EE 415 / EE 418 Game theory

Lecture 5 Dynamic Games of Complete information continued

- Leontif model of Wages and employment determination
- Rubinstein's sequential bargaining
- Read ch. 2 section 2.1C, 2.1D

2.1.C Wage Employment in a Unionized Firm

- Two players: firm (controlling employment) and labor union (monopoly seller of labor to the firm, controlling wage)
- The union's utility function is $U(w,L)$; Where

  w is the wage the union demands from the firm

  L is employment
- Assume that $U(w,L)$ increases in both w and L
- The firm's profit function is

$$\pi(w,L) = R(L) - wL$$

  where $R(L)$ is the revenue the firm can earn if employs L workers
- Assume $R(L)$ is increasing and concave

- Suppose the timing of game is:

(1) The union makes a wage demand, w

(2) The firm observes (and accepts) w and then chooses employment, L

(3) Payoffs are $U(w, L)$ and $\pi(w, L)$

- First, we can characterize the firm's best response in stage (2), $L^*(w)$, to an arbitrary wage demanded by the union in stage (1), w.

Given w, the firm chooses $L^*(w)$ to solve

$$\max_{L \geq 0} \pi(w, L) = \max_{L \geq 0} R(L) - wL$$

FOC; $\quad R'(L) - w = 0$

To guarantee that FOC; $R'(L) - w = 0$ has a solution, assume that $R'(0) = \infty$ And that $R'(\infty) = 0$, as suggested in Figure 2.1.1.

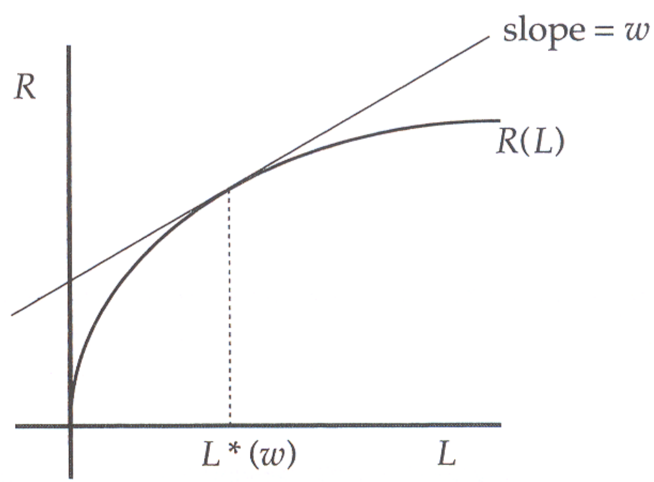


Figure 2.1.1.

Figure 2.1.2 plots $L^*(w)$ as a function of w

And illustrates that $L^*(w)$ cut each of the firm's isoprofit curves at its maximum

Holding L fixed, the firm does better when w is lower, so lower isoprofit curves represent higher profit level.

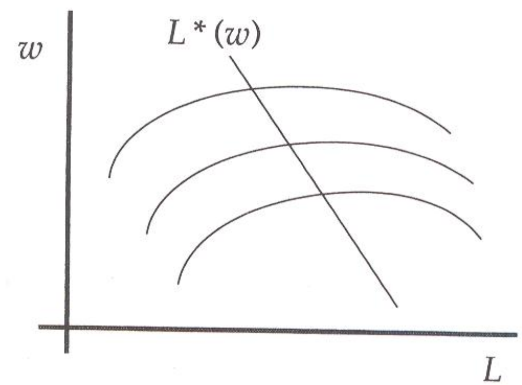
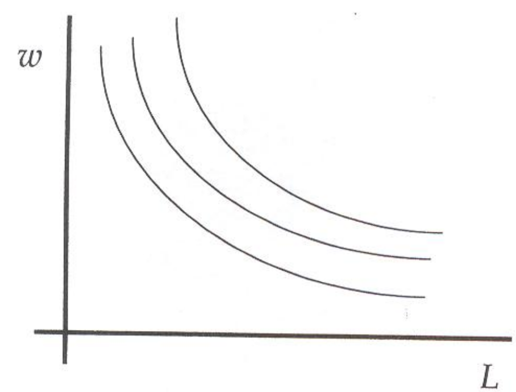


