Chapter 15

Risk and Information

Chapter Fifteen Overview

1. Introduction: *Amazon.com*

2. Describing Risky Outcome – Basic Tools
    - *Lotteries and Probabilities*
    - *Expected Values*
    - *Variance*

3. Evaluating Risky Outcomes
    - *Risk Preferences and the Utility Function*

4. Avoiding and Bearing Risk
    - *The Demand for Insurance and the Risk Premium*
    - *Asymmetric Information and Insurance*
    - *The Value of Information and Decision Trees*

Tools for Describing Risky Outcomes

Definition: A **lottery** is any event with an uncertain outcome.

Examples: Investment, Roulette, Football Game.

Definition: A **probability** of an outcome *(of a lottery)* is the likelihood that this outcome occurs.

Example: The probability often is estimated by the historical frequency of the outcome.

Probability Distribution

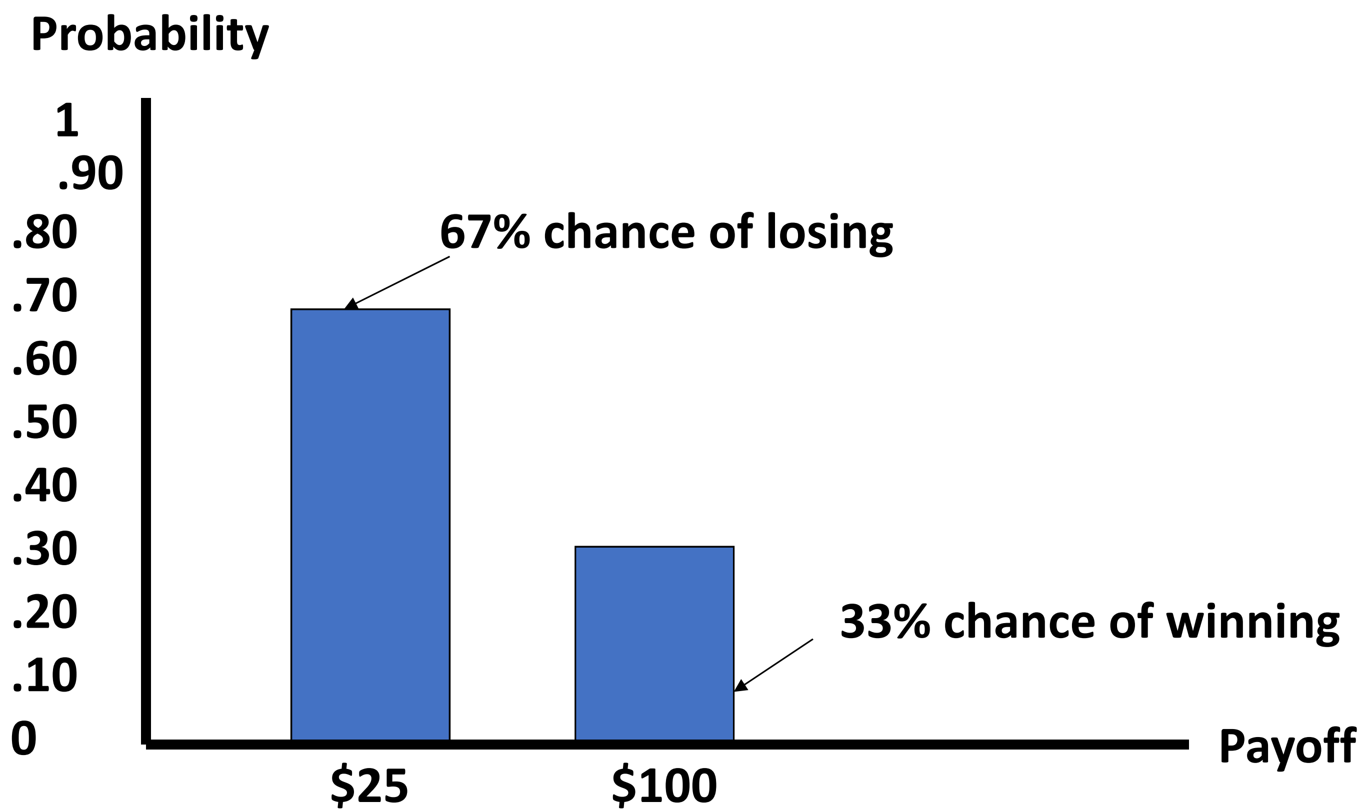
Definition: The **probability distribution** of the lottery depicts all possible payoffs in the lottery and their associated probabilities.

Property:

- The probability of any particular outcome is between 0 and 1
- The sum of the probabilities of all possible outcomes equals 1.

Definition: Probabilities that reflect subjective beliefs about risky events are called **subjective probabilities**.

Probability Distribution



Expected Value and Expected Utility

Definition: The **expected value** of a lottery is a measure of the average payoff that the lottery will generate.

Let X be a lottery with three possible outcomes A, B, and C.

$$E[X] = Pr(A) \cdot A + Pr(B) \cdot B + Pr(C) \cdot C$$

Definition: The **expected utility** of a lottery is the average value of the utility levels that the lottery will generate.

$$E[U(X)] = Pr(A) \cdot U(A) + Pr(B) \cdot U(B) + Pr(C) \cdot U(C)$$

Expected Value

In our example lottery, which pays \$25 with probability .67 and \$100 with probability 0.33, the expected value is:

EV = .67 x \$25 + .33 x 100 = \$50.

Notice that the expected value need not be one of the outcomes of the lottery.

