Sept 26th 2024

# Extensions of the Two-Variable Linear Regression Model
# Part II

# Extensions of the Two-Variable Linear Regression Model
# Part II

# Functional Forms of Regression Models

- The log-linear model
- Semilog models
- Reciprocal models
- The logarithmic reciprocal model

$$Y_i = \beta_1 + \beta_2 X_{2i} + u_i$$
$$= \underbrace{\hat{\beta}_1 + \hat{\beta}_2 X_{2i}}_{\hat{Y}_i} + \hat{u}_i$$

## The log-linear model

The exponential regression model:

$$Y_i = \beta_1 X_i^{\beta_2} e^{u_i} \qquad (1)$$

Which may be expressed alternatively as

$$\ln Y_i = \ln \beta_1 + \beta_2 \ln X_i + u_i \qquad (2)$$

$$\ln Y_i = \alpha_1 + \beta_2 \ln X_i + u_i \qquad (3)$$

$$where\ \alpha = \ln \beta_1$$

$$\ln(AB) = \ln A + \ln B$$

$$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$$

$$\ln(A^k) = k \ln A$$

Assuming that A and B are positive, and where $k$ is some constant.

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

$$\frac{dY}{dX} = \beta_2$$

$$\ln Y = \beta_1 + \beta_2 \ln X$$

$$\frac{1}{Y}\frac{dY}{dX} = \frac{\beta_2}{X}\frac{dX}{dX}$$

$$\beta_2 = \frac{X}{Y}\frac{dY}{dX} = \frac{dY/Y}{dX/X} = elasticity$$

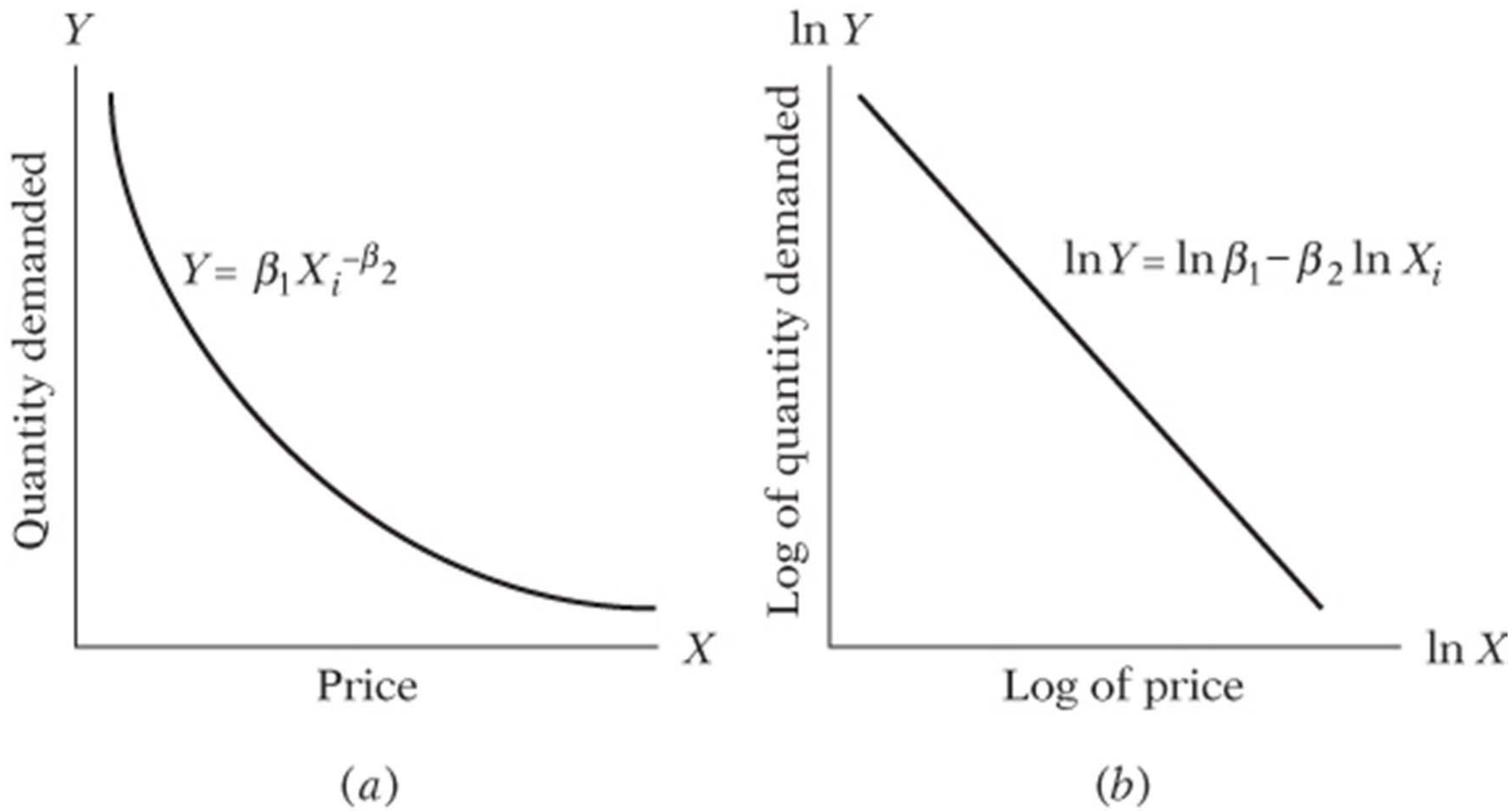
# The log-linear model

- The slope coefficient $\beta_2$ measures the elasticity of Y with respect to X, that is, the percentage change in Y for a given (small) percentage change in X

- If Y represents the quantity of a commodity demanded and X its unit price, $\beta_2$ measures the price elasticity of demand.