Figure 2.1.2

Figure 2.1.3 depicts the union's indifferent curve. Holding L fixed, the union does better when w Is higher,

so higher indifference curves represent higher utility level for union



union's indifference curves

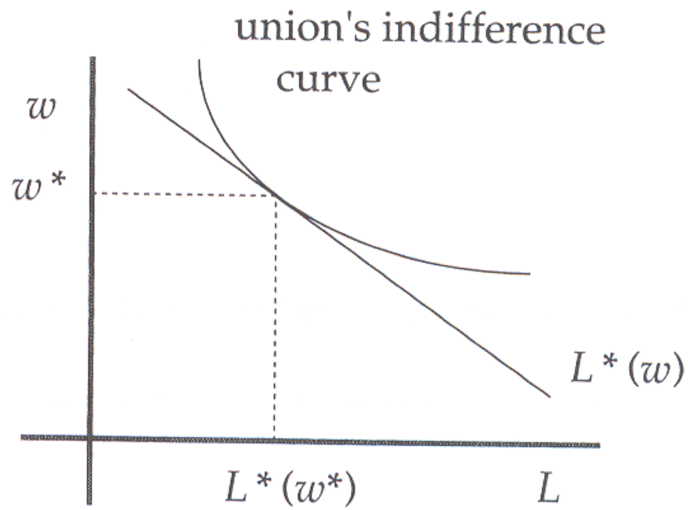
Figure 2.1.3

Thus, the union's problem at the first stage amounts to

$$\max_{w \geq 0} U(w, L^*(w))$$

In term of the indifference curve plotted in Figure 2.1.3, the union would like to choose the wage demand w that yields the outcome $(w, L^*(w))$ that is on the highest possible indifference curve. The solution to the union's problem is w^*, the wage demand such that the union's indifference curve through the point $(w^*, L^*(w^*))$ is tangent to $L^*(w)$ At that point; see Figure 2.1.4.

Figure 2.1.4



Thus, $(w^*, L^*(w^*))$ is the backwards-induction outcome of this wage-and-employment game.

It is straightforward to see that $(w^*, L^*(w^*))$ is inefficient: both the union's utility and the firm's profit would be increased if w and L were in the shaded region in Figure 2.1.5.

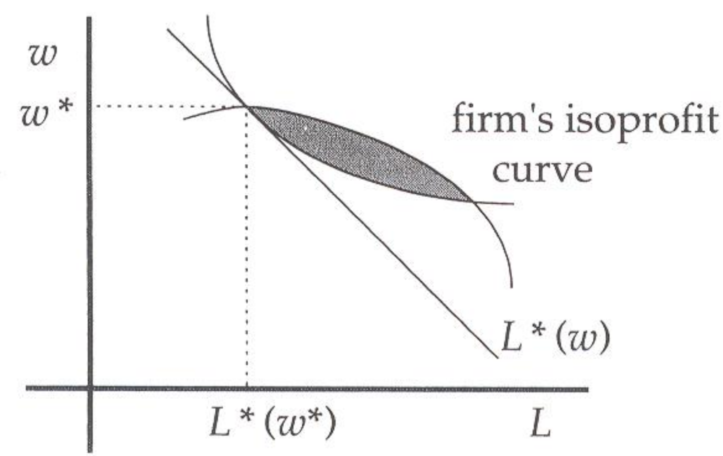


Figure 2.1.5

2.1.D Sequential Bargaining

- Players 1 and 2 are bargaining over one dollar. They alternate in market offer: first player 1 makes a proposal that player 2 can accept or reject; if 2 rejects then 2 makes a proposal that 1 can accept or reject; and so on.
- Once an offer has been rejected, it ceases to be binding and is irrelevant to the subsequent play of the game. Each offer takes one period, and the players are impatient: they discount payoffs received in later periods by the factor δ per period, where $0 < \delta < 1$

- A more detailed description of the timing of the three-period bargaining game as follows.

(1a) At the beginning of the first period, player 1 proposes to take a share s_1 of the dollar, leaving $1-s_1$ for player 2.

(1b) Player 2 either accepts the offer (in which case the game ends and the payoffs s_1 to player1 and $1-s_1$ to player 2 are immediately received) or rejects the offer(in which case play continues to the second period).

(2a) At the beginning of the second period, player 2 proposes than player 1 take a share s_2 of the dollar, leaving $1-s_2$ for player 2.