Variance & Standard Deviation

Definition: The **variance** of a lottery is the sum of the probability-weighted squared deviations between the possible outcomes of the lottery and the expected value of the lottery. It is a measure of the lottery's riskiness.

$$Var = (A - EV)^2(Pr(A)) + (B - EV)^2(Pr(B)) + (C - EV)^2(Pr(C))$$

Definition: The **standard deviation** of a lottery is the square root of the variance. It is an alternative measure of risk

Variance & Standard Deviation

For the example lottery

The squared deviation of winning is:

- $(\$100 - \$50)^2 = 50^2 = 2500$

The squared deviation of losing is:

- $(\$25 - \$50)^2 = 25^2 = 625$

The variance is:

- (2500 x .33)+ (625 x .67) = 1250

Evaluating Risky Outcomes

<u>Example:</u> Work for IBM or Amazon.com?

Suppose that individuals facing risky alternatives attempt to maximize expected utility, i.e., the probability-weighted average of the <u>utility</u> from each possible outcome they face.

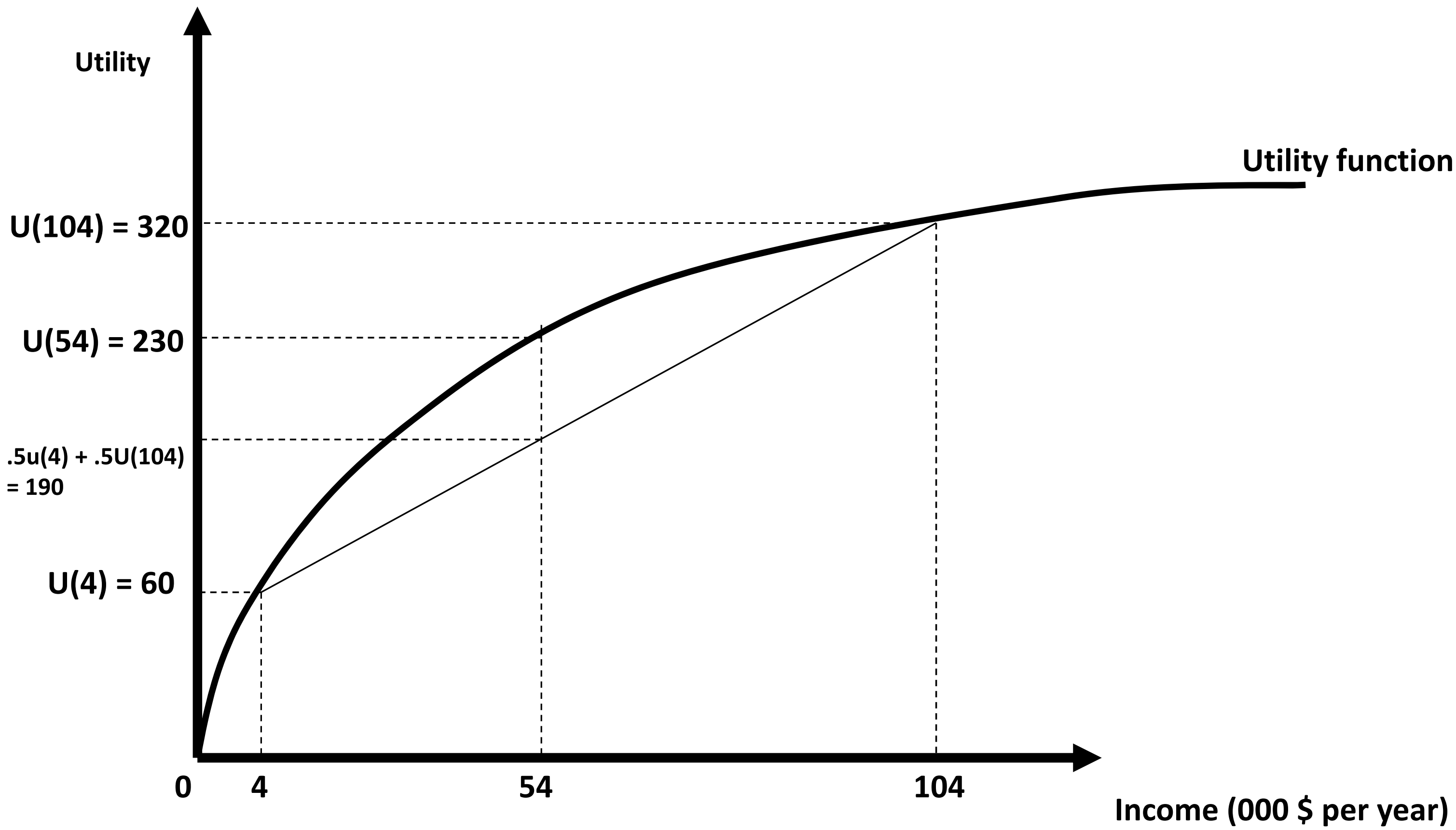
U(IBM) = U($54,000) = 230

U(Amazon) =

.5xU($4,000) + .5xU($104,000) = .5(60) + .5(320) = 190

Note:
EV(Amazon) =
.5($4000)+.5($104,000) = $54,000

Evaluating Risky Outcomes



Risk Preferences

Notes:

- Utility as a function of yearly income only
- Diminishing marginal utility of income

Definition: The risk preferences can be classified as follows:

An individual who prefers a sure thing to a lottery with the same expected value is **risk averse**

An individual who is indifferent about a sure thing or a lottery with the same expected value is **risk neutral**

An individual who prefers a lottery to a sure thing that equals the expected value of the lottery is **risk loving** (or **risk preferring**)

Risk Preferences

Definition: The risk preferences can be classified as follows:

An individual who prefers a sure thing to a lottery with the same expected value is **risk-averse.**

An individual who is indifferent about a sure thing or a lottery with the same expected value is **risk-neutral.**

An individual who prefers a lottery to a sure thing that equals the expected value of the lottery is **risk-loving.**

Risk Preferences

Let X be a lottery.