- If the relationship between quantity demanded and price is as shown in figure (a), the double-log transformation as shown in figure (b) will then give the estimate of the price elasticity $(-\beta_2)$

# The log-linear model



# Percentage change vs. Percentage point change

Example –The unemployment rate

The unemployment rate of 6%, if this rate goes to 8%, we say that **the percentage point change in the unemployment rate is 2**

**The percentage change in the unemployment rate is (8-6)/6, or about 33%**

- Absolute change
- Relative or proportional change
- Percentage change or percent growth rate.

$$(X_t - X_{t-1}) \Rightarrow \text{Absolute change}$$

$$\frac{(X_t - X_{t-1})}{X_{t-1}} = \frac{X_t}{X_{t-1}} - 1 \Rightarrow \text{Relative or proportional change}$$

$$\frac{(X_t - X_{t-1})}{X_{t-1}} \times 100$$ is the percentage change or the growth rate

$X_t$ : current values of the variable X

$X_{t-1}$ : previous values of the variable X

# Example

Expenditure on durable goods in relation to total personal consumption expenditure

- We wish to find the elasticity of expenditure on durable goods with respect to total personal consumption expenditure.

$$\ln Y_t = \alpha_1 + \ln X_t$$

$Y_t$ $X_t$ $\ln Y_t$ $\ln X_t$

## TABLE 6.3

**Total Personal Expenditure and Categories (Billions of chained [2000] dollars; quarterly data at seasonally adjusted annual rates)**

Sources: Department of Commerce, Bureau of Economic Analysis. *Economic Report of the President*, 2007, Table B-17, p. 347.

| Year or quarter | EXPSERVICES | EXPDUR | EXPNONDUR | PCEXP |
|---|---|---|---|---|
| 2003-I | 4,143.3 | 971.4 | 2,072.5 | 7,184.9 |
| 2003-II | 4,161.3 | 1,009.8 | 2,084.2 | 7,249.3 |
| 2003-III | 4,190.7 | 1,049.6 | 2,123.0 | 7,352.9 |
| 2003-IV | 4,220.2 | 1,051.4 | 2,132.5 | 7,394.3 |
| 2004-I | 4,268.2 | 1,067.0 | 2,155.3 | 7,479.8 |
| 2004-II | 4,308.4 | 1,071.4 | 2,164.3 | 7,534.4 |
| 2004-III | 4,341.5 | 1,093.9 | 2,184.0 | 7,607.1 |
| 2004-IV | 4,377.4 | 1,110.3 | 2,213.1 | 7,687.1 |
| 2005-I | 4,395.3 | 1,116.8 | 2,241.5 | 7,739.4 |
| 2005-II | 4,420.0 | 1,150.8 | 2,268.4 | 7,819.8 |
| 2005-III | 4,454.5 | 1,175.9 | 2,287.6 | 7,895.3 |
| 2005-IV | 4,476.7 | 1,137.9 | 2,309.6 | 7,910.2 |
| 2006-I | 4,494.5 | 1,190.5 | 2,342.8 | 8,003.8 |
| 2006-II | 4,535.4 | 1,190.3 | 2,351.1 | 8,055.0 |
| 2006-III | 4,566.6 | 1,208.8 | 2,360.1 | 8,111.2 |

*Note:* See Table B-2 for data for total personal consumption expenditures for 1959–1989.
EXPSERVICES = expenditure on services, billions of 2000 dollars.
EXPDUR = expenditure on durable goods, billions of 2000 dollars.
EXPNONDUR = expenditure on nondurable goods, billions of 2000 dollars.
PCEXP = total personal consumption expenditure, billions of 2000 dollars.

(*Continued*)

$\ln Y_i$ $\ln X_t$

$$\ln \widehat{EXDUR_i} = -7.5417 + 1.6266 \ln PCEX_t$$

As these result shows , the elasticity of EXPDUR with respect to PCEX is about 1.63, suggesting that **if total personal expenditure goes up by 1 percent, on average, the expenditure on durable goods goes up by about 1.63 percent**

Thus, expenditure on durable goods is very responsive to changes in personal consumption expenditure. This is one reason why producers of durable goods keep a keen eye on changes in personal income and personal consumption expenditure

## Semilog Models

- Log-Lin model

$$\ln Y_i = \beta_1 + \beta_2 X_i + u_i$$

- Lin-Log model

$$Y_i = \beta_1 + \beta_2 \ln X_i + u_i$$

## The Log-Lin model

$$\ln Y_i = \beta_1 + \beta_2 X_i + u_i$$

- The rate of growth of certain economic variables such as population, GNP, money supply employment, productivity, and trade deficit

Suppose we want to find out the growth rate of personal consumption expenditure on services

$Y_t$ denote real expenditure on services at time t and $Y_0$ the initial value of the expenditure on services

Recall the following well-known compound interest formula from your introductory course in economics

$$Y_t = Y_0(1 + r)^t$$

where $r$ is the compound rate of growth of $Y$

$$Y_t = Y_0(1+r)^t$$

Taking the natural logarithm, we can write

$$\ln Y_t = \ln Y_0 + t\ln(1+r) \qquad (1)$$

Now letting

$$\beta_1 = \ln Y_0$$
$$\beta_2 = \ln(1+r)$$

We can write as

$$\ln Y_t = \beta_1 + \beta_2 t \qquad (2)$$

We obtain

$$\boxed{\ln Y_t = \beta_1 + \beta_2 t + u_t} \qquad (3)$$

$$\ln Y = \beta_1 + \beta_2 X$$

$$\frac{1}{Y}\frac{dY}{dX} = \beta_2$$

$$\beta_2 = \frac{dY/Y}{dX}$$

- Using differential calculus one can show that

$$\beta_2 = \frac{d(lnY)}{dX} = \frac{\left(\frac{1}{Y}\right)}{\left(\frac{dY}{dX}\right)} = \frac{\left(\frac{dY}{Y}\right)}{dX}$$

For a small changes in Y and X this relation may be approximated by $\dfrac{\frac{Y_t - Y_{t-1}}{Y_{t-1}}}{(X_t - X_{t-1})}$

Relative change

Absolute change

$$\beta_2 = \frac{\text{relative change in regressand}}{\text{absolute change in regressor}}$$

$\beta_2$ *is known as the semielasticity of Y with respect to X*

$100 * \beta_2$ *is known as the semielasticity of Y with respect to X*

# Example

- We want to find out the growth rate of personal consumption expenditure on services for the data
- The regression results over time (t) are as follows:

**TABLE 6.3**
**Total Personal Expenditure and Categories (Billions of chained [2000] dollars; quarterly data at seasonally adjusted annual rates)**

Sources: Department of Commerce, Bureau of Economic Analysis. *Economic Report of the President*, 2007, Table B-17, p. 347.

