1) Find Cournot equilibrium when there are 3 firms in the market

$P = a - bQ$; $Q = q_1 + q_2 + q_3$ $\Rightarrow$ $P = a - bq_1 - bq_2 - bq_3$

$C = C_1 = C_2 = C_3$

What is equilibrium price? P^*

What are firms' profit? $\pi_1 = \pi_2 = \pi_3$

$$\pi_1 = TR_1 - TC_1 = (a - bq_1 - bq_2 - bq_3)\, q_1 - C_1$$

$$\frac{d\pi_1}{dq_1} = a - 2bq_1 - bq_2 - bq_3 = 0$$

$$a - bq_2 - bq_3 = 2bq_1$$

$$\frac{a}{2b} - 0.5q_2 - 0.5q_3 = q_1 \text{ — Best response function (1)}$$

$$\pi_2 = TR_2 - TC_2 = (a - bq_1 - bq_2 - bq_3)\, q_2 - C_2$$

$$\frac{d\pi_2}{dq_2} = a - bq_1 - 2bq_2 - bq_3 = 0$$

$$a - bq_1 - bq_3 = 2bq_2$$

$$\frac{a}{2b} - 0.5q_{f_1} - 0.5q_{f_3} = q_{f_2}$$ — Best response function ②

$$\Pi_3 = TR_3 - TC_3 = (a - bq_{f_1} - bq_{f_2} - bq_{f_3})q_{f_3} - C_3$$

$$\frac{d\Pi_3}{dq_{f_3}} = a - bq_{f_1} - bq_{f_2} - 2bq_{f_3} = 0$$

$$a - bq_{f_1} - bq_{f_2} = 2bq_{f_3}$$

$$\frac{a}{2b} - 0.5q_{f_1} - 0.5q_{f_3} = q_{f_3}$$ — Best response function ③

sub q_{f_3} in q_{f_1}

$$q_{f_1} = \frac{a}{2b} - 0.5q_{f_2} - 0.5\left(\frac{a}{2b} - 0.5q_{f_1} - 0.5q_{f_2}\right)$$

$$q_{f_1} = \frac{a}{2b} - 0.5q_{f_2} - 0.5\frac{a}{2b} + 0.25q_{f_1} + 0.25q_{f_2}$$

$$q_{f_1} = \frac{a}{4b} + 0.25q_{f_1} - 0.25q_{f_2}$$

$$0.75 q_{b_1} = \frac{a}{4b} - 0.25 q_{b_2} = 0.25 \frac{a}{b} - 0.25 q_{b_2}$$

$$q_{b_1} = \frac{1}{3}\left(\frac{a}{b} - q_{b_2}\right)$$

sub q_{b_3} in q_{b_2}

$$q_{b_2} = \frac{a}{2b} - 0.5 q_{b_1} - 0.5\left(\frac{a}{2b} - 0.5 q_{b_1} - 0.5 q_{b_2}\right)$$

$$q_{b_2} = \frac{a}{2b} - 0.5 q_{b_1} - 0.5 \frac{a}{b} + 0.25 q_{b_1} + 0.25 q_{b_2}$$

$$q_{b_2} = 0.25 \frac{a}{b} - 0.25 q_{b_1} + 0.25 q_{b_2}$$

$$0.75 q_{b_2} = 0.25\left(\frac{a}{b} - q_{b_1}\right)$$

$$q_{b_2} = \frac{1}{3}\left(\frac{a}{b} - q_{b_1}\right)$$

sub q_{b_2} in q_{b_1}

$$q_{b_1} = \frac{1}{3}\left[\frac{a}{b} - \left(\frac{1}{3}\left(\frac{a}{b} - q_{b_1}\right)\right)\right]$$

$$= \frac{1}{3}\left(\frac{a}{b} - \frac{a}{3b} + \frac{q_{b_1}}{3}\right)$$

$$q_{b_1} = \frac{a}{3b} - \frac{a}{9b} - \frac{q_{b_1}}{9}$$

$$9 q_{b_1} = \frac{3a}{b} - \frac{a}{b} + q_{b_1}$$

$$8 q_{b_1} = \frac{2a}{b}$$

$$q_{b_1} = \frac{a}{4b}$$

$$q_{b_2} = \frac{a}{3b} - \frac{a}{12b}$$

$$12 q_{b_2} = \frac{4a}{b} - \frac{a}{b}$$

$$q_{b_2} = \frac{a}{4b}$$

$$q_{b_3} = \frac{a}{2b} - 0.5 \frac{a}{4b} - 0.5 \frac{a}{4b}$$

$$q_{b_3} = \frac{a}{4b}$$

$$\therefore\ P = a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right)$$

$$P = a - \frac{3}{4}a$$

$$P = 0.25a \quad \#$$

$$\pi_1 = P \cdot q_1 - C_1 = 0.25a \cdot \frac{a}{4b} - C_1 = \frac{a^2}{16b} - C_1 \quad \#$$