U(E[X]) denotes **Utility of the EV (sure thing) of the lottery X.**

E[U(X)] denotes **Expected Utility of the lottery X.**

risk-averse: **U(E[X]) > E[U(X)]**

happiness from sure thing > expected happiness from lottery

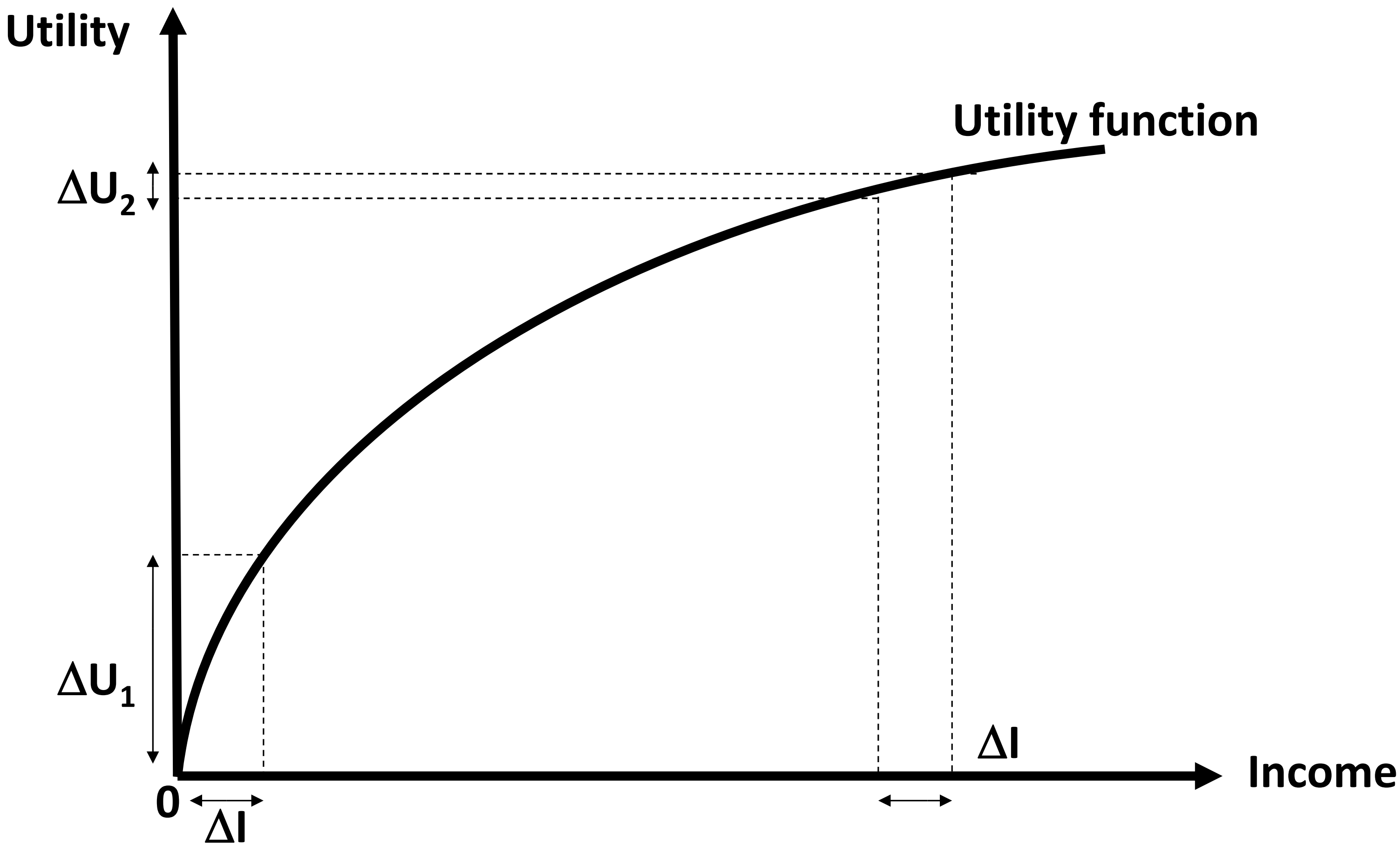
risk-neutral: **U(E[X]) = E[U(X)]**

happiness from sure thing = expected happiness from lottery

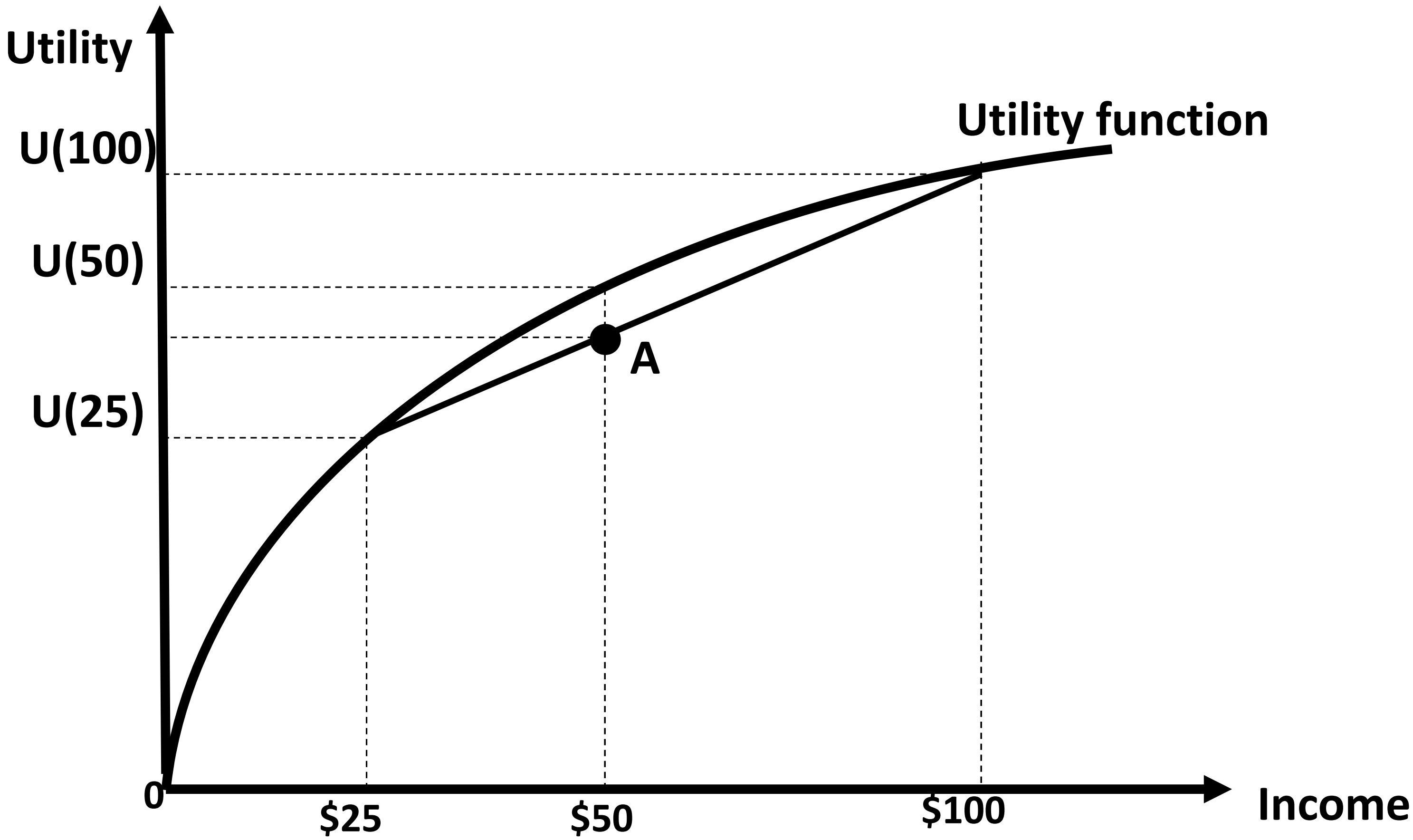
risk-loving: **U(E[X]) < E[U(X)]**

happiness from sure thing < expected happiness from lottery

Utility Function – Risk Averse Decision Maker

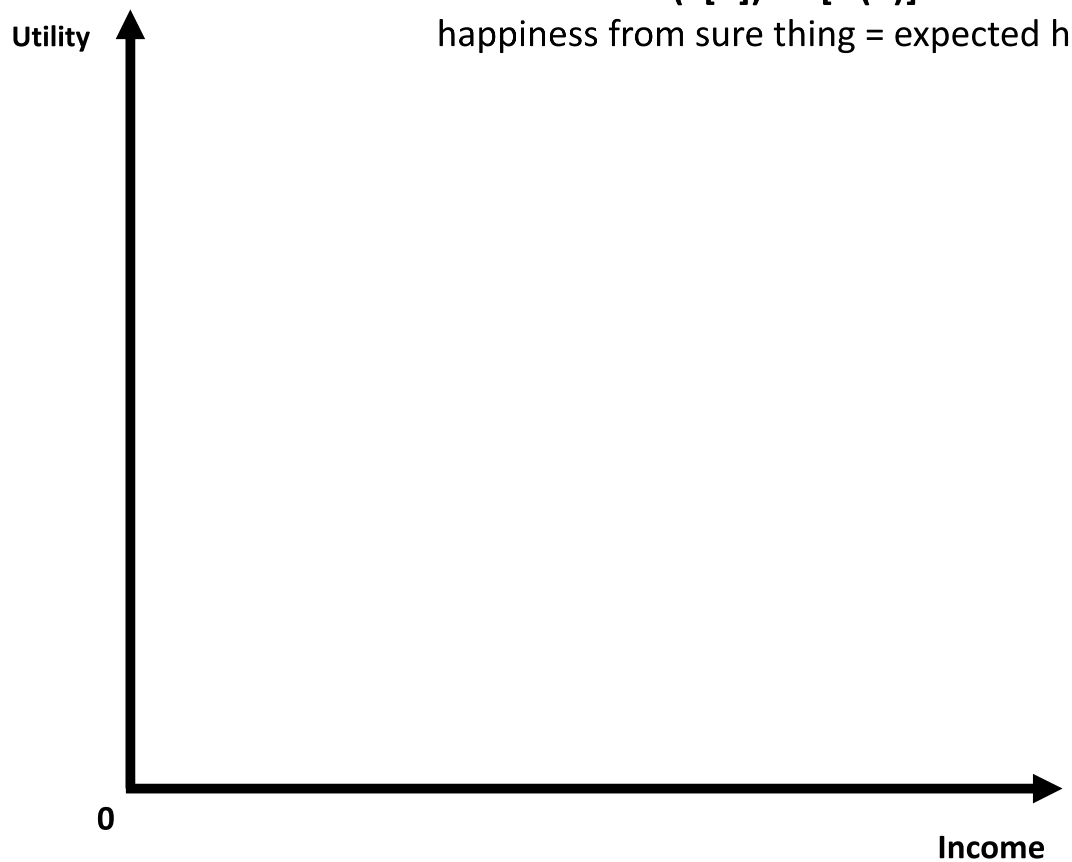


Utility Function – Risk Averse Decision Maker



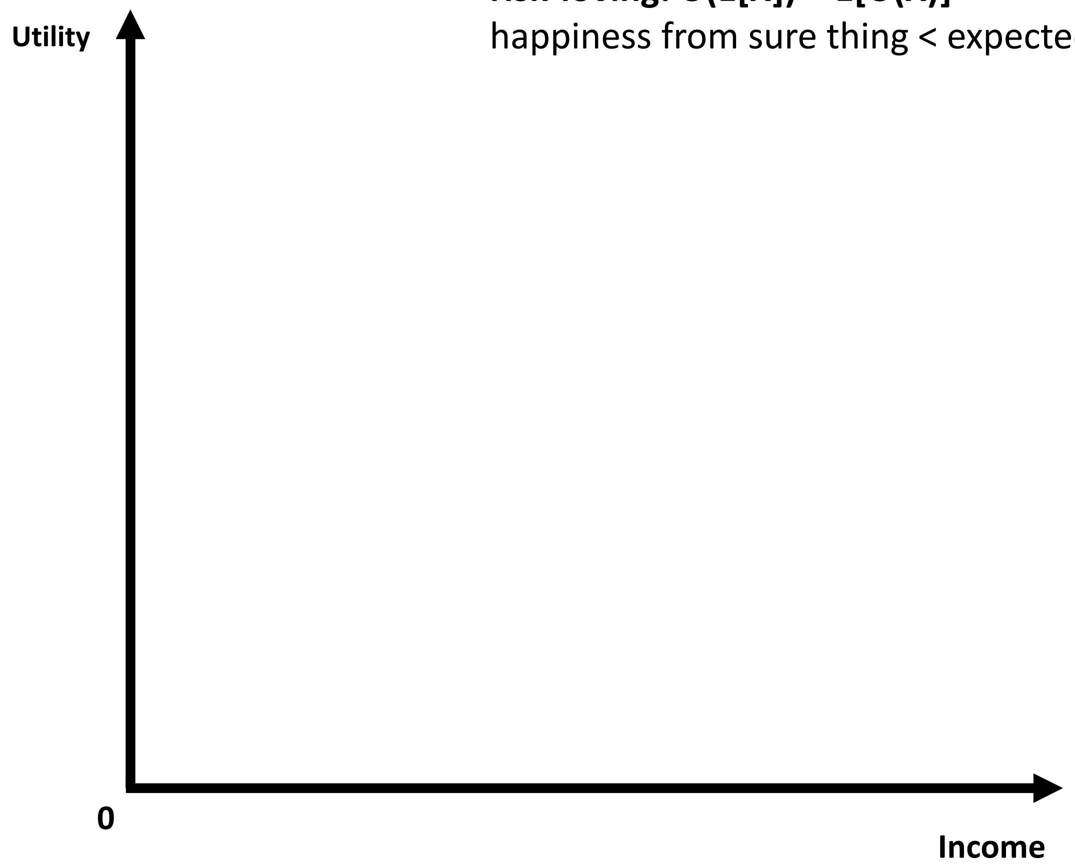
Risk-Neutral Individuals

risk-neutral: U(E[X]) = E[U(X)]
happiness from sure thing = expected happiness from lottery



Risk-Loving Individuals

risk-loving: U(E[X]) < E[U(X)]
happiness from sure thing < expected happiness from lottery



Risk Preferences

Examples

