

Solution to Price Discrimination Practice Problems.

①

$$\begin{aligned} a) \quad Q &= q_f^r + q_f = 18 - 5p + 10 - 2p \\ &= 28 - 7p \quad \text{or} \quad P = 4 - \frac{Q}{7} \end{aligned}$$

Then, use this demand curve (for 2 persons) to find price. (Using the 1-person average demand would get you the same answer)

$$\begin{aligned} \pi^{\text{mono}} &= \text{total revenue} - \text{total cost} \\ &= PQ - MCQ \\ &= (P - MC)Q \quad ; \quad MC = 2 \\ &= \left(4 - \frac{Q}{7} - 2\right)Q = 2Q - \frac{Q^2}{7} \\ &\quad \underbrace{\hspace{1.5cm}}_P \end{aligned}$$

First order condition (FOC.): $0 = \frac{d\pi^{\text{mono}}}{dQ}$,

$$0 = 2 - \frac{2Q}{7}$$

$Q = 7$ #, plug this back into demand curve

$$P = 4 - \frac{Q}{7} = 4 - \frac{7}{7} = 3 \text{ #}$$

b) π^{under}

= total revenue - total cost

$$= Pq_f^r - mcq_f^r$$

$$= (P - mc)q_f^r$$

$$= \left(\frac{18 - q_f^r}{5} - 2\right)q_f^r$$

$$= \left(\frac{8 - q_f^r}{5}\right)q_f^r$$

$$q_f^r = 18 - 5P$$

$$P = \frac{18 - q_f^r}{5}$$

$$\text{FOC: } 0 = \frac{d\pi_{\text{under}}}{dq^r} \quad (2)$$

$$= \left(\frac{8-q^r}{5}\right) \times 1 + q^r \left(-\frac{1}{5}\right)$$

$$= \frac{8-2q^r}{5}$$

$$q^r = 4 \# \Rightarrow P = \frac{18-q^r}{5} = 2.8 \#$$

π_{over}

$$= pq - mcq \quad \left| \quad \begin{array}{l} q = 10 - 2p \\ P = \frac{10-q}{2} \end{array} \right.$$

$$= (P - mc)q$$

$$= \left(\frac{10-q}{2} - 2\right)q$$

$$= \left(\frac{6-q}{2}\right)q$$

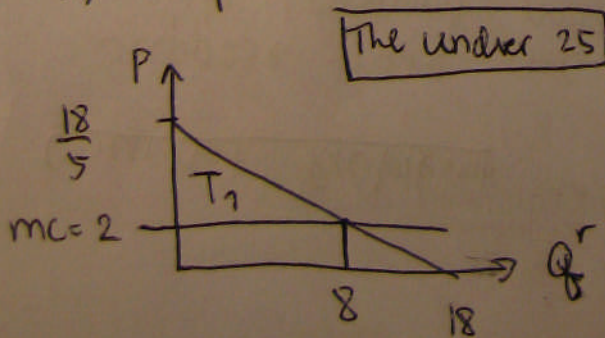
$$\text{FOC: } 0 = \frac{d\pi_{\text{over}}}{dq}$$

$$= \left(\frac{6-q}{2}\right) \times 1 + q \left(-\frac{1}{2}\right)$$

$$= \frac{6-2q}{2}$$

$$q = 3 \# \Rightarrow P = \frac{10-q}{2} = 3.5 \#$$

c) Separate cover charge for each group.



$$q^r = 18 - 5P$$

$$P = \frac{18}{5} - \frac{1}{5}q^r$$

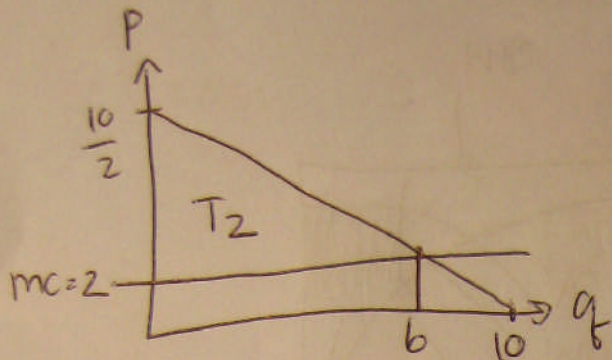
$$\text{cover charge} = T_1$$

$$\text{price/unit} = mc = 2 \#$$

$$T_1 = \frac{1}{2} \times \text{height} \times \text{base} = \frac{1}{2} \times \left(\frac{18}{5} - 2\right) \times 8 = 16.4 \#$$

The over 25

(3)



$$q_f = 10 - 2P$$

$$P = \frac{10}{2} - \frac{q_f}{2}$$

cover charge = T_2
 Price/unit = $MC = \$2$

$$T_2 = \frac{1}{2} \times \text{height} \times \text{base} = \frac{1}{2} \left(\frac{10}{2} - 2 \right) \times 6 = \$9$$

d) charge cover charge = $T_1 = \$6.4$
 price per drink = $MC = \$2$

if cover charge exceeds \$6.4, the under 25 will not buy because the cover charge exceeds their surplus.

