ —— 3/week.

Price of Toyota 1 m฿ you buy 1. car.
1,000,010.฿ —— 1 car.

∴ Can we say. D. for Toyota is not as sensitive to change in Price compared to D for Pepsi.

No! — We have to consider %Δ in P
and the resulting %Δ in Q_D

Price Elasticity of Demand.

$$\eta_D = \frac{\%\Delta Q_D}{\%\Delta P} = \frac{\text{Percentage Change in } Q_D}{\text{Percentage change in Price}}$$

Example If the price increases by 10% $(\%\Delta P = 10\%)$
the quantity demanded decreases by 25%
$(\%\Delta Q_D = -25\%)$

$$\therefore \eta_D = \frac{-25\%}{10\%} = -2.5$$

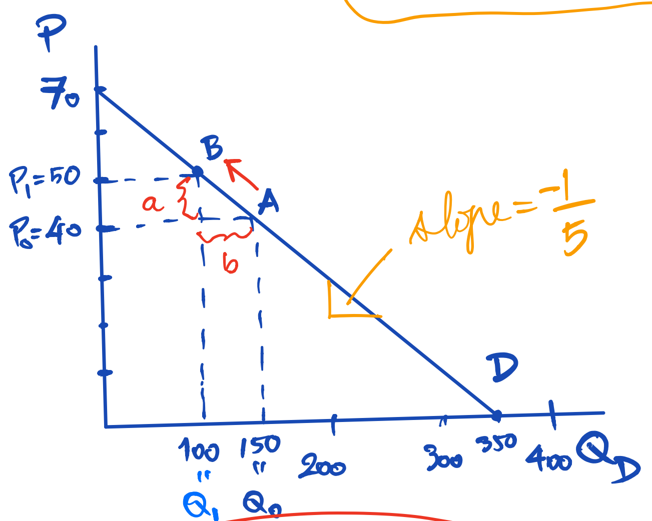
- negative sign reflects the Law of Demand.
- η_D has no unit.

(decreases)

- $\eta_D = -2.5$ means if the price increases by 1% the quantity demanded decreases by 2.5% (increases) -

η_D = Percentage change in Q_D per 1 percentage change in Price.

Example. $D: \; P = 70 - \frac{1}{5} Q_D$



At A. $P_0 = 40, Q_0 = 150$

B. $P_1 = 50, Q_1 = 100$

From A to B.

$\Delta P = 50 - 40 = P_1 - P_0$

$\Delta Q = 100 - 150 = Q_1 - Q_0$

$$\% \Delta P = \frac{10}{40} \times 100 = \boxed{\frac{P_1 - P_0}{P_0} \times 100} = 25\%$$

$$\% \Delta Q_D = \frac{-50}{150} \times 100 = \boxed{\frac{Q_1 - Q_0}{Q_0} \times 100} = -33.33\%$$

$\therefore$ From A to B, $\eta_D = \dfrac{\% \Delta Q_D}{\% \Delta P} = \dfrac{-33.33\%}{25\%} = -1.33$

Check! From B to A, $\eta_D = -1.33$?

$$\eta_D = \frac{\% \Delta Q_D}{\% \Delta P} = \frac{(Q_1 - Q_0)/Q_0 \cancel{\times 100}}{(P_1 - P_0)/P_0 \cancel{\times 100}}$$

$$= \frac{(Q_1 - Q_0)}{(P_1 - P_0)} \cdot \frac{P_0}{Q_0} = \frac{b}{a} \cdot \frac{P_0}{Q_0}$$