$$\pi_2 = P \cdot q_2 - C_2 = 0.25a \cdot \frac{a}{4b} - C_2 = \frac{a^2}{16b} - C_2 \quad \#$$

$$\pi_3 = P \cdot q_3 - C_3 = 0.25a \cdot \frac{a}{4b} - C_3 = \frac{a^2}{16b} - C_3 \quad \#$$

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2 If there are N firm

$$q_i^* = f(N),\ P = (N),\ \pi_i = f(N)$$

$$P = a - bQ\ ;\ Q = q_N$$

$$C_N = C_n = C_n = C$$

$$\pi_i = P(Q)\,q_i - C_i q_i$$
$$= (a - b(q_N))\,q_i - C_i q_i$$

Assume $q_{f_1} + q_{f_2} + \dots + q_{f_n} = A$

$$P = a - b(q_{f_1} + q_{f_2} + \dots + q_{f_n})$$

$$P = a - bq_{f_1} - bq_{f_2} - \dots - bq_{f_n}$$

$$\pi_1 = (a - bq_{f_1} - bq_{f_2} - \dots - bq_{f_n})\, q_{f_1} - C_1$$

$$\pi_n = (a - bq_{f_1} - bq_{f_2} - \dots - bq_{f_n})\, q_{f_n} - C_n$$

$$\frac{d\pi_1}{dq_{f_1}} = a - 2bq_{f_1} - bq_{f_2} - \dots - bq_{f_n} = 0$$

$$\frac{a}{2b} - 0.5(q_{f_2} + q_{f_3} + \dots + q_{f_n}) = q_{f_1}$$

$$\frac{a}{2b} - 0.5(q_{f_1} + q_{f_2} + \dots + q_{f_{n-1}}) = q_{f_n}$$

rearrange:

$$q_{f_1} - 0.5q_{f_1} = \frac{a}{2b} - 0.5(q_{f_1} + q_{f_2} + \dots + q_{f_n})$$

$$0.5q_{f_1} = \frac{a}{2b} - 0.5A$$

$$q_{f_1} = \frac{a}{b} - A$$

$$q_{f_2} = \frac{a}{b} - A$$

$$q_{f_1} + q_{f_2} + \dots + q_{f_n} \Rightarrow A = n\left(\frac{a}{b} - A\right)$$

$$A = n\frac{a}{b} - nA$$

$$(n+1)A = n\frac{a}{b}$$

$$A = \frac{na}{(n+1)b}$$

$$q_i = \frac{a}{(n+1)b} \ \#$$

$$P = a - b(A) = a - b\left(\frac{na}{(n+1)b}\right)$$

$$P = a - \frac{n}{n+1}a = \frac{a(n+1) - na}{n+1} = \frac{na + a - na}{n+1}$$

$$P = \frac{a}{n+1} \ \#$$

$$\pi_i = P \cdot q_i - C_i = \frac{a}{n+1} \cdot \frac{a}{(n+1)b} - C_i$$

$$\pi_i = \frac{a^2}{(n+1)^2 b} - C_i \ \#$$

3 From 2 What happen if $N \to \infty$, $N = 1$

if $n \to \infty$; $q_i = \frac{a}{(n+1)b} \to 0$ $\therefore$ each firm will $\to$ 0 unit

: $A = nq_i \to \infty$ $\therefore$ every firms collude at A which $\to \infty$ unit

: $P = \frac{a}{n+1} \to 0$ $\therefore$ supply $\uparrow \Rightarrow$ Price $\downarrow$ to close to 0

: $\pi_i = \frac{a^2}{(n+1)^2 b} - C_i = -C_i$ $\therefore$ each firm will loss, fixed cost $= C$

if $n \to 1$: $q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$ $\therefore$ Monopoly will sell less

$a = \frac{a}{2b} < a = \frac{na}{(n+1)b}$

: $A = nq_i = a \to n = 1$ $\therefore$ firm is monopoly

: $P = \frac{a}{n+1} = \frac{a}{2}$ $\therefore$ $P_M = \frac{a}{2} > P = \frac{a}{n+1}$

: $\pi_i = \frac{a^2}{(n+1)^2 b} - C_i = \frac{a^2}{4b} - C_i$: $\pi_M > \pi_i = \frac{a^2}{(n+1)^2 b} - C_i$

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