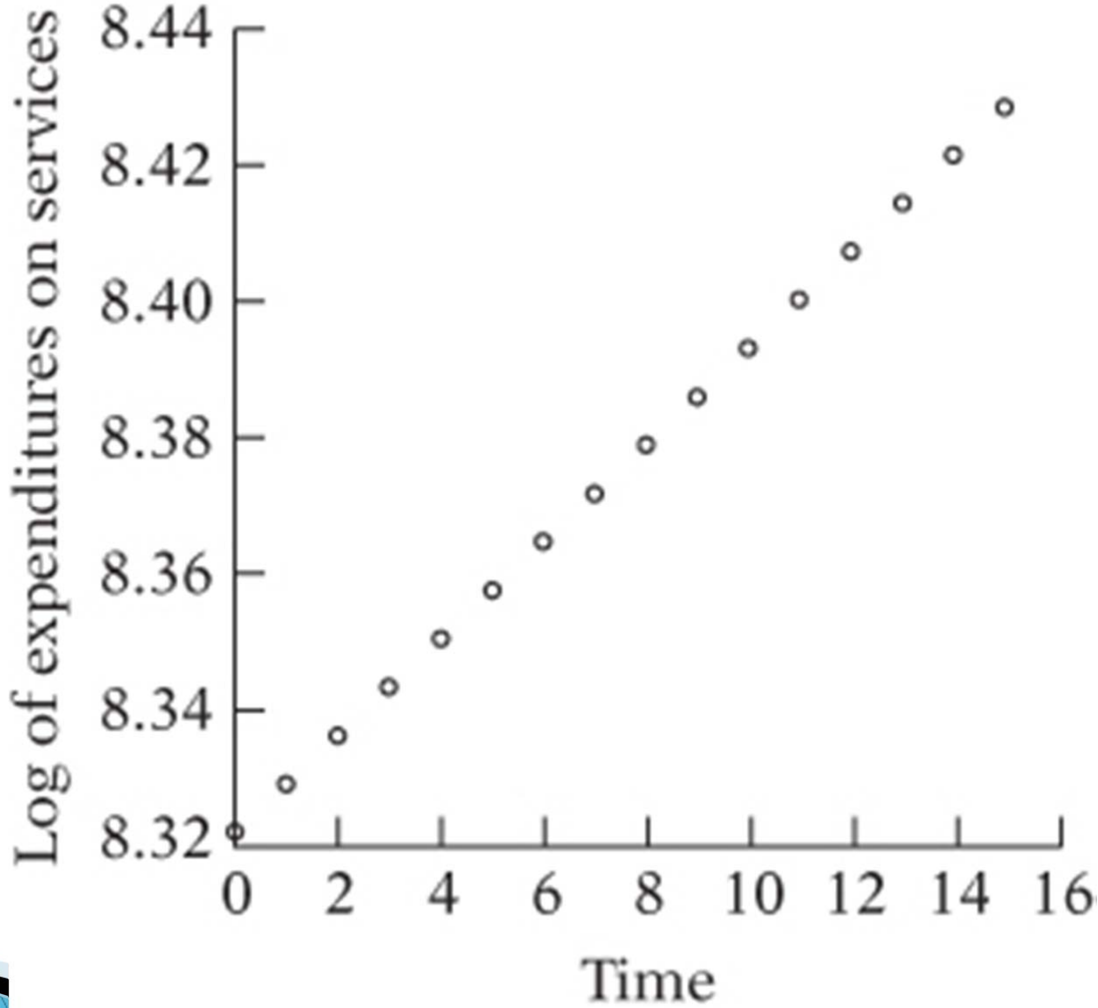
$Y_t$

| Year or quarter | EXPSERVICES | EXPDUR | EXPNONDUR | PCEXP |
|---|---|---|---|---|
| 2003-I | 4,143.3 | 971.4 | 2,072.5 | 7,184.9 |
| 2003-II | 4,161.3 | 1,009.8 | 2,084.2 | 7,249.3 |
| 2003-III | 4,190.7 | 1,049.6 | 2,123.0 | 7,352.9 |
| 2003-IV | 4,220.2 | 1,051.4 | 2,132.5 | 7,394.3 |
| 2004-I | 4,268.2 | 1,067.0 | 2,155.3 | 7,479.8 |
| 2004-II | 4,308.4 | 1,071.4 | 2,164.3 | 7,534.4 |
| 2004-III | 4,341.5 | 1,093.9 | 2,184.0 | 7,607.1 |
| 2004-IV | 4,377.4 | 1,110.3 | 2,213.1 | 7,687.1 |
| 2005-I | 4,395.3 | 1,116.8 | 2,241.5 | 7,739.4 |
| 2005-II | 4,420.0 | 1,150.8 | 2,268.4 | 7,819.8 |
| 2005-III | 4,454.5 | 1,175.9 | 2,287.6 | 7,895.3 |
| 2005-IV | 4,476.7 | 1,137.9 | 2,309.6 | 7,910.2 |
| 2006-I | 4,494.5 | 1,190.5 | 2,342.8 | 8,003.8 |
| 2006-II | 4,535.4 | 1,190.3 | 2,351.1 | 8,055.0 |
| 2006-III | 4,566.6 | 1,208.8 | 2,360.1 | 8,111.2 |

| $t$ | $\ln Y_t$ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| ⋮ | |

*Note:* See Table B-2 for data for total personal consumption expenditures for 1959–1989.
EXPSERVICES = expenditure on services, billions of 2000 dollars.
EXPDUR = expenditure on durable goods, billions of 2000 dollars.
EXPNONDUR = expenditure on nondurable goods, billions of 2000 dollars.
PCEXP = total personal consumption expenditure, billions of 2000 dollars.

(*Continued*)


Log of expenditures on services
8.44
8.42
8.40
8.38
8.36
8.34
8.32
0
2
4
6
8
10
12
14
16
Time

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$\hat{\beta}_2 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}$$

$$\ln \widehat{EXS_t} = 8.3226 + 0.00705t$$

Over the quarterly period 2003-I to 2006-III, expenditures on services increased at the (quarterly) rate of 0.705 percent. Roughly, this is equal to an annual growth rate of 2.82 percent

$0.705 \times 4$

$\hat{\beta}_2 * 100 = 0.00705 \times 100 = 0.705$

## Linear Trend Model

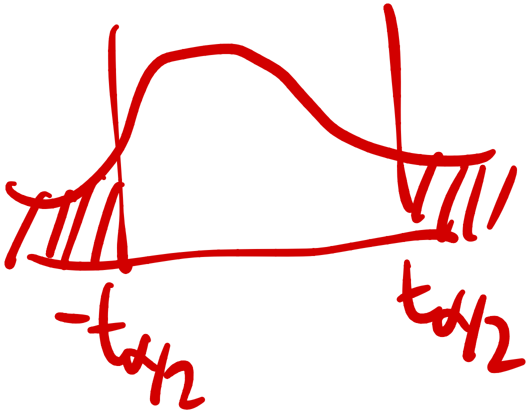
n=15
df=13

① $H_0: \beta_2=0$
$H_1: \beta_2 \neq 0$

$\pm 1.771$

② $\alpha = 0.10$ $t_{critical}$:

$$Y_t = \beta_1 + \beta_2 t + u_t$$




③ $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} = 44.4671$

$$\widehat{EXS}_t = 4111.545 + 30.674t$$

$$t \quad = (655.5628) \;\; (44.4671)$$

④

⑤

$$r^2 = 0.9935$$

Over the quarterly period 2003-I to 2006-III, on average, expenditure on services increased at the absolute rate of about 30 billion dollars per quarter. That is, there was an upward trend in the expenditure on services