Suppose that an individual must decide between buying one of two stocks: the stock of an Internet firm and the stock of a Public Utility. The values that the shares of the stock may take (and, hence, the income from the stock, I) and the associated probability of the stock taking each value are:

Internet firm		**Public Utility**	
I	Probability	I	Probability
$80	.3	$80	.1
$100	.4	$100	.8
$120	.3	$120	.1

Risk Preferences

Examples

Which stock should the individual buy if she has utility function U = (100I)1/2? Which stock should she buy if she has utility function U = I?

EU(Internet) = .3U(80) + .4U(100) + .3U(120)

EU(P.U.) = .1U(80) + .8U(100) + .1U(120)

a. U = (100I)1/2:

- U(80) = (8000)1/2 = 89.40
- U(100) = (10000)1/2 = 100
- U(120) = (12000)1/2 = 109.5

Risk Preferences

Examples

→ EU(Internet) = .3(89.40)+.4(100)+.3(109.50) = 99.70

→ EU(P.U.) =

.1(89.40) + .8(100) + .1(109.50) = 99.9

The individual should purchase the public utility stock

Risk Preferences

Examples

U = I:

→ EU(Internet) = .3(80)+.4(100)+.3(120)=100

→ EU(P.U.)

.1(80) + .8(100) + .3(120) = 100

This individual is indifferent between the two stocks.

LEARNING-BY-DOING EXERCISE 15.1

Computing the Expected Utility for Two Lotteries

Consider the two lotteries depicted in Figure 15.3. They have the same expected value, but the first (investing in the Internet company's stock) has a larger variance than the second (investing in the public utility company's stock). This tells us that the first lottery is riskier than the second lottery. Suppose that a risk-averse decision maker has the utility function $U(I) = \sqrt{100I}$, where I denotes the payoff of the lottery.

Problem Which lottery does the decision maker prefer—that is, which one has the bigger expected utility?

One decision maker is risk neutral, with the utility function $U(I) = 100I$, while the other is risk loving, with the utility function $U(I) = 100I^2$, where I denotes the payoff of the lottery.

Problem

(a) Which lottery does the risk-neutral decision maker prefer?

(b) Which lottery does the risk-loving decision maker prefer?

