Chapter 5
How Do Risk and Term Structure Affect Interest Rates?

■ Answers to End-of-Chapter Questions

1. In investment, a corporate bond rating represents the creditworthiness of the corporate bond. The ratings are published by credit rating agencies and used by investment professionals to assess the likelihood the debt will be repaid. Generally, the lower is the corporate bond rating, the higher the likelihood of default and thus the higher the bond's yield. Risk premium is defined as the return in excess of the risk-free rate of return that an investment is expected to yield. An asset's risk premium is a form of compensation for investors to tolerate the extra risk – compared to that of a risk free investment. Lower corporate bond ratings mean that the yield of the corporate bonds will be higher and thus causes the risk premium (bond yield – risk-free rate) to increase.

2. Bonds and Treasury bills can be compared based on the following criteria:

   Ratings – Both corporate bonds and Treasury bills are given ratings but Treasury bills are considered safer as it is backed by the government.

   Compensation to investors – U.S. bonds pay out fixed coupons on a periodic basis depending on the maturity period of the bond. Treasury bills however compensate usually on a zero-coupon basis which is the difference between the purchase price of the bill and the face value at maturity.

   Volatility – Treasury bills are safer and less volatile compared to corporate bonds

   Liquidity – Treasury bills are more liquid compared to bonds as there are more buyers and sellers.

3. Corporate bonds and stocks can be a good or bad combination based on the timing of the investments concerned. During times of increasing interest rates, the bond value will go down, so it is better to purchase stocks and vice versa.

   Bonds and equity shares have different characteristics that can complement each other in a portfolio held by an investor. Equity shares are more risky compared to government bonds in which bonds can generate a fixed income. A portfolio should be balanced and should include a combination of risky and less risky assets.

4. Bond prices and the interest rates have an inverse relationship. When interest rates go up bond prices drop, and vice versa. During the economic recession, interest rates in general decline during recessions, increasing bond prices. The main reason for this is the sign of flight to quality. This was seen in the rise of Treasury bond prices in the wake of the Global Financial Crisis of 2008.

5. If yield curves on average were flat, this would suggest that the risk premium on long-term relative to short-term bonds would equal zero and we would be more willing to accept the pure expectations theory.

6. The flat yield curve at shorter maturities suggests that short-term interest rates are expected to fall moderately in the near future, while the steep upward slope of the yield curve at longer maturities indicates that interest rates further into the future are expected to rise. Because interest rates and expected inflation move together, the yield curve suggests that the market expects inflation to fall moderately in the near future but to rise later on.

7. The steep upward-sloping yield curve at shorter maturities suggests that short-term interest rates are expected to rise moderately in the near future because the initial, steep upward slope indicates that the average of expected short-term interest rates in the near future is above the current short-term interest rate. The downward slope for longer maturities indicates that short-term interest rates are eventually expected to fall sharply. With a positive risk premium on long-term bonds, as in the liquidity premium theory, a downward slope of the yield curve occurs only if the average of expected short-term interest rates is declining, which occurs only if short-term interest rates far into the future are falling. Since interest rates and expected inflation move together, the yield curve suggests that the market expects inflation to rise moderately in the near future but fall later on.

8. Corporate bond generates yield which is tax deductible and defined as net redemption yield. However the quoted yield on corporate bonds are quoted gross and defined as 'gross redemption yield'. The link between the income tax rates and the yield to maturity is that the net redemption yield reflects the tax chargeable to an individual.

9. The government guarantee will reduce the default risk on corporate bonds, making them more desirable relative to Treasury securities. The increased demand for corporate bonds and decreased demand for Treasury securities will lower interest rates on corporate bonds and raise them on Treasury bonds.

10. Lower brokerage commissions for corporate bonds would make them more liquid and thus increase their demand, which would lower their risk premium.

11. Abolishing the tax-exempt feature of municipal bonds would make them less desirable relative to Treasury bonds. The resulting decline in the demand for municipal bonds and increase in demand for Treasury bonds would raise the interest rates on municipal bonds, while the interest rates on Treasury bonds would fall.

■ Quantitative Problems

1. a. The yield to maturity would be 5% for a one-year bond, 6% for a two-year bond, 6.33% for a three-year bond, 6.5% for a four-year bond, and 6.6% for a five-year bond.

b. The yield to maturity would be 5% for a one-year bond, 4.5% for a two-year bond, 4.33% for a three-year bond, 4.25% for a four-year bond, and 4.2% for a five-year bond.

The upward-sloping yield curve in (a) would be even steeper if people preferred short-term bonds over long-term bonds because long-term bonds would then have a positive risk premium. The downward-sloping yield curve in (b) would be less steep and might even have a slight positive upward slope if the long-term bonds have a positive risk premium.