e) before midnight's linear pricing = same price for all units with no cover charge

$$Q = \frac{2}{7} q^r + \frac{7}{7} q$$

only $\frac{2}{7}$ of the under. \uparrow all of the over 25

$$= \frac{2}{7} (18 - 5P) + \frac{7}{7} (10 - 2P)$$

$$= \frac{36 - 10P + 70 - 14P}{7} = \frac{106 - 24P}{7}$$

$$Q = \frac{106 - 24P}{7} \Rightarrow P = \frac{106 - 7Q}{24}$$

Π_{before} = total revenue - total cost
 = $PQ - MCQ$
 = $(P - MC)Q$

$$\begin{aligned}
 &= \left(\frac{106 - 7Q}{24} - 2 \right) Q \\
 \text{FOC: } 0 &= \frac{d\pi}{dQ} \text{ before} \\
 &= \left(\frac{106 - 7Q - \cancel{48}}{24} \right) + Q \left(\frac{-7}{24} \right) \\
 &= \frac{58 - 14Q}{24} \\
 Q &= \frac{4.14}{\cancel{4.28}} \Rightarrow P = \frac{106 - 7 \times Q}{24} \approx \frac{3.21}{\cancel{3.17}}
 \end{aligned}$$

AFTER Midnight \rightarrow only the under 25 stays.
 so, from part a) the linear (uniform) monopoly price is \$ 2.8 / unit for the under 25.

Compare profits

in part d) nightclub charges \$ 6.4 as a cover charge per person. Then it charges \$ 2 per unit to cover the cost.
 Thus profit / person = \$ 6.4.

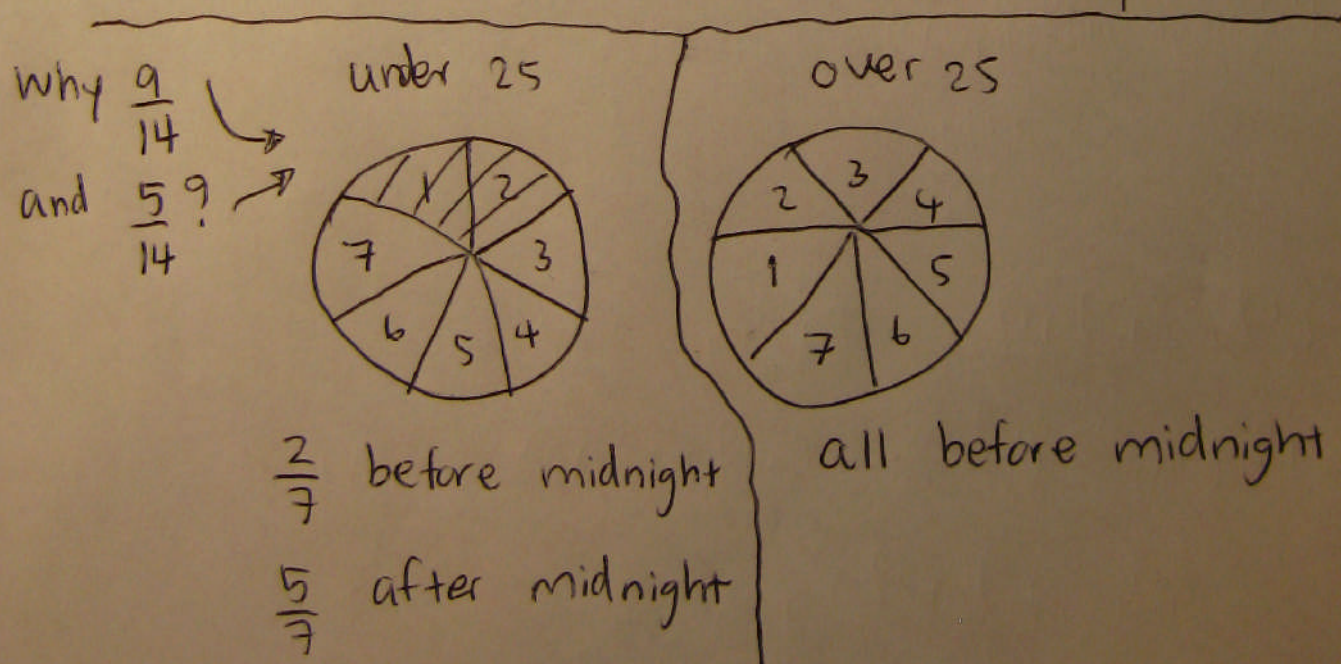
in part e) $\pi^{\text{before}} = (P - mc)Q$ ⑤
 $= (3.77 - 2) 4.28$
 $= \$ 5$

but this is for $\frac{2}{7} + \frac{7}{7} = \frac{9}{7}$ person > 1 person.

Thus π^{before} per person $= 5 \times \frac{7}{9} \approx 3.88 / \text{person}$

$\pi^{\text{After}} = (P - mc)Q$
 $= (2.8 - 2) 4$
 $= \$ 3.2 / \text{person.}$

Thus, average $\pi = 3.88 \times \frac{9}{14} + 3.2 \times \frac{5}{14}$
 $= 2.49 + 1.14$
 $= \$ 3.63 / \text{person.}$



There are 14 portions of customers