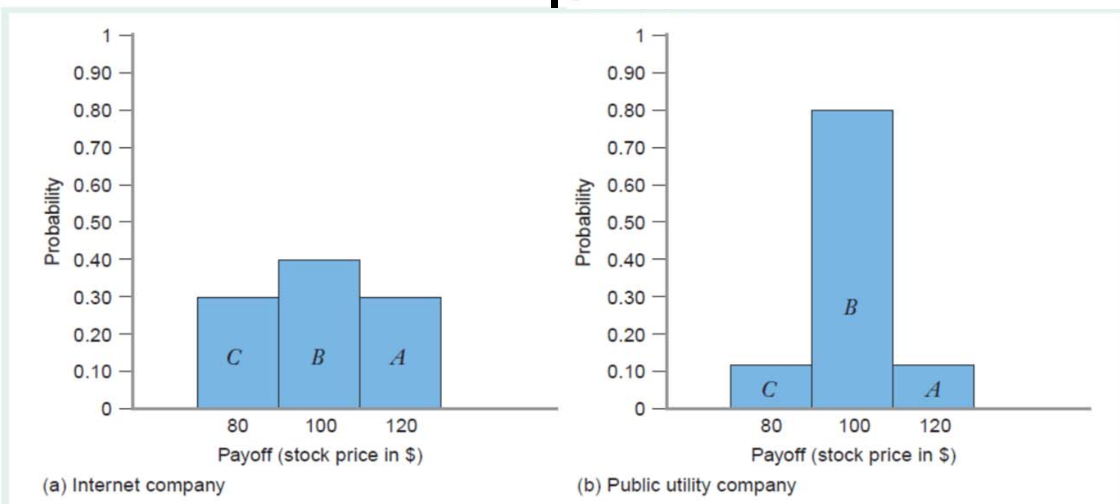


FIGURE 15.3 Probability Distributions, Riskiness, and Variances

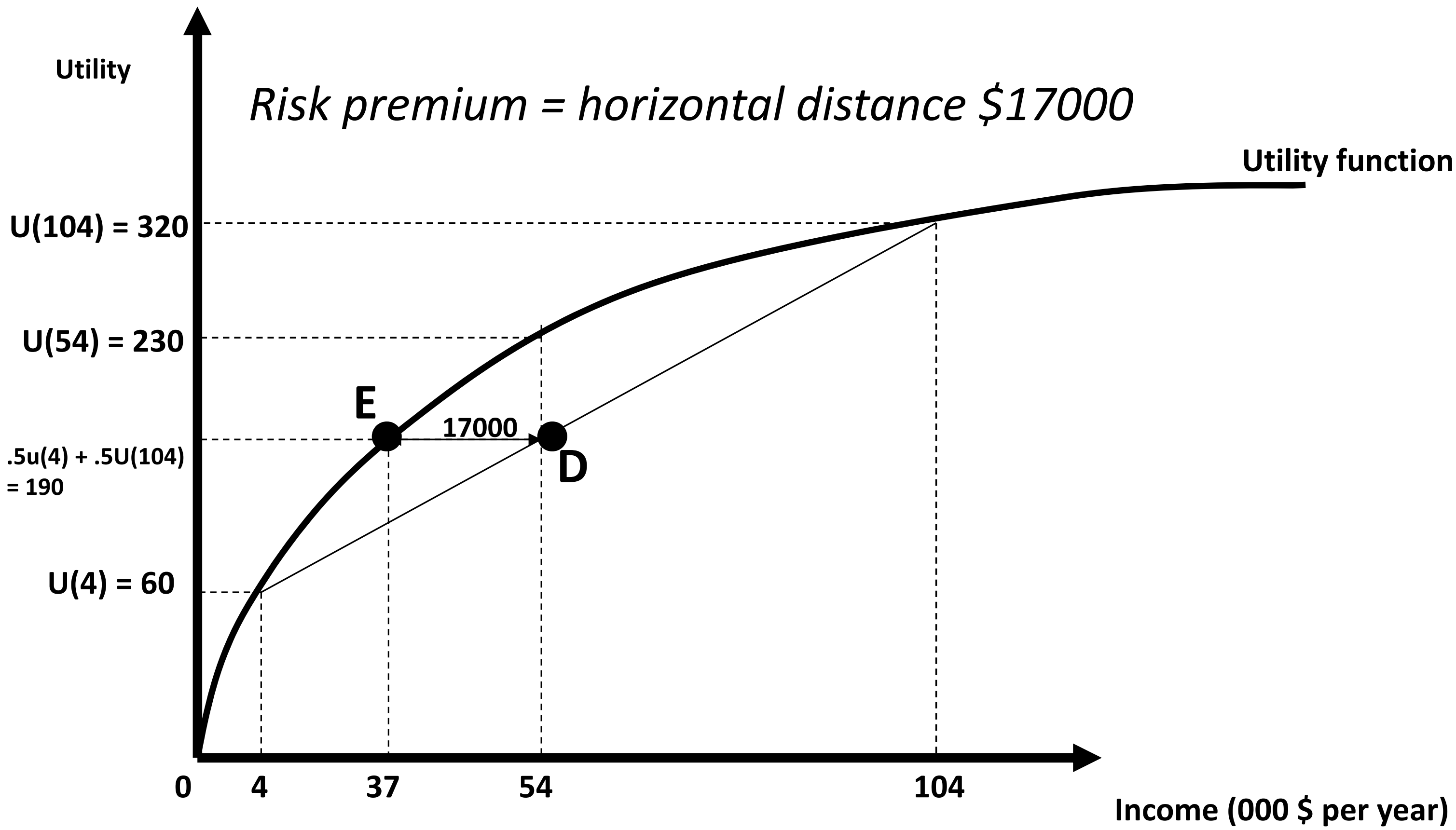
Risk Premium

Definition: The **risk premium** of a lottery is the necessary difference between the expected value of a lottery and the sure thing so that the decision maker is indifferent between the lottery and the sure thing.

