Quiz 6

1. Let

$$f(x,y,z) = \ln(xz)\sqrt{\cos(3x^2+2y)}.$$

Find $\dfrac{\partial f}{\partial x}$.

Solution By the product rule,

$$\begin{aligned}
\frac{\partial f}{\partial x} &= \ln(xz)\frac{\partial}{\partial x}[\cos(3x^2+2y)]^{1/2} + [\cos(3x^2+2y)]^{1/2}\frac{\partial}{\partial x}\ln(xz) \\
&= \ln(xz)\frac{1}{2}[\cos(3x^2+2y)]^{-1/2}\frac{\partial}{\partial x}[\cos(3x^2+2y)] + [\cos(3x^2+2y)]^{1/2}\frac{1}{xz}\frac{\partial}{\partial x}(xz) \\
&= \ln(xz)\frac{1}{2}[\cos(3x^2+2y)]^{-1/2}[-\sin(3x^2+2y)]\frac{\partial}{\partial x}(3x^2+2y) + [\cos(3x^2+2y)]^{1/2}\frac{1}{xz}\frac{\partial}{\partial x}(xz) \\
&= \ln(xz)\frac{1}{2}[\cos(3x^2+2y)]^{-1/2}[-\sin(3x^2+2y)][6x] + [\cos(3x^2+2y)]^{1/2}\frac{1}{xz}[z] \\
&= \ln(xz)\frac{1}{2}[\cos(3x^2+2y)]^{-1/2}[-\sin(3x^2+2y)]\frac{\partial}{\partial x}(3x^2+2y) + [\cos(3x^2+2y)]^{1/2}\frac{1}{xz}\frac{\partial}{\partial x}(xz) \\
&= \ln(xz)\frac{1}{2}[\cos(3x^2+2y)]^{-1/2}[-\sin(3x^2+2y)][3x] + [\cos(3x^2+2y)]^{1/2}\frac{1}{xz}[z] \\
&= -3x\ln(xz)[\cos(3x^2+2y)]^{-1/2}\sin(3x^2+2y) + \frac{[\cos(3x^2+2y)]^{1/2}}{x}.
\end{aligned}$$

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