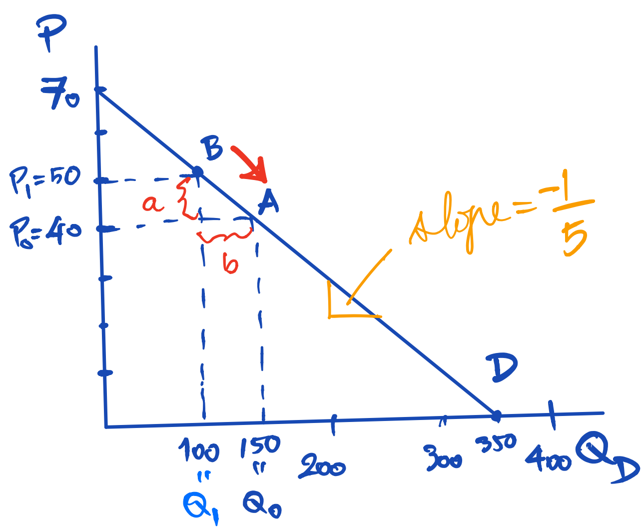
$$= \frac{1}{\text{slope}} \cdot \frac{P_0}{Q_0} = \frac{1}{-1/5} \cdot \frac{40}{150} = -\frac{4}{3}$$

slope of graph with Q_D on X-axis.

η_D from A to B is given by

$$\eta_D = \frac{1}{\text{slope}} \cdot \frac{P_0}{Q_0}\,,$$ where P_0, Q_0 are the price & Q_D at A.

From B to A.



$\Delta P = -10 = 40 - 50 = P_0 - P_1$

$\% \Delta P = \frac{-10}{50} = -20\% = \frac{P_0 - P_1}{P_1} \times 100$

$\Delta Q = 50 = 150 - 100 = Q_0 - Q_1$

$\% \Delta Q = \frac{50}{100} = 50\% = \frac{Q_0 - Q_1}{Q_1} \times 100$

$\eta_D = \frac{50\%}{-20\%} = -2.5$

$$= \frac{(Q_0 - Q_1)/Q_1}{(P_0 - P_1)/P_1}$$

from B to A $\longrightarrow$ $= \frac{1}{\text{slope}} \cdot \frac{P_1}{Q_1}$ at B.

From $A \to B$, $\eta_D = \frac{1}{\text{slope}} \frac{P_0}{Q_0}$

$B \to A$, $\eta_D = \frac{1}{\text{Slope}} \frac{P_1}{Q_1}$

η_D by Arc Approach.

If we want to find η_D between A & B (without specifying from where to where) we use

average price $\frac{P_0+P_1}{2}$

average quantity $\frac{Q_0+Q_1}{2}$

$$\eta_D = \frac{1}{slope} \frac{(P_1+P_2)/2}{(Q_1+Q_2)/2} = \frac{1}{slope} \frac{(P_1+P_2)}{(Q_1+Q_2)}$$

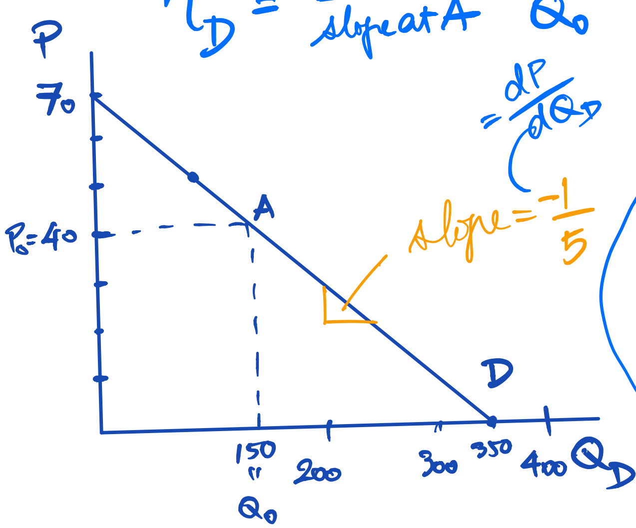
Arc Elasticity η_D by Midpoint method.

Point Elasticity of Demand.

at a single point A

$A = (Q_0, P_0)$

$$\eta_D = \frac{1}{slope\ at\ A} \cdot \frac{P_0}{Q_0}$$



$$= \frac{1}{-1/5} \frac{P_0}{Q_0} = -5\left(\frac{40}{150}\right) = -1.33$$

$$\boxed{\begin{array}{l} D: P = 70 - \frac{1}{5} Q_D \\ D: \ Q_D = 350 - 5P \end{array}}$$