$$pU(I_1) + (1-p)U(I_2) = U(pI_1 + (1-p)I_2 - RP)$$

The larger the variance of the lottery, the larger the risk premium

Avoiding Risk - Insurance



Computing Risk Premium

Example: Computing a Risk Premium

- $U = I^{(1/2)}$; $p = .5$
- $I_1 = \$104,000$
- $I_2 = \$4,000$

Computing Risk Premium

$pU(I_1) + (1-p)U(I_2) = U(pI_1 + (1-p)I_2 - RP)$

A. Verify that the risk premium for this lottery is approximately $17,000

.5(104,000)1/2 + .5(4,000)1/2 = (.5(104,000) + .5(4,000) - RP)1/2

$192.87 = ($54,000 - RP)1/2

$37,198 = $54,000 - RP

RP = $16,802

Computing Risk Premium

B. Let I_1 = \$108,000 and I_2 = \$0. What is the risk premium now?

.5(108,000)1/2 + 0 = (.5(108,000) + 0 - RP)1/2

.5(108,000)1/2 = (54,000 - RP)1/2

RP = \$27,000

(Risk premium rises when variance rises, EV the same…)

The Demand for Insurance

Lottery (If you do not buy the insurance)

\$50,000 if no accident (p = .95)
\$40,000 if accident (1-p = .05)

(i.e. "Endowment" is that income in the good state is 50,000 and income in the bad state is 40,000)

EV = .95(\$50000)+.05(\$40000) = \$49,500

The Demand for Insurance

Insurance:

Coverage = $10,000
Price = $500
$49,500 sure thing.

Why?

In a good state, receive 50000-500 = 49500

In a bad state, receive 50000-500-10000+10000=49500

The Demand for Insurance

If you are risk averse, you prefer to insure this way over no insurance. *Why?*

Full coverage (⇔ no risk so prefer all else equal)

Definition: A **fairly priced** insurance policy is one in which the insurance premium (price) **equals** the expected value of the promised payment. i.e.,

500 = .05(10,000) + .95(0)

LEARNING-BY-DOING EXERCISE 15.4

The Willingness to Pay for Insurance

Your current disposable income is $90,000. Suppose that there is a 1 percent chance that your house may burn down, and if it does, the cost of repairing it will be $80,000, reducing your disposable income to $10,000. Suppose, too, that your utility function is $U = \sqrt{I}$.

Problem

(a) Would you be willing to spend $500 to purchase an insurance policy that fully insures you against your loss?

(b) What is the highest price that you would be willing to pay for an insurance policy that fully insures you in the event that your house burns down?

The Supply of Insurance

Insurance company expects to break even and assumes all risk – why would an insurance company ever offer this policy?

Definition: **Asymmetric Information** is a situation in which one party knows more about its own actions or characteristics than another party.

Adverse Selection & Moral Hazard

Definition: **Adverse Selection** is *opportunism* characterized by an informed person's benefiting from trading or otherwise contracting with a less informed person who does not know about an *unobserved characteristic* of the informed person.

Definition: **Moral Hazard** is *opportunism* characterized by an informed person's taking advantage of a less informed person through an *unobserved action.*

Adverse Selection & Moral Hazard

Adverse Selection / Hidden Information:

- Good and Bad cars in second-hand garages.
- Healthy and Unhealthy people in health insurance markets.

Moral Hazard / Hidden Action:

- People drive carelessly after buying car insurances.
- Employers fails to observe lazing employees.
- Doctors over-prescribe treatments for patients with health insurances.

Decision Trees

Definition: A **decision tree** is a diagram that describes the options available to a decision maker, as well as the risky events that can occur at each point in time.

Key Elements

1. Decision Nodes
2. Chance Nodes
3. Probabilities
4. Payoffs

We analyze decision problems by working backward along the decision tree to decide what the optimal decision would Be.