(2b) Player 1 either accepts the offer (in which case the game ends and the payoffs s_2 to player 1 and $1-s_2$ to player 2 are immediately received) or rejects the offer(in which case play continues to the third period).

(3) At the beginning of the third period, player 1 receives a share s of the dollar, leaving $1-s$ for player 2, where

$$0<\delta<1$$

- In this three-period model, the third-period settlement $(s,1-s)$ is given exogenously.
- To solve for the backward-induction outcome of this three-period game, we first compute player 2's optimal offer if the second period is reached.
- Player 1 can receive s in the third period by rejecting player 2's offer of s_2 this period, but the value this period receiving s next period is only δs

- Thus, player 1 will accept s_2 if and only if $s_2 \geq \delta s$

- Player 2's second-period decision problem therefore amounts to choosing between receiving $1-\delta s$ this period and receiving $1-s$ next period.

- The discounted value of the latter option is $\delta(1-s)$, which is less than the $1-\delta s$, so player 2's optimal second-period offer is $s_2^* = \delta s$

- Thus, if play reaches the second period, player 2 will offer s_2^* and player 1 will accept.

- Since player 1 can solve Player 2's second-period problem as well as player 2 can, player 1 knows that player 2 can receive $1-s_2^*$ in the second period by rejecting player 1's offer of s_1 this period, but the value of this period of receiving $1-s_2^*$ next period is only $\delta(1-s_2^*)$.

- Thus, player 2 will accept $1-s_1$ if and only if $1-s_1 \geq \delta(1-s_2^*)$ or $s_1 \leq 1-\delta(1-s_2^*)$

- Player 1's first-period decision problem therefore amounts to choosing between receiving $1-\delta(1-s_2^*)$ this period (by offering $1-s_1 = \delta(1-s_2^*)$ to player 2) and receiving s_2^* next period.

- The discounted value of the latter option is $\delta s_2^* = \delta^2 s$, which is less than the $1-\delta(1-s_2^*) = 1-\delta(1-\delta s)$ available from the former option, so player 1's optimal first-period offer is

$$s_1^* = 1-\delta(1-s_2^*) = 1-\delta(1-\delta s)$$

- Thus, in the backward-induction outcome of this three-period game, player 1 offers the settlement $(s_1^*, 1-s_1^*)$ to player 2, who accepts.

Infinite-horizon case

- The timing is as described previously, except that the exogenous settlement in step(3) is replaced by an infinite sequence of step (3a),(3b),(4a),(4b), and so on.
- Player 1 makes the offer in odd-numbered periods, player 2 in even-numbered; bargaining continues until one player accepts an offer.

Because the game could go on infinitely,

- Suppose that there is a backward-induction outcome of the game as a whole in which player 1 and 2 receive the payoffs s and $1-s$.
- We can use these payoffs in the game beginning in the third period, then work backward to first period to compute a new backward-induction outcome for the game as a whole.

- In this new backward-induction outcome , player 1 will offer the settlement $(f(s), 1-f(s))$ in the first period and player 2 will accept, where $f(s)=1-\delta(1-\delta s)$ is the share taken by player 1 in the first period of the three-period model above when the settlement $(s, 1-s)$ is exogenously imposed in the third period.

- Let s_H be the highest payoff player1 can achieve.
- Imagine using s_H as the third-period payoff to player 1, as previously described:

- This will produce a new backward-induction outcome in which player 1's first-period payoff is $f(s_H)$ since $f(s) = 1 - \delta + \delta^2 s$; $f(s_H)$ is the highest possible first-period payoff because s_H is the highest possible third-period payoff.

- But s_H is also the highest possible first-period payoff, so

$$f(s_H) = s_H$$

- Parallel arguments show that $f(s_L) = s_L$, where s_L is lowest payoff player 1 can achieve in any backward-induction outcome of the game as a whole. The only value of s that satisfies $f(s) = s$ is $1/(1+\delta)$, which we will denote by s^*
- Thus, $s_H = s_L = s^*$, so there is a unique backward-induction outcome of the game as a whole: in the first period, player 1 offers the settlement $(s^* = 1/(1+\delta), 1 - s^* = \delta/(1+\delta))$ to player 2, who accepts.