

Oligopoly and Game Theory

1. A monopolist can produce at constant average (marginal) cost of $AC = MC = 5$. It faces a market demand curve given by $Q = 53 - P$.
 - a) Calculate the profit-maximizing price and quantity for this monopolist. Also calculate its profits.
 - b) Suppose the second firm enters the market. Let Q_1 be the output of the first firm and Q_2 be the output of the second. Market demand is now given by $Q_1 + Q_2 = 53 - P$.
Assuming that this second firm has the same costs as the first, write the profits of each firm as functions of Q_1 and Q_2 .
 - c) Suppose (as in the Cournot model) that each firm chooses its profit-maximizing level of output on the assumption that its competitor's output is fixed. Find each firm's reaction curve.
 - d) Calculate the Cournot equilibrium. What are the resulting market price and profits of each firm.
 - e) Find the Stackleberg equilibrium when the first firm is the leader and the new entrant is the follower.
2. Consider two firms (B and L) that produce the same good, but with product differentiation. Each firm facing the following differentiated demand curves:
 $Q_B = 9.6 - 2P_B + P_L$ and $Q_L = 9.6 - 2P_L + P_B$
 In addition, $MC_B = MC_L = 1.2$.
 - a) If there were no product differentiation (i.e. both demands become $Q = 12 - 2P$), what would be the Bertrand equilibrium price in this market?
 - b) Now with product differentiation, find the Bertrand-Nash equilibrium prices, outputs, and profits.
 - c) If both firms form a cartel, find the collusive price, quantities, and profits.
3. Two computer firms, A and B, are planning to market network systems for office information management. Each firm can develop either a fast, high-quality system (High), or a slower, low quality system (Low). Market research indicates that the resulting profits to each firm for the alternative strategies are given by the following payoff matrix:

		Firm B	
		High	Low
Firm A	High	30 , 30	50 , 35
	Low	40 , 60	20 , 20

- a) If both firms make their decisions at the same time and follow maximin (low-risk) strategies, what will the outcome be?
- b) Suppose both firms try to maximize profits, but Firm A has a head start in planning and can commit first. Now what will the outcome be? What will the outcome be if Firm B has the head start in planning and can commit first?
- c) Getting a head start costs money (you have to gear up a large engineering team). Now consider the two-stage game in which first, each firm decides how much

money to spend to speed up its planning, and second, it announces which product (H or L) it will produce. Which firm will spend more to speed up its planning? How much will it spend? Should the other firm spend anything to speed up its planning? Explain.

4. Two firms are in the chocolate market. Each can choose to go for the high end of the market (high quality) or the low end (low quality). Resulting profits are given by the following payoff matrix:

		Firm 2	
		Low	High
Firm 1	Low	-20 , -30	900 , 600
	High	100 , 800	50 , 50

- What outcomes, if any, are Nash equilibria?
- If the managers of both firms are conservative and each follows a maximin strategy, what is the cooperative outcome?
- Which firm benefits most from the cooperative outcome? How much would that firm need to offer the other to persuade it to collude?