$$= \frac{1}{dP/dQ_D} \cdot \frac{P_0}{Q_0}$$

$$= \frac{dQ_D}{dP} \cdot \frac{P_0}{Q_0}$$

$dP \quad \infty$

when $|\eta_D| > 1 \Rightarrow$ Demand is Elastic

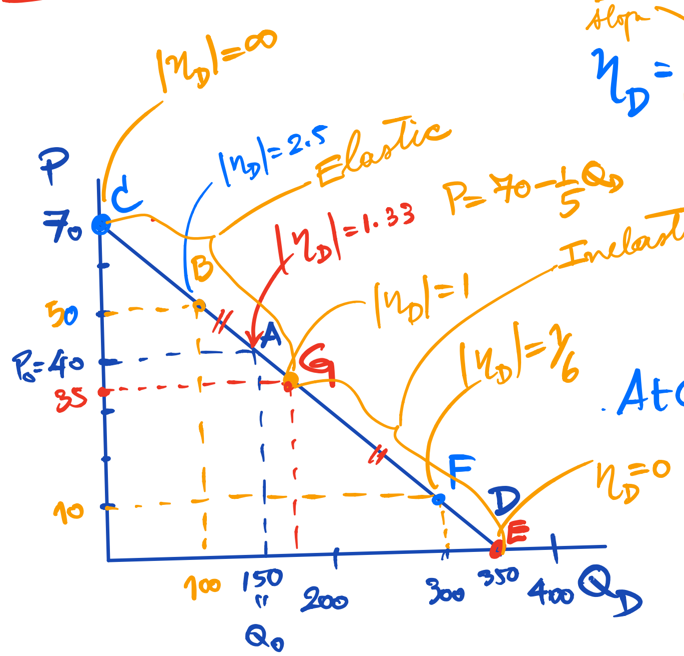
$|\eta_D| < 1 \Rightarrow$ Demand in Inelastic.

$|\eta_D|$ depends on the availability of substitutes.

- Demand for Pork ← more elastic
  Demand for meat.
- Demand in Short Run & Long Run.

$$\eta_D = \underbrace{\frac{dQ_D}{dP}}_{\frac{1}{\text{slope}}} \cdot \frac{P}{Q}$$

Price & Q at the point where we calculate the point Elasticity



at A, $|\eta_D| = 1.33$

B, $|\eta_D| = (5)\frac{50}{100} = 2.5$

At C, $P = 70$, $Q_D = 0$, so $\eta_D = (-5) \cdot \frac{70}{0}$

$= -\infty$

(perfectly Elastic)

At E, $P = 0$, $Q_D = 350$

$\eta_D = (-5)\left(\frac{0}{350}\right) = 0$

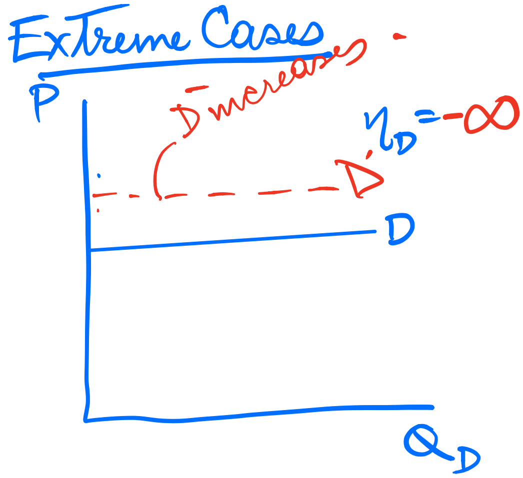
(perfectly inelastic)

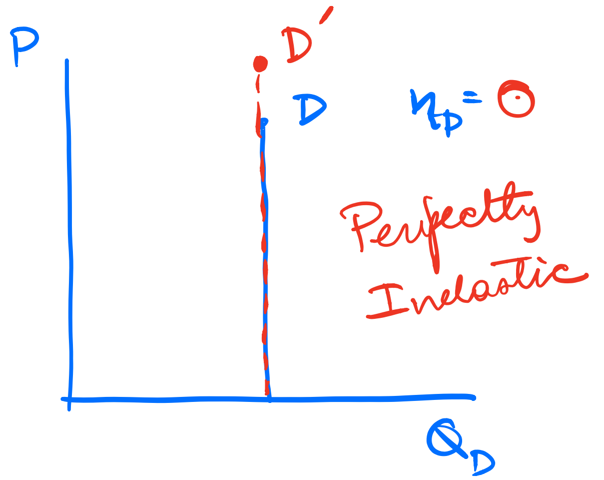
At F, $P = 10$, $Q_D = 300$, $\eta_D = (-5)\frac{10}{300} = -\frac{1}{6}$

At G, $P = 35$, $Q_D = 175$, $\eta_D = (-5) \cdot \frac{35}{175} = -1$

Unit Elastic.