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} \quad \text{or} \quad \frac{\hat{\beta}_1 - \beta_1}{se(\hat{\beta}_1)}$$

$H_0: \beta_i = 0$

$H_1: \beta_i \neq 0$

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

- The choice between the growth rate model and the linear trend model will depend upon whether one is interested in the relative or absolute change in the expenditure on services, although for comparative purposes it is the relative change that is generally more relevant

- We cannot compare the $r^2$ values of models because the regressands in the two models are different

## Lin-Log models

The absolute change in Y for a percentage change in X

$$Y_i = \beta_1 + \beta_2 \ln X_i + u_i$$

$$Y = \beta_1 + \beta_2 \ln X$$

$$\frac{dY}{dX} = \beta_2 \frac{1}{X}$$

$$\beta_2 = \frac{dY}{dX / X}$$

- Using differential calculus one can show that

$$\frac{dY}{dX} = \beta_2 \left(\frac{1}{X}\right)$$

Therefore,

$$\beta_2 = \frac{dY}{\frac{dX}{X}} \quad \begin{matrix} \leftarrow \text{Absolute change} \\ \leftarrow \text{Relative change} \end{matrix}$$

$\frac{1}{100}$ or 0.01

$$\beta_2 = \frac{Change\ in\ Y}{relative\ change\ in\ X}$$

The absolute change in Y for a percentage change in X

**

If $\Delta X / X$ changes by 0.01 unit or 1%, the absolute change in Y is

$$0.01 * \beta_2$$

Engel expenditure models

Engel postulated that “the total expenditure that is devoted to food tends to increase in arithmetic progression as total expenditure increases in geometric progression”

# Example

- Food expenditure in India example

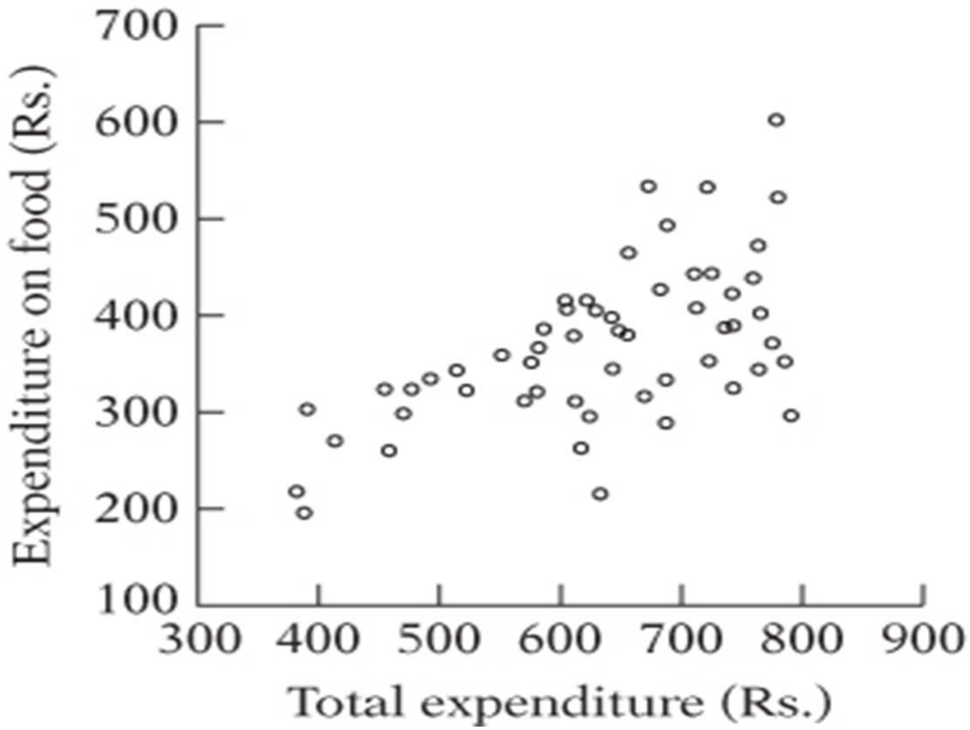


**TABLE 2.8** **Food and Total Expenditure (Rupees)**

| Observation | Food Expenditure | Total Expenditure | Observation | Food Expenditure | Total Expenditure |
|---|---|---|---|---|---|
| 1 | 217.0000 | 382.0000 | 29 | 390.0000 | 655.0000 |
| 2 | 196.0000 | 388.0000 | 30 | 385.0000 | 662.0000 |
| 3 | 303.0000 | 391.0000 | 31 | 470.0000 | 663.0000 |
| 4 | 270.0000 | 415.0000 | 32 | 322.0000 | 677.0000 |
| 5 | 325.0000 | 456.0000 | 33 | 540.0000 | 680.0000 |
| 6 | 260.0000 | 460.0000 | 34 | 433.0000 | 690.0000 |
| 7 | 300.0000 | 472.0000 | 35 | 295.0000 | 695.0000 |
| 8 | 325.0000 | 478.0000 | 36 | 340.0000 | 695.0000 |
| 9 | 336.0000 | 494.0000 | 37 | 500.0000 | 695.0000 |
| 10 | 345.0000 | 516.0000 | 38 | 450.0000 | 720.0000 |
| 11 | 325.0000 | 525.0000 | 39 | 415.0000 | 721.0000 |
| 12 | 362.0000 | 554.0000 | 40 | 540.0000 | 730.0000 |
| 13 | 315.0000 | 575.0000 | 41 | 360.0000 | 731.0000 |
| 14 | 355.0000 | 579.0000 | 42 | 450.0000 | 733.0000 |
| 15 | 325.0000 | 585.0000 | 43 | 395.0000 | 745.0000 |
| 16 | 370.0000 | 586.0000 | 44 | 430.0000 | 751.0000 |
| 17 | 390.0000 | 590.0000 | 45 | 332.0000 | 752.0000 |
| 18 | 420.0000 | 608.0000 | 46 | 397.0000 | 752.0000 |
| 19 | 410.0000 | 610.0000 | 47 | 446.0000 | 769.0000 |
| 20 | 383.0000 | 616.0000 | 48 | 480.0000 | 773.0000 |
| 21 | 315.0000 | 618.0000 | 49 | 352.0000 | 773.0000 |
| 22 | 267.0000 | 623.0000 | 50 | 410.0000 | 775.0000 |
| 23 | 420.0000 | 627.0000 | 51 | 380.0000 | 785.0000 |
| 24 | 300.0000 | 630.0000 | 52 | 610.0000 | 788.0000 |
| 25 | 410.0000 | 635.0000 | 53 | 530.0000 | 790.0000 |
| 26 | 220.0000 | 640.0000 | 54 | 360.0000 | 795.0000 |
| 27 | 403.0000 | 648.0000 | 55 | 305.0000 | 801.0000 |
| 28 | 350.0000 | 650.0000 | | | |