2. Government economists have forecasted one-year T-bill rates for the following five years as follows:

Year	1-year rate
1	4.25%
2	5.15%
3	5.50%
4	6.25%
5	7.10%

You have liquidity premium 0.25% for the next two years and 0.50% thereafter. Would you be willing to purchase a 4-year T-bond at a 5.75% interest rate?

Solution: Your required interest rate on a 4-year bond = Average interest on four 1-year bonds + Liquidity Premium

= (4.25% + 5.15% + 5.50% + 6.25%)/4 + 0.5%

= 5.29% + 0.50% = 5.79%

At a rate of 5.75%, the T-bond is just below your required rate.

3. What is the yield on a $1,000,000 municipal bond with a coupon rate of 8%, paying interest annually, versus the yield of a $1,000,000 corporate bond with a coupon rate of 10% paying interest annually? Assume that you are in the 25% tax bracket.

Solution: Municipal bond coupon payments equal $80,000 per year. No taxes are deducted; therefore, the yield would equal 8%.

The coupon payments on a corporate bond equal $100,000 per year. But you only keep $75,000 because you are in the 25% tax bracket. Therefore your after-tax yield is only 7.5%

4. Consider the decision to purchase either a 5-year corporate bond or a 5-year municipal bond. The corporate bond is a 12% annual coupon bond with a par value of $1,000. It is currently yielding 11.5%. The municipal bond has an 8.5% annual coupon and a par value of $1,000. It is currently yielding 7%. Which of the two bonds would be more beneficial to you? Assume that your marginal tax rate is 35%.

Solution: *Municipal Bond*

Purchase Price	= $1,061.50
After-tax Coupon Payment	= $85
Par Value	= $1,000
Calculated YTM	= 7%

Corporate Bond

Purchase Price	= $1,018.25
After-tax Coupon Payment	= $78
Par Value	= $1,000
Calculated YTM	= 7.35%

The corporate bond offers a higher yield and is the better buy.

5. Debt issued by Southwest Airways currently yields 24%. A municipal bond of equal risk currently yields 16%. At what marginal tax rate would an investor be indifferent between these two bonds?

Solution:

Corporate bonds (1-tax rate) = Municipal bonds

24% × (1- tax rate) = 16%

1− tax rate = (16% / 24%)

1 – tax rate = 0.67

Tax rate = 1 – 0.67

Tax rate = 0.33

6. One-year T-bill rates are expected to steadily increase by 250 basis points per year over the next nine years. Determine the required interest rate on a five-year T-bond and a nine-year T-bond if the current one-year interest rate is 15.5%. Assume that the expectations hypothesis for interest rates holds.

    1. **Solution:**

    2. 5-year bond:

Year 1 interest rate = 15.5%

Year 2 interest rate = 18.0%

Year 3 interest rate = 20.5%

Year 4 interest rate = 23%

Year 5 interest rate = 25.5%

Number of years = 5

$r_5 = (15.5\% + 18.0\% + 20.5\% + 23\% + 25.5\%)/5 = 20.5\%$

9-year bond:

Year 1 interest rate = 15.5%

Year 2 interest rate = 18.0%

Year 3 interest rate = 20.5%

Year 4 interest rate = 23%

Year 5 interest rate = 25.5%

Year 6 interest rate = 28%

Year 7 interest rate = 30.5%

Year 8 interest rate = 33%

Year 9 interest rate = 35.5%

Number of years = 9

$r_9 = (15.5\% + 18.0\% + 20.5\% + 23\% + 25.5\% + 28\% + 30.5\% + 33\% + 35.5\%)/9 = 25.5\%$

7. The one-year interest rate over the next eight years will be 4%, 5.5%, 6%, 8.5%, 10%, 11.5%, 14%, and 15.5%. Using the expectations theory, what will be the interest rates on a four-year bond, a six-year bond, and an eight-year bond?

Solution:

$0.355 = (1.04 \times 1.05 \times 1.06 \times 1.065 \times 1.08) - 1 + \text{LP}$

$0.355 = 0.3314 + \text{LP}$
$\text{LP} = 0.0236 \text{ or } 2.36\%$

8. Using the information from the previous question, now assume that the investor prefers holding short-term bonds. A liquidity premium of 10 basis points is required for each year of a bond's maturity. What will be the interest rates on a 3-year bond, 6-year bond, and 9-year bond?