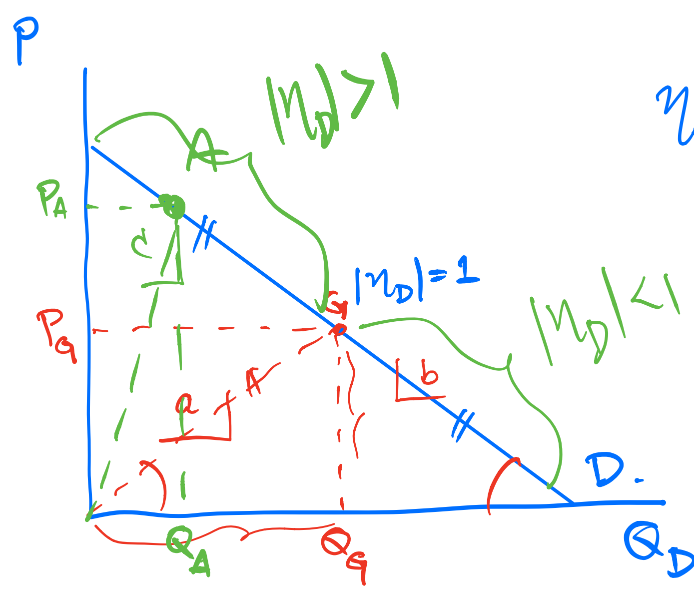
Extreme Cases





perfectly Elastic

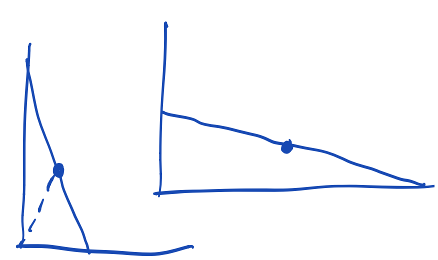
$$\eta_D = \frac{1}{\text{slope}} \frac{P}{Q_D}$$



$$\eta_D = \frac{1}{\text{slope}_G} \cdot \frac{P \text{ at } G}{Q \text{ at } G}$$

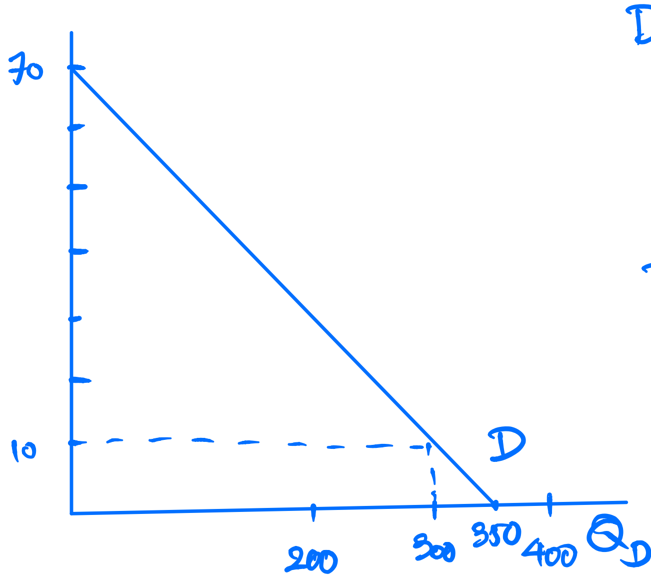
At G. $|\eta_D| = 1$

$$\eta_D = \frac{a}{b}$$



at A, $|\eta_D| = \frac{c}{b}$

Price Elasticity of D and Total Revenue (TR)



$D: \; P = 70 - \frac{1}{5} Q_D$

at $P = 10$, $Q = 300$

$TR = \overset{\uparrow 1\%}{P} \cdot \overset{\downarrow -.167\%}{Q_D} = 10 \times 300 = 3{,}000$

$\eta_D = -\frac{1}{\text{slope}} \cdot \frac{P}{Q_D}$

$= -\frac{1}{1/5} \cdot \frac{10}{300} = -\frac{1}{6}$

$= -0.167$

if the price increases by 1%,
the quantity demanded decreases by 0.167%
inelastic

$\therefore$ TR increases.

