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1. $P = a - bQ$; $Q = q_1 + q_2 + q_3$

$c_1 = c_2 = c_3 = c$

$P = a - bq_1 - bq_2 - bq_3$

$\pi_1 = TR_1 - TC_1$

$= (a - bq_1 - bq_2 - bq_3)q_1 - c_1$

$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 = 0$

$a - bq_2 - bq_3 = 2bq_1$

$q_1 = \frac{a}{2b} - \frac{1}{2}q_2 - \frac{1}{2}q_3$

$\pi_2 = (a - bq_1 - bq_2 - bq_3)q_2 - c_2$

$\frac{\partial \pi_2}{\partial q_2} = a - bq_1 - 2bq_2 - bq_3 = 0$

$q_2 = \frac{a}{2b} - \frac{1}{2}q_1 - \frac{1}{2}q_3$

$\pi_3 = (a - bq_1 - bq_2 - bq_3)q_3 - c_3$

$\frac{\partial \pi_3}{\partial q_3} = a - bq_1 - bq_2 - 2bq_3 = 0$

$q_3 = \frac{a}{2b} - \frac{1}{2}q_1 - \frac{1}{2}q_2$

$q_1 = \frac{a}{2b} - \frac{1}{2}q_2 - \frac{1}{2}\left(\frac{a}{2b} - \frac{1}{2}q_1 - \frac{1}{2}q_2\right)$

$= \frac{a}{4b} + \frac{1}{4}q_1 - \frac{1}{4}q_2$

$\frac{3}{4}q_1 = \frac{a}{4b} - \frac{1}{4}q_2$

$= \frac{1}{4}\frac{a}{b} - \frac{1}{4}q_2$

$q_1 = \frac{1}{3}\left(\frac{a}{b} - q_2\right)$

$q_2 = \frac{a}{2b} - \frac{1}{2}q_1 - \frac{1}{2}\left(\frac{a}{2b} - \frac{1}{2}q_1 - \frac{1}{2}q_2\right)$

$= \frac{1}{4}\frac{a}{b} - \frac{1}{4}q_1 + \frac{1}{4}q_2$

$\frac{3}{4}q_2 = \frac{1}{4}\left(\frac{a}{b} - q_1\right)$

$q_2 = \frac{1}{3}\left(\frac{a}{b} - q_1\right)$

$q_1 = \frac{1}{3}\left[\frac{a}{b} - \frac{1}{3}\left(\frac{a}{b} - q_1\right)\right]$

$= \frac{1}{3}\left(\frac{a}{b} - \frac{a}{3b} + \frac{q_1}{3}\right)$

$= \frac{a}{3b} - \frac{a}{9b} + \frac{q_1}{9}$

$q_1 = \frac{a}{4b}$

$q_2 = \frac{a}{3b} - \frac{a}{12b}$

$= \frac{a}{4b}$

$q_3 = \frac{a}{2b} - \frac{1}{2}\frac{a}{4b} - \frac{1}{2}\frac{a}{4b}$

$= \frac{a}{4b}$

$P = a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right)$

$= a - \frac{3}{4}a$

$= \frac{1}{4}a$

$\pi_1 = Pq_1 - c_1$

$= \frac{1}{4}a \cdot \frac{a}{4b} - c_1$

$= \frac{a^2}{16b} - c_1$

$\pi_2 = Pq_2 - c_2$

$= \frac{1}{4}a \cdot \frac{a}{4b} - c_2$

$= \frac{a^2}{16b} - c_2$

$\pi_3 = Pq_3 - c_3$

$= \frac{1}{4}a \cdot \frac{a}{4b} - c_3$

$= \frac{a^2}{16b} - c_3$

2. $q_i^* = f(N)$, $P = (N)$, $\pi_i = f(N)$

$P = a - bq$; $Q = q_N$

$c_N = c_N = c_N = c$

$\pi_i = P(Q)q_i - c_i q_i$

$= [a - b(q_N)]q_i - c_i q_i$

$q_1 = \frac{a}{2b} - \frac{1}{2}(q_2 + q_3 + \dots + q_N)$

$q_N = \frac{a}{2b} - \frac{1}{2}(q_1 + q_2 + \dots + q_{N-1})$

$q_1 - \frac{1}{2}q_1 = \frac{a}{2b} - \frac{1}{2}(q_1 + q_2 + \dots + q_N)$

$\frac{1}{2}q_1 = \frac{a}{2b} - \frac{1}{2}A$

$q_1 = \frac{a}{b} - A$

$q_2 = \frac{a}{b} - A$

$q_n = \frac{a}{b} - A$

$A = n(\frac{a}{b} - A)$

$= n\frac{a}{b} - nA$

$(n+1)A = n\frac{a}{b}$

$= \frac{na}{(n+1)b}$

$q_1 = \frac{a}{(n+1)b}$

$P = a - b(A)$

$= a - b(\frac{na}{(n+1)b})$

$= a - \frac{n}{n+1}a$

$= \frac{a(n+1) - na}{n+1}$

$= \frac{a}{n+1}$

$\pi_i = Pq_i - c_i$

$= \frac{a}{n+1} \cdot \frac{a}{(n+1)b} - c_i$

$= \frac{a^2}{(n+1)^2 b} - c_i$

3. If $N \to \infty$

$q_i = \frac{a}{(n+1)b} \to 0$ each firm sells at q almost 0 unit

$A = \frac{na}{(n+1)b} \to \infty$ every firm's q combined is almost 0 unit

$P = \frac{a}{n+1} \to 0$ supply increases as P decreases approaching 0

$\pi_i = \frac{a^2}{(n+1)^2 b} - c_i \to -c_i$ every firm loses profit

If $N = 1$

$q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$ monopolist firm decides to sell less output

$A = \frac{na}{(n+1)b} = q$ firms will become monopolist

$P = \frac{a}{n+1} = \frac{a}{2}$ monopolist firm would set higher price

$\pi_i = \frac{a^2}{(n+1)^2 b} - c_i = \frac{a^2}{4b} - c_i$ monopolist firm gains more profit