Question 1 (25 points)

Consider the problem of an investor trying to determine the best portfolio of two risky assets (stocks) and a risk-free asset (T-bill). Let the two risky assets be Boeing and Microsoft, the riskfree asset be a one-year T-bill and suppose the investment horizon is one year. Boeing has an expected return of 15% and standard deviation of 25% while Microsoft has an expected return of 35% and standard deviation 40%. Both stocks have a correlation coefficient of -0.5. The riskfree rate is 5%.

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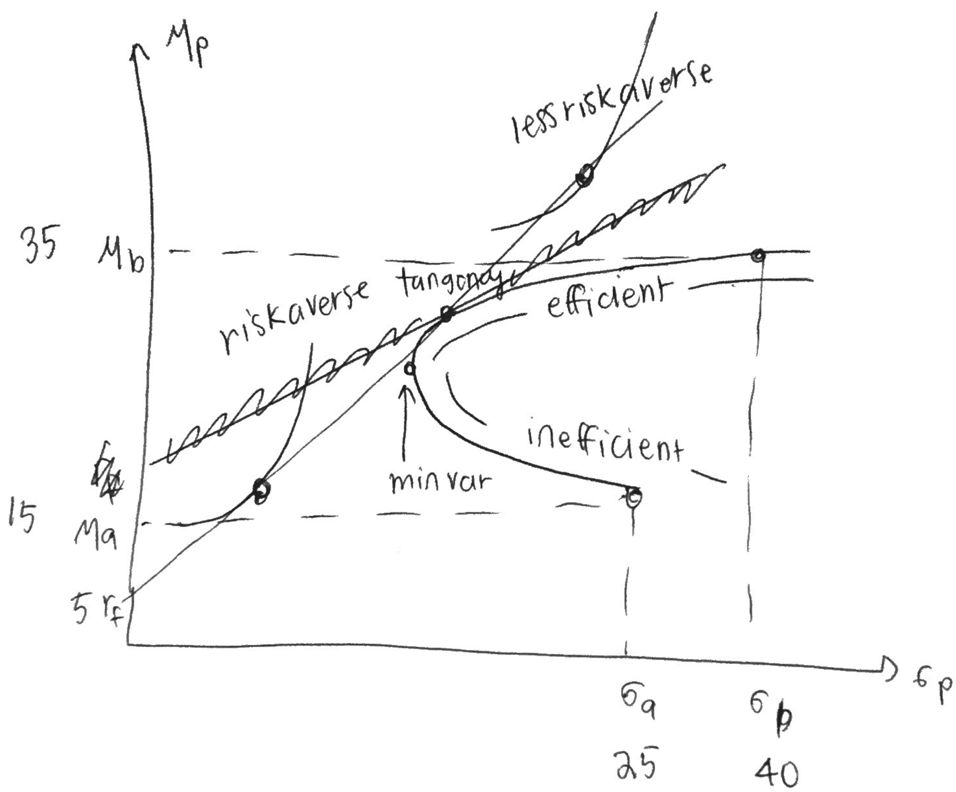
a) Draw the investment opportunity set consisting of the two risky assets. On your diagram, identify the sets of inefficient and efficient portfolios. Do not forget to fully label your graph.



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b) On the above graph, identify the tangency portfolio, minimum variance portfolio, and the set of efficient combinations of T-bills and the two risky assets. On the efficient set, indicate the asset allocation of the portfolios that a very risk averse investor would choose to hold and the portfolios that a very risk tolerant investor would choose to hold.

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c) Assume that the investor does not want to short-sell stocks and wants to achieve a minimum expected return of 20%. Write out (but do not solve) the maximization problem to determine the tangency portfolio. Briefly explain each condition of your optimization problem in words.

$$\min_{w_i} \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{j>1}^{n} w_i w_j \sigma_{ij}$$ 3

$$\text{st} \sum_{i=1}^{n} w_i = 1$$ 1

$i = 2$

$$\sum_{i=1}^{n} w_i \mu_i = 20\%$$ 3

$$w_i \geq 0$$ no shortselling 3

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$$\max \frac{E(r_P) - r_f}{\sigma_P} = \frac{w_A E(r_A) + w_B E(r_B) - r_f}{\sqrt{w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 \cdot w_A w_B \rho_{AB} \sigma_A \sigma_B}}$$