Ex. $P = 10$, $Q_D = 300 \Rightarrow TR = 3{,}000$
$P = 11$, $Q_D = 295 \Rightarrow TR = 3{,}245$

TR increase.

Q_D can change because of

1) Price $\Rightarrow$ Price Elasticity of D $= \eta_D = \dfrac{\% \Delta Q_D}{\% \Delta P}$

2) Income $\Rightarrow$ Income Elasticity of D $= \eta_I = \dfrac{\% \Delta Q_D}{\% \Delta I}$

3) Price of other goods $\Rightarrow$ Cross Price Elasticity

$$\eta_{xy} = \frac{\% \Delta Q_x}{\% P_y}$$

Income Elasticity $\quad \eta_I = \dfrac{\% \Delta Q_D}{\% \Delta I}$

I	Q_D
30,000	100
40,000	120

price is unchanged.

$\% \Delta Q = 20\% = \dfrac{120-100}{100} \times 100$

$\% \Delta I = 33.33\% = \dfrac{40{,}000-30{,}000}{30{,}000} \times 100$

$$\eta_D = \frac{20\%}{33.33\%} = 0.6$$

normal		Inferior.
$\eta_I \geq 0$		$\eta_I < 0$
$0 \leq \eta_I \leq 1$ Necessary	$\eta_I > 1$ Luxury.	

Even with higher price for cooking gas, the consumption still remains almost the same because cooking gas is ~~necessary product~~.

Necessary product is defined by η_I where we change income not price

Cross Price Elasticity of D.

(Q Pepsi) Q_x	(Price of Coke) P_y
30	10
35	12

X = Pepsi
Y = Coke

$$\eta_{xy} = \frac{\% \Delta Q_x}{\% \Delta P_y} = \frac{(35-30)/30}{(12-10)/10} = \frac{1/6}{2/10} = \frac{5}{6}$$

$\eta_{xy} > 0$ — X + Y are substitutes.

$\eta_{xy} < 0$ — X + Y are complementary

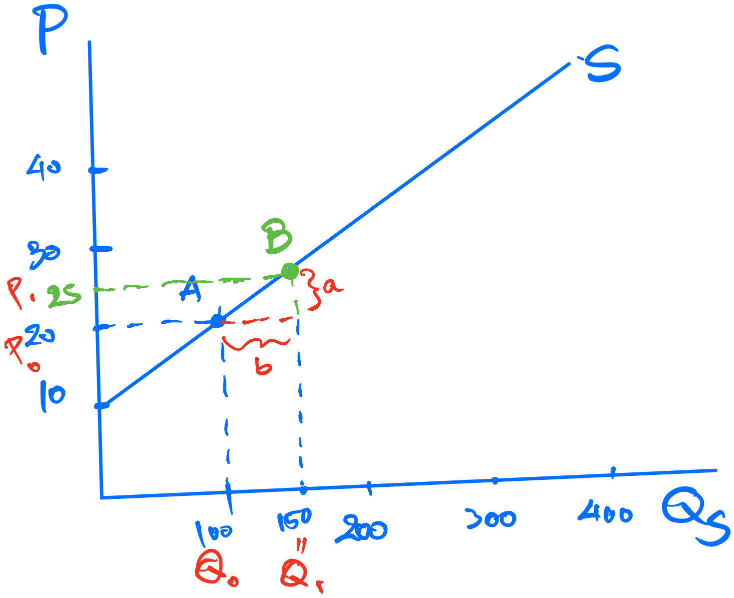
Price Elasticity of Supply

$$\eta_S = \frac{\% \Delta Q_S}{\% \Delta P}$$

$S: \; P = 10 + \frac{1}{10} Q_S$

A: $P_0 = 20, \; Q_0 = 100$

B: $P_1 = 25, \; Q_1 = 150$



Arc Elasticity of S

from A to B.