Source: Chandan Mukherjee, Howard White, and Marc Wuyts, *Econometrics and Data Analysis for Developing Countries,* Routledge, New York, 1998, p. 457.

ln Total exp.

$$\overbrace{Foodexp_i}^{Y_i} = -1{,}283.912 + 257.2700\,\overset{\ln X_i}{\underline{lnTotalexp_i}}$$

- As this figure suggests, food expenditure increases more slowly as total expenditure increases, perhaps giving credence to Engel's law

- The slope coefficient of about 257 means that an increase in the total ~~food~~ expenditure of 1 percent, on average, leads to about 2.57 rupees increase in the expenditure on food of the 55 families included in the sample

  (Note: we have divided the estimated slope coefficient by 100)

$$0.01 \times \beta_2 = 0.01 \times 257.2700$$
$$= 2.57$$

# Reciprocal models

$$Y_i = \beta_1 + \beta_2\left(\frac{1}{X_i}\right) + u_i$$

As X increases indefinitely, in term $\beta_2\left(\frac{1}{X_i}\right)$ approaches zero and Y approaches the limiting or asymptotic value $\beta_1$

$$Y_i = \beta_1 + \beta_2 \left( \frac{1}{X} \right)$$

$$\frac{dY}{dX} = -\beta_2 \left( \frac{1}{X^2} \right)$$

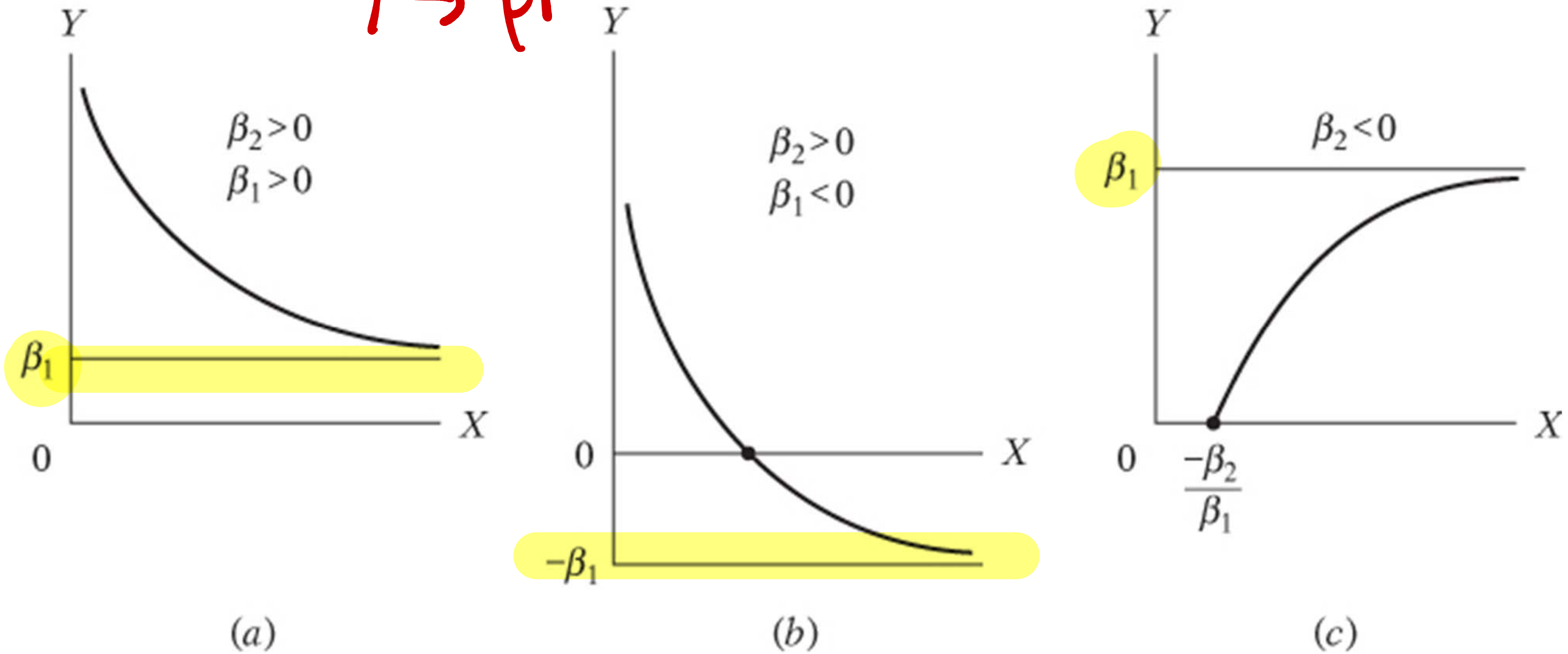
if $\beta_2$ is positive, the slope is negative throughout