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Question 2 (20 points)

The following output is based on estimating the CAPM regression for IBM and an equally weighted portfolio of 15 stocks using monthly return data over the period January 1978 to December 1982. Note that the values in parentheses represent the standard error of the intercept and slope estimates, and σ_ε is the standard error of the residuals.

$$E(r_{IBM}) = \underset{(0.006)}{0.002} + \underset{(0.103)}{0.657} * (E(r_M) - r_f), \sigma_\varepsilon = 0.1472$$

$$E(r_{Port}) = \underset{(0.004)}{0.002} + \underset{(0.068)}{0.777} * (E(r_M) - r_f), \sigma_\varepsilon = 0.0311$$

a) What are the estimated values of beta for IBM and the portfolio? According to the results above, the beta for the portfolio is larger in magnitude than the beta for IBM. Why do you think this is the case?

0.657 and 0.777

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Beta of portfolio ~~was~~ is larger as the portfolio moves more with the market 4

b) Based on the standard deviation of the residuals, what can you say about the risk characteristics of IBM compared to the portfolio? Does it make sense that σ_ε of the portfolio is smaller than that of IBM? Explain.

Standard deviation of residuals larger for IBM

3 pts for mentioning firm specific risk

4 pts for diversify

Less idiosyncratic risk as portfolio diversifies it away

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c) Does CAPM hold for IBM and the portfolio? Justify your answer by testing null hypotheses.

IBM tstat $\frac{0.002}{0.006} = 1/3 < 1.96$

port $\frac{0.002}{0.004} = 1/2 < 1.96$

7

~~not statis~~ Ho $\alpha = 0$

cannot reject null that $\alpha = 0$

yes CAPM holds

Question 3 (25 points)

The following is a list of prices for zero coupon bonds of various maturities. The face value of the bond is $1000.

Maturity (years)	Price of zero coupon bond
1	965.6
2	924.8
3	884.2
4	846.5

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a) Calculate the spot rates associated with each bond as well as the implied one year forward rates.

$r_{01} = \left(\frac{1000}{965.6}\right) - 1 = 0.0356$

$f_{12} = \frac{(1+r_{02})^2}{(1+r_{01})} - 1 = 0.0441$

$r_{02} = \left(\frac{1000}{924.8}\right)^{1/2} - 1 = 0.0399$

$f_{23} = \frac{(1+r_{03})^3}{(1+r_{02})^2} - 1 = 0.0450$

$r_{03} = \left(\frac{1000}{884.2}\right)^{1/3} - 1 = 0.0419$

$f_{34} = \frac{(1+r_{04})^4}{(1+r_{01})^3} - 1 = 0.044$

$r_{04} = \left(\frac{1000}{846.5}\right)^{1/4} - 1 = 0.0425$

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b) Plot the term structure of interest rates. If the expectation hypothesis, what does the information in the term structure say about the course of future interest rates?

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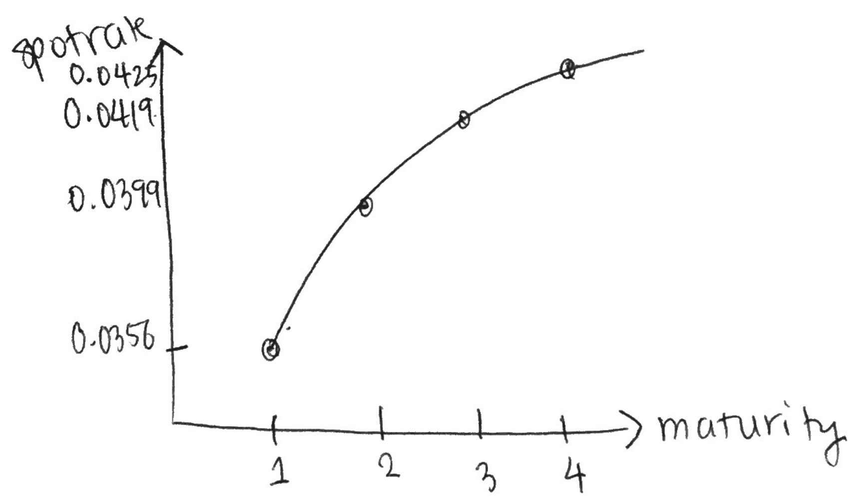

The expectations hypothesis implies that the implied forward rates are the best forecasts of future spot rates. ~~Using the~~ The EH predicts slightly rising one year spot rates for 2 years then a slight decrease.