$$\eta_S = \frac{\% \Delta Q_S}{\% \Delta P} = \frac{(Q_1 - Q_0)/Q_0}{(P_1 - P_0)/P_0} = \underbrace{\frac{Q_1 - Q_0}{P_1 - P_0}}_{\frac{1}{\text{slope } S}} \cdot \frac{P_0}{Q_0}$$

$$= \frac{(150-100)/100}{(25-20)/20}$$

$$= \frac{50/100}{5/20} = 2$$

from B to A

$$\eta_S = \frac{\% \Delta Q_S}{\% \Delta P}$$

$$= \frac{(Q_0 - Q_1)/Q_1}{(P_0 - P_1)/P_1}$$

$$= \frac{1}{\text{slope } S} \cdot \frac{P_1}{Q_1}$$

$$= \frac{1}{1/10} \cdot \frac{25}{150} = \frac{5}{3}$$

Arc Elasticity by Mid Point Method between A & B

$$\eta_s = \frac{\% \Delta Q_s}{\% \Delta P}$$

$$= \frac{(Q_1 - Q_0)/(Q_1 + Q_0)}{(P_1 - P_0)/(P_1 + P_0)}$$

$$= \frac{1}{\text{slope}} \frac{(P_1 + P_0)}{(Q_1 + Q_0)} = \frac{1}{1/10} \cdot \frac{(25+20)}{(150+100)} = 1.8$$