   Solution: To solve this problem, you will need to use the following equation:

$$i_{nt} = l_{nt} + \frac{i_t + i^e_{t+1} + i^e_{t+2} + \cdots + i^e_{t+n-1}}{n}$$

   3-year bond = (0.30) + [(3 + 4.5 + 6)]/(3) = 4.8%
   6-year bond = (0.60) + [(3 + 4.5 + 6 + 7.5 + 9 + 10.5)]/(6) = 7.35%
   9-year bond = (0.90) + [(3 + 4.5 + 6 + 7.5 + 9 + 10.5 + 13 + 14.5 + 16)]/(9) = 10.233%

9. Which bond would produce a greater return if the pure expectations theory was to hold true, a 2-year bond with an interest rate of 15% or two 1-year bonds with sequential interest payment of 13% and 17%?

   Solution: Both of the bonds would produce the same return.

   Two 2-year bonds:
   = (13% + 17%)/2
   = 15%

10. Little Monsters Inc. borrowed $1,000,000 for two years from NorthernBank Inc. at an 11.5% interest rate. The current risk-free rate is 2% and Little Monsters's financial condition warrants a default risk premium of 3% and a liquidity risk premium of 2%. The maturity risk premium for a two-year loan is 1%, and inflation is expected to be 3% next year. What does this information imply about the rate of inflation in the second year?

    Solution: If inflation were expected to remain constant at 3% over the life of the loan, the interest rate on the two-year loan would be 11%. Since the actual two-year interest rate is 11.5%, the one-year interest rate in year 2 must be 12%, since 11.5 = (11 + 12)/2.

    The required rate of 12% = R_f + DRP + LP + MRP + Inflation Premium
    = 2% + 3% + 2% + 1% + Inflation Premium

    So, the Inflation Premium in year 2 is 4%. But this is an average premium over two years.
    Inflation Premium 4% = (Year 1 Inflation + Year 2 Inflation)/2
    = (3% + x)/2

    or

    $x = 5\%$

11. One-year T-bill rates are 2% currently. If interest rates are expected to go up after 3 years by 2% every year, what should be the required interest rate on a 10-year bond issued today?

Solution: I (10-year bond) $= \dfrac{2+2+2+2(1.02)+2(1.02)^2+\cdots+2(1.02)^7}{10}$

$= 21.165/10 = 2.1165\%$

12. One-year T-bill rates over the next 4 years are expected to be 3%, 4%, 5%, & 5.5%. If 4-year T-bonds are yielding 4.5%, what is the liquidity premium on this bond?

Solution:

$4.5\% = (3\% + 4\% + 5\% + 5.5\%)/4 + LP$
$4.5\% = 4.375\% + LP$
$0.125\% = LP$

13. At your favorite bond store, Bonds-R-Us, you see the following prices:
    a. 1-year $100 zero selling for $90.19
    b. 3-year 10% coupon $1000 par bond selling for $1000
    c. 2-year 10% coupon $1000 par bond selling for $1000

    Assume that the pure expectations theory for the term structure of interest rates holds, no liquidity or maturity premium exists, and the bonds are equally risky. What is the implied 1-year rate two years from now?

Solution: From (a), you know that the 1-year rate today is 10.877%.

Using this information, (c) tells you that:

$1000 = 100/1.10877 + 1100/(1 + \text{2-year rate})^2$

So, the 2-year rate today is 9.95%.

Using these two rates, (b) tells you that:

$1000 = 100/1.10877 + 100/1.0995^2 + 1100/(1 + \text{3-year rate})^3$

So, the 3-year rate today is 9.97%

1-year rate 2 years from now = $(3 \times 9.97\% - 2 \times 9.95\%) = 10.01\%$

14. You observe the following market interest rates, for both borrowing and lending:

    one-year rate = 5%

    two-year rate = 6%

    one-year rate one year from now = 7.25%

    How can you take advantage of these rates to earn a riskless profit? Assume that the Pure Expectation Theory for interest rates holds.

Solution: Borrow $100 today at the two-year rate. You will be required to payback $\$100 \times (1.06)^2$, or $112.36 in two years.

Simultaneously, lend the $100 you borrowed at the one-year rate of 5%. A year later, you will have $105. Again, lend the $105 at the one-year rate of 7.25%. At the end of that year, you will have $\$105 \times (1.0725) = \$112.6125$.

Use the $112.6125 to payoff the $112.36 you owe. This leaves $0.2525 in profit. Recall that this required no cash upfront. As long as the future rate of 7.25% is guaranteed, you can risklessly make a profit.

15. The expected one-year interest rate two years from now is

$$i^e_{t+2} = [(1 + i_{3t} - k_{3t})^3/(1 + i_{2t} - k_{2t})^2] - 1$$

$$= [(1 + 0.06 - 0.0035)^3/(1 + 0.05 - 0.0025)^2] - 1$$
$$= 0.075 = 7.5\%.$$