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c) What is the price of a 2 year coupon bond making annual coupon payments with an annual coupon rate of 3% and a face value of $1000? Write out (but do not solve) the equation you would use to find the yield to maturity of this 2 year coupon bond.

3 $PV = C_0 + \frac{C_1}{(1+r_{01})} + \frac{C_2}{(1+r_{02})^2} = \frac{30}{1.0356} + \frac{1030}{(1+0.0399)^2} = 981.51$

3 Solve $981.51 - \frac{30}{(1+r)} - \frac{1030}{(1+r)^2} = 0$

Question 4 (25 points)

The stock of Heavy Metal (HM) is currently selling at $40. It changes price only once a month: with equal probability the price either goes up by 20 percent or it falls by 16.7 percent. The risk free interest rate is 1 percent per month. Consider a one-month call option of HM stock with an exercise price of $40.

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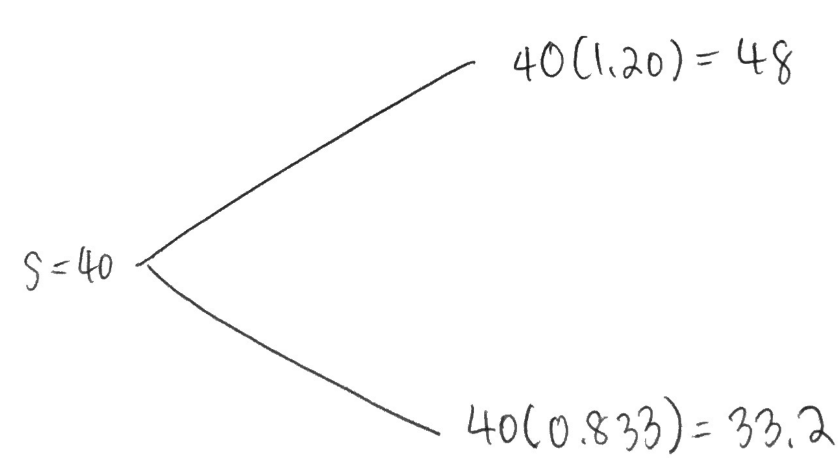
a) Graph the tree diagram showing the current stock price of HM and the two possible values of the stock price next month.



b) At the expiration date of the option, what is the value of the call if the stock price goes up and what is the value if the stock price goes down?

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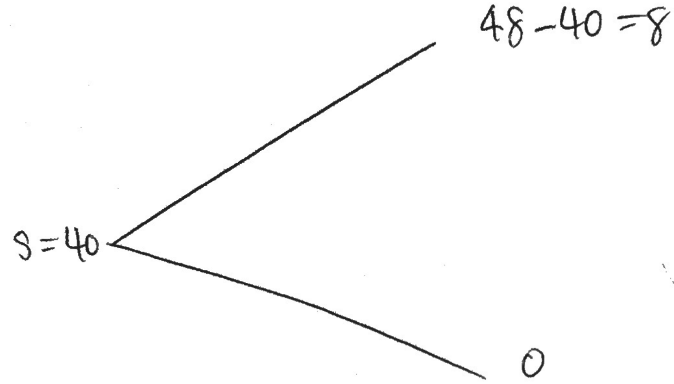

c) What is the current value of the call option? Show and explain your working.