Point Elasticity of S. at P_0 and Q_0

$$\eta_s = \frac{1}{\text{slope}} \cdot \frac{P_0}{Q_0}$$

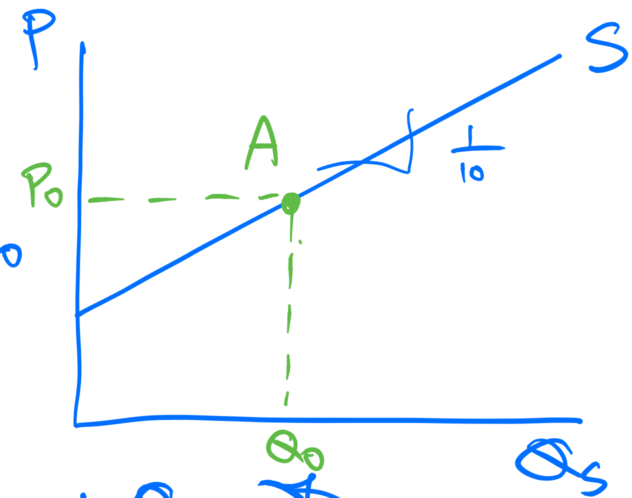
at A, $P_0 = 20$, $Q_0 = 100$

$$= \frac{1}{1/10} \cdot \frac{20}{100} = 2.$$

Find η_s at B, $P_1 = 25$, $Q_1 = 150$

$$\eta_s = \frac{dQ_s}{dP} \cdot \frac{P}{Q}.$$

$$\frac{dQ_s}{dP} = \frac{1}{dP/dQ_s} = \frac{1}{1/10} = 10$$



S: $P = 10 + \frac{1}{10}Q_s$

S: $Q_s = -100 + 10P$

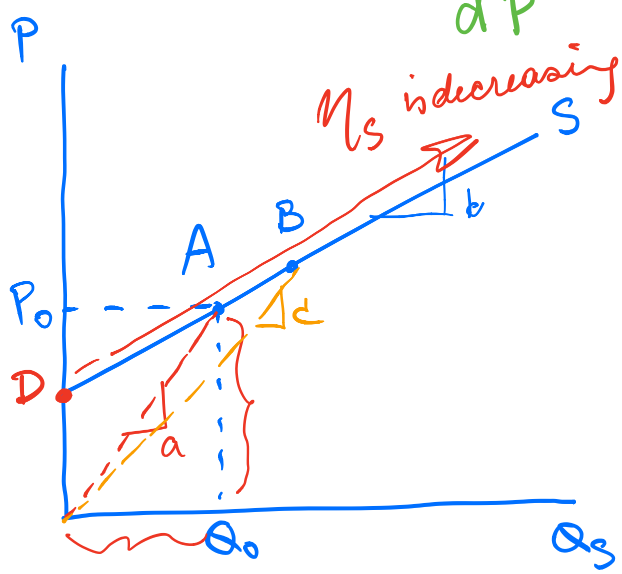
$$\frac{dQ_s}{dP} = 10$$

At A, $\eta_s = \frac{1}{\text{slope at } A} \cdot \frac{P_0}{Q_0}$; P_0 & Q_0 are price & quantity at A

$$= \frac{dQ_s}{dP} \cdot \frac{P_0}{Q_0}$$

$$= \frac{\text{slope of line from origin to } A}{\text{slope of } S \text{ at } A}$$

η_s is decreasing, but always > 1



$$\eta_s = \frac{1}{\text{slope at } A} \cdot \frac{P_0}{Q_0}$$

$$= \frac{a}{b} > 1$$

(Supply is elastic)

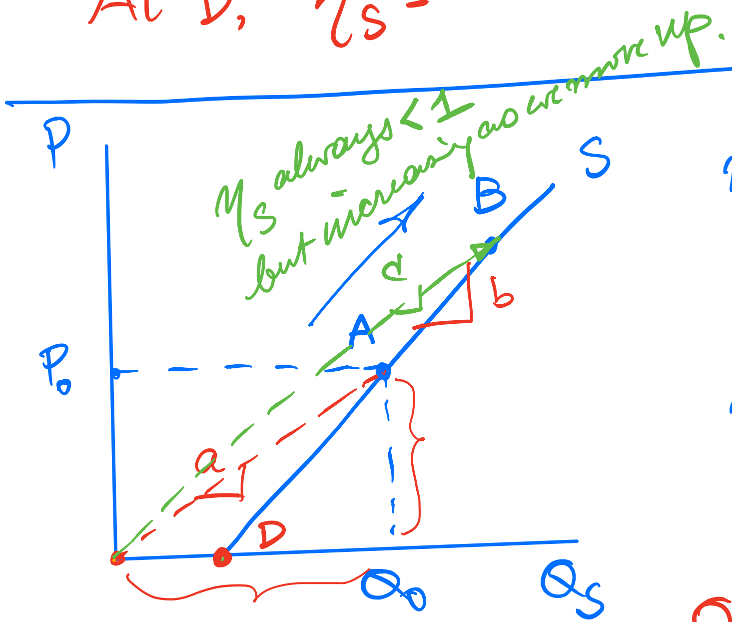
At B, $\eta_s = \frac{c}{b} > 1$

$$\eta_s^{(A)} > \eta_s^{(B)} > 1$$

$$\frac{a}{b} > \frac{c}{b}$$

perfectly elastic

At D, $\eta_s = \infty$ because $Q_s = 0$.



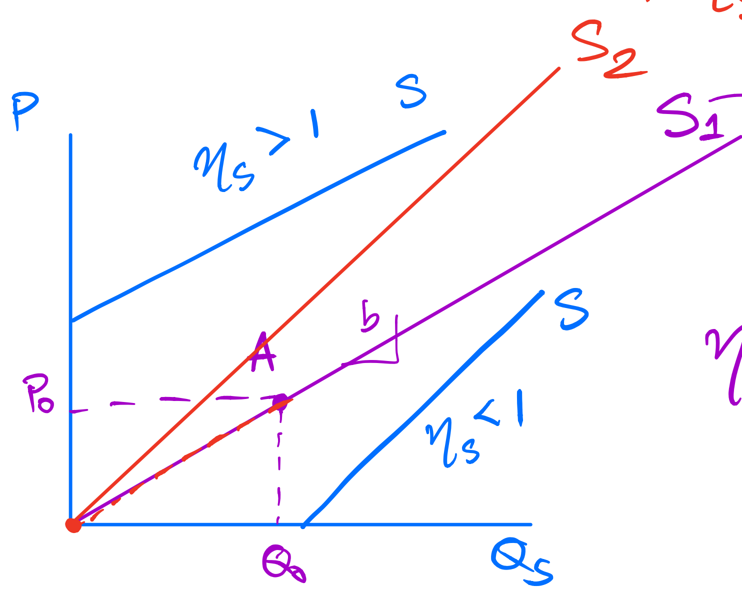
$$\eta_s^{(A)} = \frac{1}{\text{slope at } A} \cdot \frac{P_0}{Q_0}$$

