Student No..................................

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EE431/438 Economics of Financial Markets and Institutions

Problem Set 6: Arbitrage Pricing Theory

Please submit at the BE office, 5th floor department of Economics building.

Deadline of submission : November 1th, 2013, before 15.00 hrs.

Late submission will not be accepted.

1. Assuming that the market is complete and perfect, what is the arbitrage equilibrium price of asset D, E, G, H in the example below? Hint: At equilibrium, no arbitrage opportunity exists.

	State 1	State 2	Price
Asset A	1	0	0.4
Asset B	0	1	0.5
Asset C	2	0	= (...2..×..0.4..) + (..0..×..0..) =.....0.8....
Asset D	0	2	= (.......×.......) + (.......×.......) =............
Asset E	2	2	= (.......×.......) + (.......×.......) =............
Asset G	3	1	= (.......×.......) + (.......×.......) =............
Asset H	1	1	= (.......×.......) + (.......×.......) =............

Read the following note.

- Notice that, at equilibrium, price of an asset $= p_1Q_1 + p_2Q_2$, where
- p_1 is the price of security paying \$1 if state 1 occurs and nothing if any other state occurs (the price of security A in this example),
  - p_2 is the price of security paying \$1 if state 2 occurs and nothing if any other state occurs (the price of security B in this example),
  - Q_1 is the quantity of dollars the asset paid if state 1 occurs,
  - Q_2 is the quantity of dollars the asset paid if state 2 occurs.
- If there are n states, price of an asset $=\sum_{i=1}^{n} p_iQ_i$, where
  - p_i is the price of security paying \$1 if state i occurs and nothing if any other state occurs,
  - Q_i is the quantity of dollars the asset paid if state i occurs.
- Security that pays \$1 if a specified state occurs and nothing if any other state occurs is called "a pure security". In this example, security A and security B are pure securities.
- It is not necessary that $p_1 + p_2 = 1$. There are more than 2 states in this example. The given information is sufficient to calculate the price of any asset which pays some money when state 1 and/or state 2 occur(s).

2. Using the information given in question 1, is there any arbitrage opportunity if the price of security C is equal to 1? If yes, explain how an investor can make an arbitrage profit.

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3. Using the information given in question 1,is there any arbitrage opportunity if the price of security C is equal to 0.6? If yes, how an investor can make an arbitrage profit? Explain.

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4. You are given the following information.

Security	Payoff		Security Prices
	State 1	State 2	
j	\$0	\$8	4
k	9	4	5
Asset 1	1	0	?
Asset 2	0	1	?
Asset C	5	4	?

(a) What are the prices of asset 1 and asset 2? Hint: See the note in question 1.

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(b) What is the equilibrium price of a third security C? Hint: See the note in question 1.

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5. The returns on stocks A and B are determined by the following one-factor model:

$ER_A = 0.05 + 0.5F_1$; asset A has $\beta_{A1} = 0.5$.

$ER_B = 0.02 - 0.25F_1$; asset B has $\beta_{B1} = -0.25$.

Notation: β_{j1} and β_{j1} for j = A; B denote the responses of the rates of return on assets A and B to the factor 1 (F_1).

(a) Determine the portfolio weights you need to place on A and B in order to construct a portfolio which has zero exposure to the factor 1 (F_1) (a risk-free portfolio). What is the expected return of the risk-free portfolio?

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(b) From (a), is there any abritrage opportunity exist if the market risk-free interest rate is 1%? If yes, explain how an investor can make an arbitrage profit.

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(c) Determine the portfolio weights you need to place on A and B in order to construct a portfolio which has a unit exposure to the factor 1 (F_1). What is the expected return of the portfolio which has a unit exposure to the factor 1?

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(d) Derive an equation for the equilibrium expected rate of return of an asset K which has $\beta_{K1} = b_K$. (Hint: $ER_i = R_f + (\lambda_1 - R_f)\beta_{i1}$, where λ_1 is the expected rate of return of the portfolio which as a unit exposure to the factor 1.)

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6. Suppose $R_i = 0.05 + 0.5F_1 = 0.075$. Find the value of F_1.

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7. The following information is provided for a stock market in which asset returns respond to one factor:

Asset i	β_{i1}	Expected Return on Asset i ($E(R_i)$)
Asset M	1	$0.10 = E(R_m) = \lambda_1$
Asset R_f	0	$0.06 = R_f$
Asset A	0.2	?

Find the expected rate of return on asset A. (Hint: $ER_i = R_f + (\lambda_1 - R_f)\beta_{i1}$, where λ_1 is the expected rate of return of the portfolio which as a unit exposure to the factor 1 ($\beta_{i1} = 1$).)

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Note that $(\lambda_1 - R_f)$ is the risk premium for factor 1.

8. Suppose that the table below reflects information for a stock market in which assets' rates of return respond to **two factors**:

	Expected rate of return on asset	Rate of change of the return on asset with respect to factor 1 (β_1)	Rate of change of the return on asset with respect to factor 2 (β_2)
Asset 1	0.02	0	0
Asset 2	0.17	1	0
Asset 3	0.13	1	1
Asset 4	?	0	1

Assuming that the conditions of Arbitrage Pricing Theory (APT) hold, calculate the expected rate of return of the asset 4.

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