$\beta_2$ is negative, the slope is positive throughout

$X \to \infty$

$\beta_2\left(\frac{1}{X}\right) \to 0$

$Y \to \beta_1$



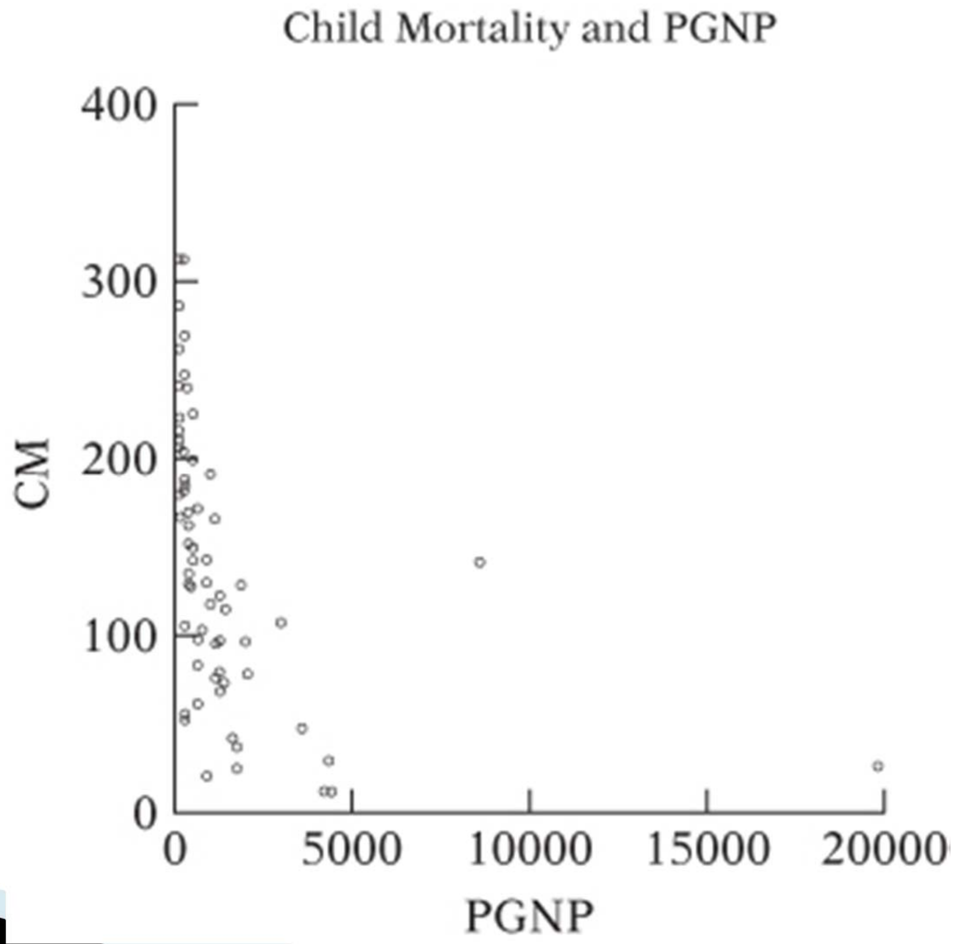
$$CM_i = \beta_1 + \beta_2 \left(\frac{1}{PGNP_i}\right) + u_i$$

## Example

- Child mortality and per capita GNP – 64 countries

in 1980

The number of deaths of children under age 5 in a year per 1000 live births



PGNP

**TABLE 6.4** Fertility and Other Data for 64 Countries

| Observation | CM | FLFP | PGNP | TFR | Observation | CM | FLFP | PGNP | TFR |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 128 | 37 | 1870 | 6.66 | 33 | 142 | 50 | 8640 | 7.17 |
| 2 | 204 | 22 | 130 | 6.15 | 34 | 104 | 62 | 350 | 6.60 |
| 3 | 202 | 16 | 310 | 7.00 | 35 | 287 | 31 | 230 | 7.00 |
| 4 | 197 | 65 | 570 | 6.25 | 36 | 41 | 66 | 1620 | 3.91 |
| 5 | 96 | 76 | 2050 | 3.81 | 37 | 312 | 11 | 190 | 6.70 |
| 6 | 209 | 26 | 200 | 6.44 | 38 | 77 | 88 | 2090 | 4.20 |
| 7 | 170 | 45 | 670 | 6.19 | 39 | 142 | 22 | 900 | 5.43 |
| 8 | 240 | 29 | 300 | 5.89 | 40 | 262 | 22 | 230 | 6.50 |
| 9 | 241 | 11 | 120 | 5.89 | 41 | 215 | 12 | 140 | 6.25 |
| 10 | 55 | 55 | 290 | 2.36 | 42 | 246 | 9 | 330 | 7.10 |
| 11 | 75 | 87 | 1180 | 3.93 | 43 | 191 | 31 | 1010 | 7.10 |
| 12 | 129 | 55 | 900 | 5.99 | 44 | 182 | 19 | 300 | 7.00 |
| 13 | 24 | 93 | 1730 | 3.50 | 45 | 37 | 88 | 1730 | 3.46 |
| 14 | 165 | 31 | 1150 | 7.41 | 46 | 103 | 35 | 780 | 5.66 |
| 15 | 94 | 77 | 1160 | 4.21 | 47 | 67 | 85 | 1300 | 4.82 |
| 16 | 96 | 80 | 1270 | 5.00 | 48 | 143 | 78 | 930 | 5.00 |
| 17 | 148 | 30 | 580 | 5.27 | 49 | 83 | 85 | 690 | 4.74 |
| 18 | 98 | 69 | 660 | 5.21 | 50 | 223 | 33 | 200 | 8.49 |
| 19 | 161 | 43 | 420 | 6.50 | 51 | 240 | 19 | 450 | 6.50 |
| 20 | 118 | 47 | 1080 | 6.12 | 52 | 312 | 21 | 280 | 6.50 |
| 21 | 269 | 17 | 290 | 6.19 | 53 | 12 | 79 | 4430 | 1.69 |
| 22 | 189 | 35 | 270 | 5.05 | 54 | 52 | 83 | 270 | 3.25 |
| 23 | 126 | 58 | 560 | 6.16 | 55 | 79 | 43 | 1340 | 7.17 |
| 24 | 12 | 81 | 4240 | 1.80 | 56 | 61 | 88 | 670 | 3.52 |
| 25 | 167 | 29 | 240 | 4.75 | 57 | 168 | 28 | 410 | 6.09 |
| 26 | 135 | 65 | 430 | 4.10 | 58 | 28 | 95 | 4370 | 2.86 |
| 27 | 107 | 87 | 3020 | 6.66 | 59 | 121 | 41 | 1310 | 4.88 |
| 28 | 72 | 63 | 1420 | 7.28 | 60 | 115 | 62 | 1470 | 3.89 |
| 29 | 128 | 49 | 420 | 8.12 | 61 | 186 | 45 | 300 | 6.90 |
| 30 | 27 | 63 | 19830 | 5.23 | 62 | 47 | 85 | 3630 | 4.10 |
| 31 | 152 | 84 | 420 | 5.79 | 63 | 178 | 45 | 220 | 6.09 |
| 32 | 224 | 23 | 530 | 6.50 | 64 | 142 | 67 | 560 | 7.20 |

*Note:* CM = Child mortality, the number of deaths of children under age 5 in a year per 1000 live births.
FLFP = Female literacy rate, percent.
PGNP = per capita GNP in 1980.
TFR = total fertility rate, 1980–1985, the average number of children born to a woman, using age-specific fertility rates for a given year.