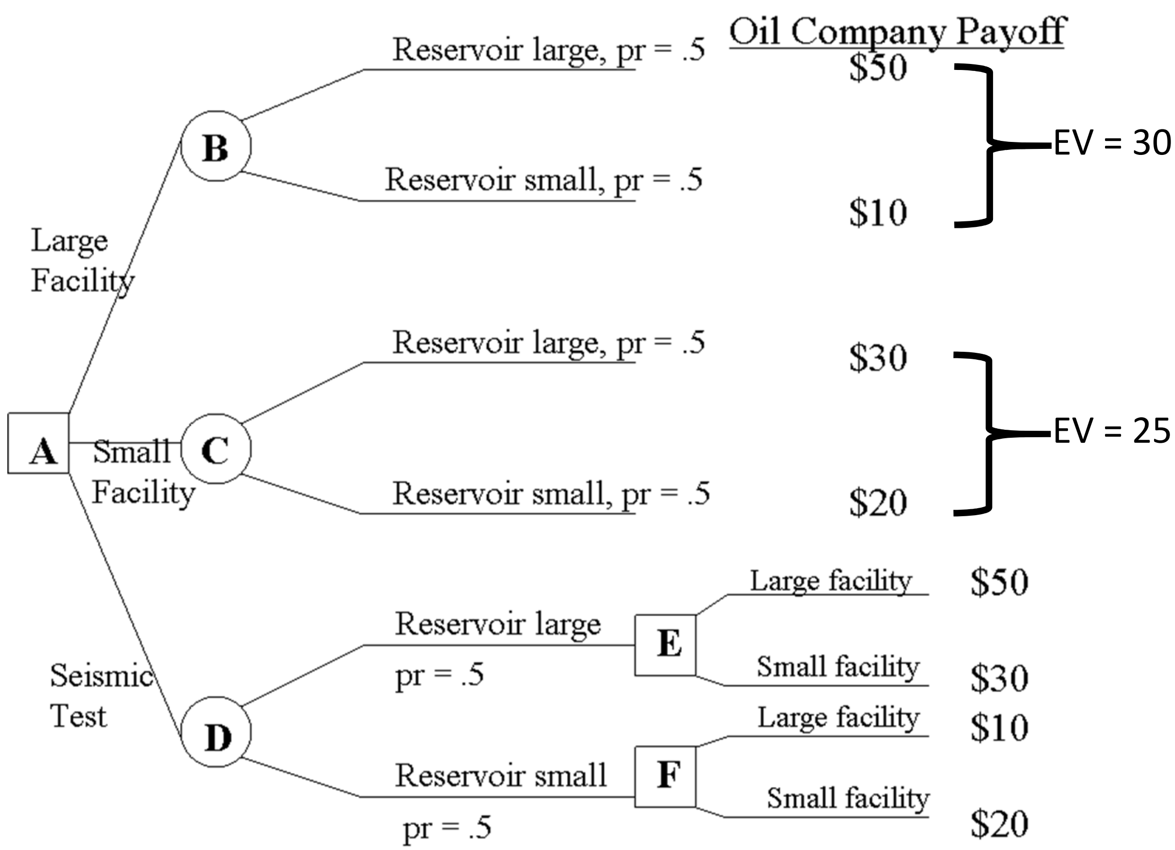
Decision Trees

- An oil company is considering whether it should build a large facility (good if the oil reservoir is large) or a small facility (good if the oil reservoir is small).

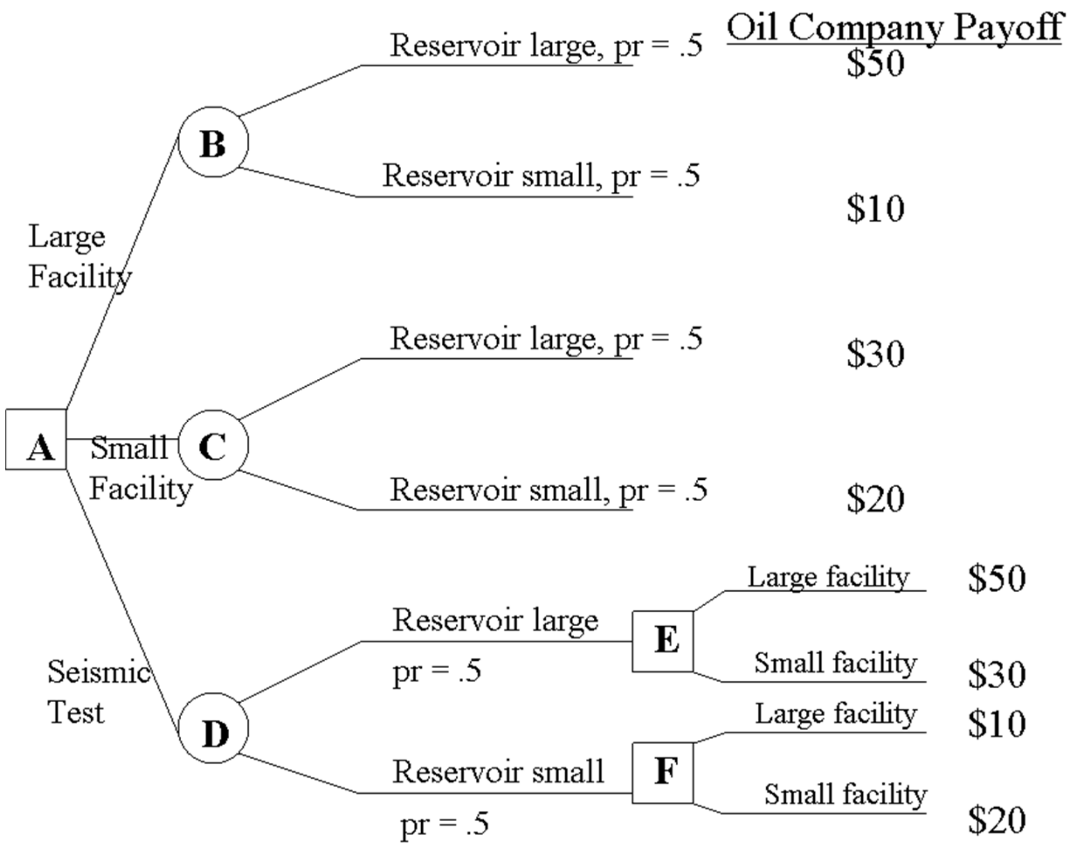
- Pr(Small Reservoir) = 0.5 and Pr(Large Reservoir) = 0.5.

- However, it can implement “free seismic test” before building a facility to determine if the reservoir is large or small.

Decision Trees



Decision Trees



Decision Trees

Steps in constructing and analyzing the tree:

1. Map out the decision and event sequence
2. Identify the alternatives available for each decision
3. Identify the possible outcomes for each risky event
4. Assign probabilities to the events
5. Identify payoffs to all the decision/event combinations
6. Find the optimal sequence of decisions

Perfect Information

Definition: The **value of perfect information** is the increase in the decision maker's expected payoff when the decision maker can -- at no cost -- obtain information that reveals the outcome of the risky event.

Perfect Information

Example:

- Expected payoff to conducting test: $35M
- Expected payoff to not conducting test: $30M

The value of information: $5M

The value of information reflects the value of being able to tailor your decisions to the conditions that will actually prevail in the future. It should represent the agent's willingness to pay for a "crystal ball".

Summary

1. We can think of risky decisions as lotteries.

2. We can think of individuals maximizing expected utility when faced with risk.

3. Individuals differ in their attitudes towards risk: those who prefer a sure thing are risk averse. Those who are indifferent about risk are risk neutral. Those who prefer risk are risk loving.

4. Insurance can help to avoid risk. The optimal amount to insure depends on risk attitudes.

Summary

5. The provision of insurance by individuals does not require risk lovers.

6. Adverse Selection and Moral Hazard can cause inefficiency in insurance markets.

7. We can calculate the value of obtaining information in order to reduce risk by analyzing the expected payoff to eliminating risk from a decision tree and comparing this to the expected payoff of maintaining risk.

8. The main types of auctions are private values auctions and common values auctions.