$$= \frac{a}{b} < 1$$ - inelastic

$$\eta_s^{(B)} = \frac{c}{b} < 1$$

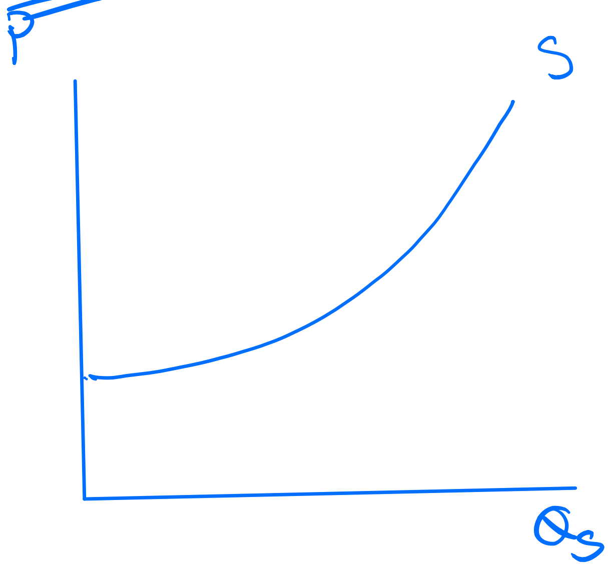
$$0 < \eta_s^{(A)} < \eta_s^{(B)} < 1$$

at D, $\eta_s^{(D)} = 0$ – perfectly inelastic.



$\eta_s = 1$ everywhere

H.W.



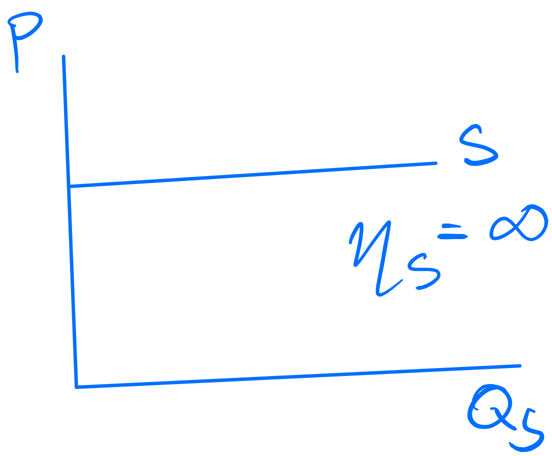
Find the points on S that has

a) $\eta_s > 1$

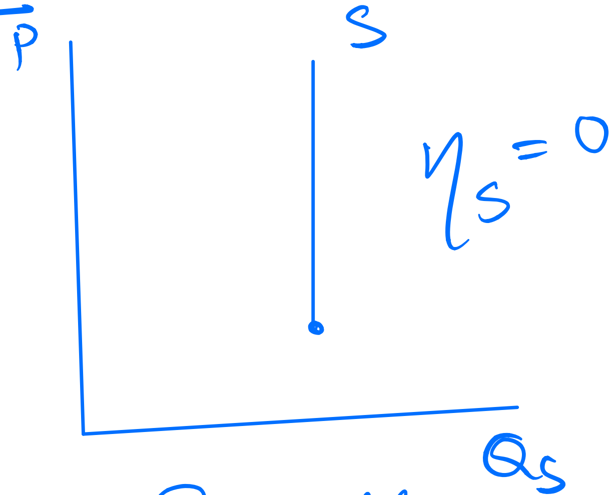
b) $\eta_s < 1$

c) $\eta_s = 1$

Extreme cases of S.



Perfectly elastic



Perfectly Inelastic.