Source: Chandan Mukherjee, Howard White, and Marc Whyte, *Econometrics and Data Analysis for Developing Countries,* Routledge, London, 1998, p. 456.

$$\widehat{CM}_i = \underset{\hat{\beta}_1}{81.7944} + \underset{\hat{\beta}_2}{27{,}237.17}\left(\frac{1}{PGNP_i}\right)$$

- As per capita GNP increases, one would expect child mortality to decrease because people can afford to spend more on health care, assuming all other factors remain constant

- The relationship is not a straight line one: As per capita GNP increases, initially there is a dramatic drop in child mortality but the drop tapers of as per capita GNP continues to increase

As per capita GNP increases indefinitely, child mortality approaches its asymptotic value of about 82 deaths per thousand

Macroeconomics

Using the data on percent rate of change of money wages (Y) and the unemployment rate (X) of U.K. for the period 1861-1957.

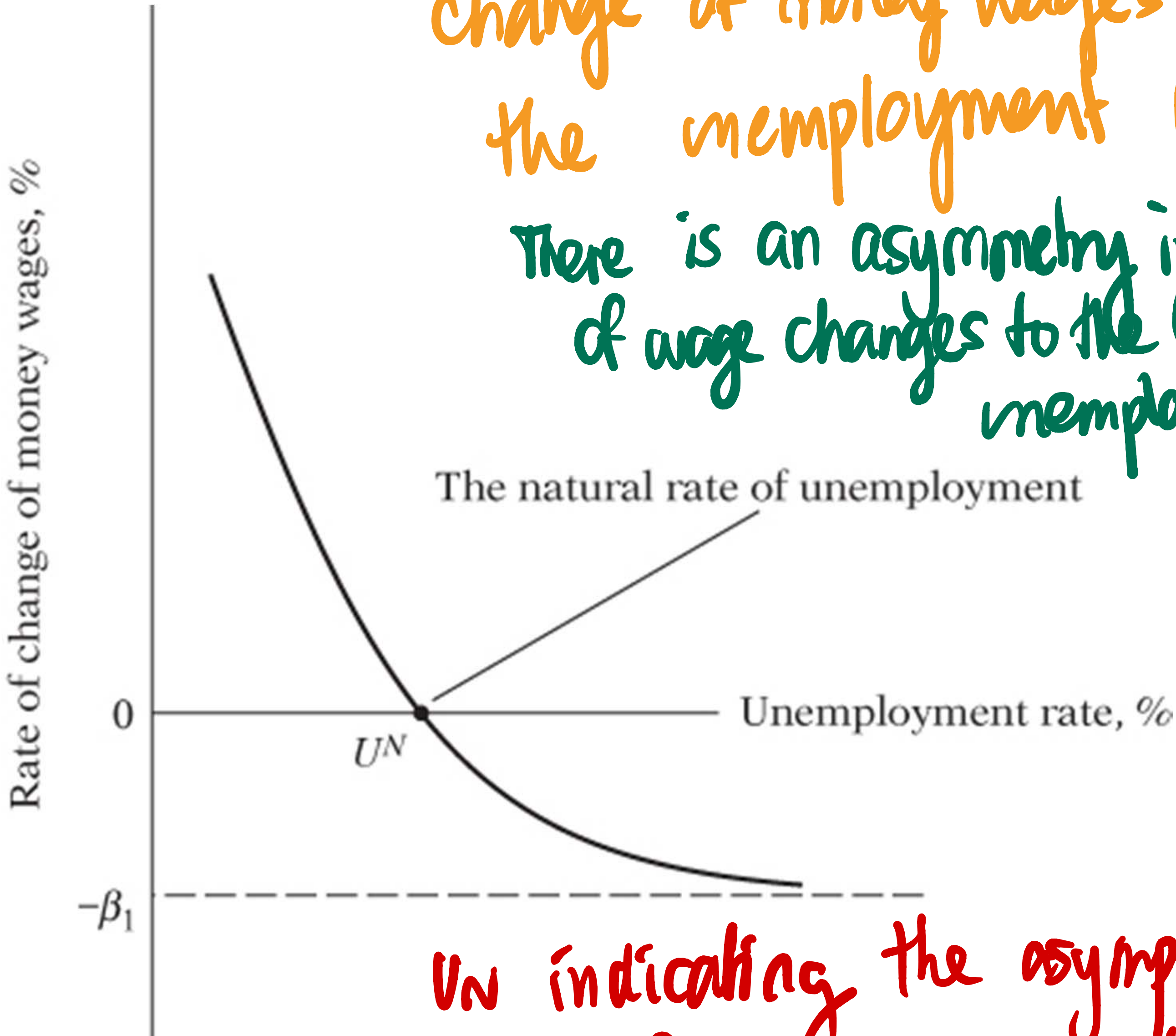
# Example
# The Phillips curve

Reading assignment ☺

# Figure 6.8

Relationship between percent rate of change of money wages (Y) and the unemployment rate (X)

There is an asymmetry in the response of wage changes to the level of the unemployment rate ~




UN indicating the asymptotic floor or $-\beta_1$ for the wage change.

The natural rate of unemployment defined as the rate of unemployment required to keep (wage) inflation constant

- Using the data on percent rate of change of money wages (Y) and the unemployment rate (X) for the United Kingdom for the period 1861-1957, Phillips obtained a curve whose general shape resembled figure 6.8

- An asymmetry in the response of wage changes to the level of the unemployment rate: Wages rise faster for a unit change in unemployment if the unemployment rate is below $U^N$, which is called **the natural rate of unemployment**, and then they fall slowly for an equivalent change when the unemployment rate is above the natural rate , indicating the asymptotic floor, or $-\beta_1$ for wage change

- This particular feature of the Phillips curve may be due to institutional factors such as union bargaining power, minimum wages, unemployment compensation.

- A comparative recent formulation is provided by Olivier Blanchard (1997)

$$\pi_t - \pi_t{}^e = \beta_2(UN_t - U^N) + u_t$$

Where

$\pi_t$=actual inflation rate at time t Percentage change in the price level as measured by a representative price index such as CPI

$\pi_t{}^e$= expected inflation rate at time t, the expectation being formed in year (t-1)

$UN_t$= actual unemployment rate prevailing at time t

$U^N$= natural rate of unemployment

$u_t$= stochastic error term

A priori, $\beta_2$ is expected to be negative
$\beta_1$ is expected to be positive (since $\beta_2$ is negative and $U_N$ is positive)

- Since $\pi_t{}^e$ is not directly observable, as a starting point one can make the simplifying assumption that $\pi_t{}^e = \pi_{t-1}$; that is, the inflation rate expected this year is the inflation rate that prevailing in the last year