10 Hedge ratio $\frac{Cu - Cd}{uS_0 - dS_0} = \frac{8-0}{48-33.32} = \frac{8}{14.68} = 0.54$ 3

Buy 0.54 Shares of Stock and write 1 call option

Payoff	u	d
Buy Stock (0.54 Shares)	$0.54 \times 48 = 25.92$	$0.54 \times 33.32 = 18$
Sell 1 Call option	-8	0
	$17.92 \approx 18$	18

4

$0.54(40) - C_0 = 18 / 1.01$ 3

$C_0 = \cancel{3.78}$

3.82

+

d) What will happen to the current value of the call option if the probability of a stock price increase stays at 20 percent per year, but the probability of a stock price fall doubles to 25.4 percent a year? You do not have to calculate the new stock price but analyze whether you think that the call value will go up or down, and explain why.

* this effects the volatility

* Call value will go up

If say go down then 3 pts
4

Question 5 (20 points)

Here is an example of a butterfly strategy: simultaneously buy one call with an exercise price of $100, sell two calls with an exercise price of $110, and buy one call with an exercise price of $120.

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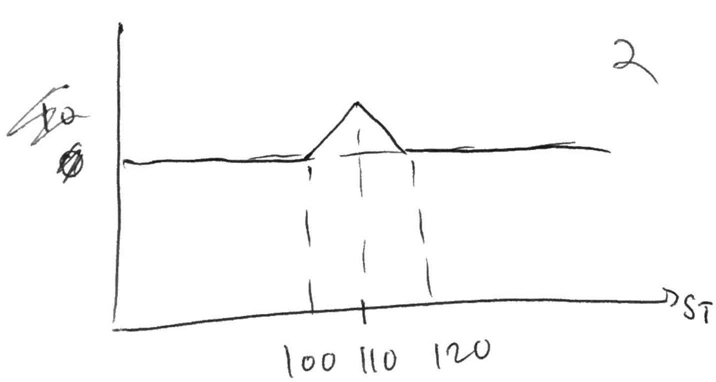
a. Draw a diagram for the butterfly, showing the pay off at expiration from the investors net position.

5 if just graph

2



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	$S_T < 100$	$100 \le S_T < 110$	$110 \le S_T \le 120$	$S_T > 120$
buy 1 call (X=100)	0	$S_T - 100$	$S_T - 100$	$S_T - 100$
sell 2 calls (X=110)	0	0	$-2(S_T - 110)$	$-2(S_T - 110)$
buy 1 call (X=120)	0	0	0	$S_T - 120$ 8
	0	$S_T - 100$	$120 - S_T$	0

b. If the investor carries out the butterfly strategy, what kind of bet is he or she making? How is this different from a straddle strategy that we discussed in class?

A butterfly is a gamble that the price will stay close to exercise price 3

A straddle is a gamble that stock price will have a large movement either up or down 3

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d) If you know the delta for each individual call option, can you find the delta of the butterfly strategy? How? What would this value reflect?

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Add / Subtract deltas of the call

Sensitivity of strategy price to change in underlying stock

Question 6 (20 points)

Consider the following:

Spot price for commodity	\$120
Futures price for commodity expiring in 1 year	\$125
Interest rate for 1 year	8%

a) Using the spots-futures parity condition, show that there is an arbitrage opportunity.

$F = P(1+0.08)$

$120(1+0.08) = 129.6 > 125$

5

b) Describe the transactions necessary to take advantage of the arbitrage opportunity and calculate the arbitrage profit.

The futures is too cheap => Buy futures

3	Sell commodity	120	$-S_T$
3	Long future	0	$S_T - 125$
3	invest in T bills	-120	$120(1.08) = 129.6$
		0	4.6

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c) Show how you would use the futures contract to create a riskless hedge against commodity price fluctuations.

Short position ~~$S_T - 125$~~ $125 - S_T$ 2

Long comm	110	120	125
Short futures	15	5	0
	125	125	125 => riskless hedge

3

5

-3 if involve T-bill

Question 7 (15 points)

According to the parity of spreads, does the futures price of a commodity with a longer maturity date always have to be greater than the futures price of the same commodity with a shorter maturity date? What conditions would it depend on? Explain your answer.

Not necessarily depends

2

Spread parity $F(T_2) = F(T_1)(1 + r_f - d)^{T_2 - T_1}$

5

Explanation

3 $r_f > d$
3 $r_f < d$
2 $\cancel{r_f = d}$ $d = 0$

8

Extra Credit (20 points)