- Substituting this assumption and writing the regression model in the standard form, we obtain

$$\pi_t - \pi_{t-1} = \beta_1 + \beta_2 UN_t + u_t$$

where $\beta_1 = -\beta_2 U^N$

Equation states that the change in the inflation rate between two time periods is linearly related to the current unemployment rate

$$\pi_t - \pi^e_t = \beta_2 (UN_t - U^N) + u_t$$

is known in the literature as the modified Phillips curve.

or the expectation-augmented Philips cure (to indicate that $\pi_{t-1}$ stands for expected inflation)

or the accelerationist Phillips curve (to suggest that a low unemployment rate leads to an increase in the inflation rate and hence an acceleration of the price

## TABLE 6.5

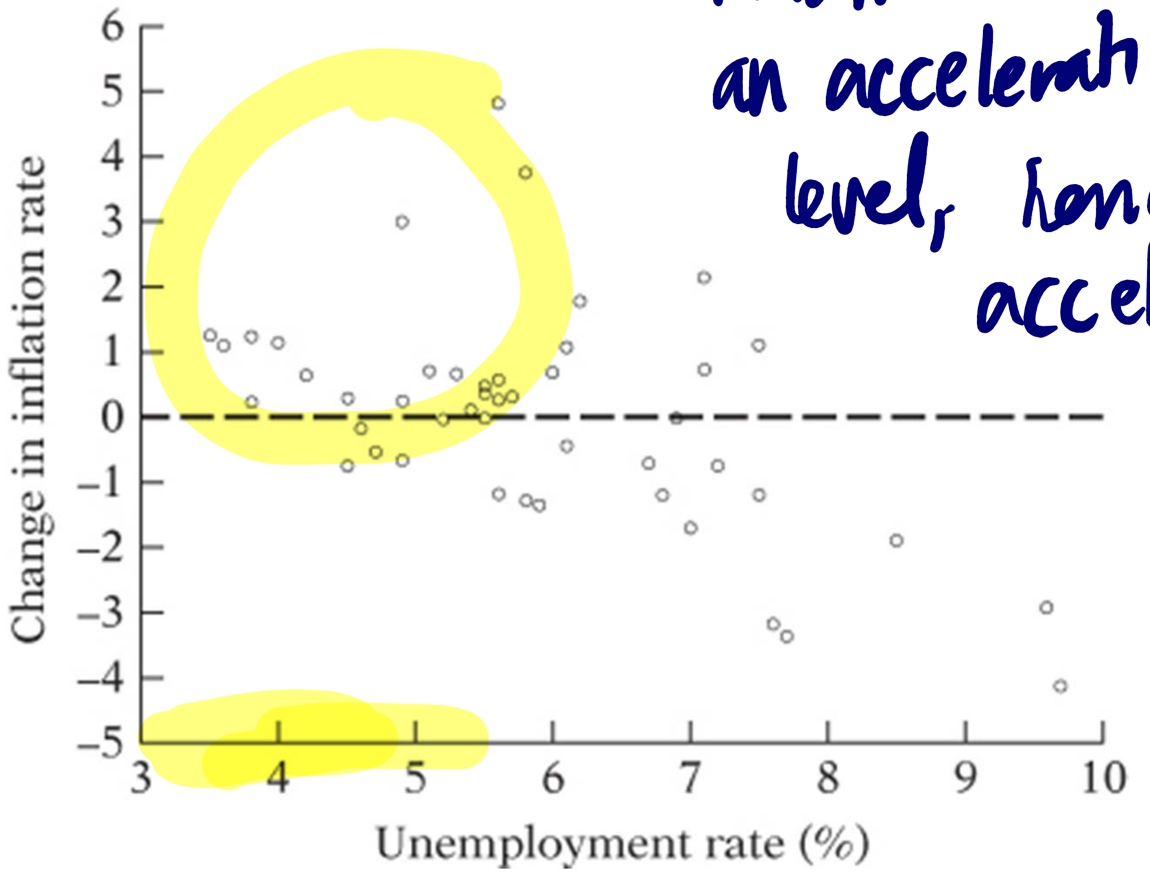
**Inflation Rate and Unemployment Rate, United States, 1960–2006 (For all urban consumers; 1982–1984 = 100, except as noted)**

Source: *Economic Report of the President,* 2007, Table B-60, p. 399, for CPI changes and Table B-42, p. 376, for the unemployment rate.

| Year | INFLRATE | UNRATE | Year | INFLRATE | UNRATE |
|---|---|---|---|---|---|
| 1960 | 1.718 | 5.5 | 1984 | 4.317 | 7.5 |
| 1961 | 1.014 | 6.7 | 1985 | 3.561 | 7.2 |
| 1962 | 1.003 | 5.5 | 1986 | 1.859 | 7.0 |
| 1963 | 1.325 | 5.7 | 1987 | 3.650 | 6.2 |
| 1964 | 1.307 | 5.2 | 1988 | 4.137 | 5.5 |
| 1965 | 1.613 | 4.5 | 1989 | 4.818 | 5.3 |
| 1966 | 2.857 | 3.8 | 1990 | 5.403 | 5.6 |
| 1967 | 3.086 | 3.8 | 1991 | 4.208 | 6.8 |
| 1968 | 4.192 | 3.6 | 1992 | 3.010 | 7.5 |
| 1969 | 5.460 | 3.5 | 1993 | 2.994 | 6.9 |
| 1970 | 5.722 | 4.9 | 1994 | 2.561 | 6.1 |
| 1971 | 4.381 | 5.9 | 1995 | 2.834 | 5.6 |
| 1972 | 3.210 | 5.6 | 1996 | 2.953 | 5.4 |
| 1973 | 6.220 | 4.9 | 1997 | 2.294 | 4.9 |
| 1974 | 11.036 | 5.6 | 1998 | 1.558 | 4.5 |
| 1975 | 9.128 | 8.5 | 1999 | 2.209 | 4.2 |
| 1976 | 5.762 | 7.7 | 2000 | 3.361 | 4.0 |
| 1977 | 6.503 | 7.1 | 2001 | 2.846 | 4.7 |
| 1978 | 7.591 | 6.1 | 2002 | 1.581 | 5.8 |
| 1979 | 11.350 | 5.8 | 2003 | 2.279 | 6.0 |
| 1980 | 13.499 | 7.1 | 2004 | 2.663 | 5.5 |
| 1981 | 10.316 | 7.6 | 2005 | 3.388 | 5.1 |
| 1982 | 6.161 | 9.7 | 2006 | 3.226 | 4.6 |
| 1983 | 3.212 | 9.6 | | | |

*Note:* The inflation rate is the percent year-to-year change in CPI. The unemployment rate is the civilian unemployment rate.

$\pi_t - \pi_{t-1}$




A low unemployment rate leads to an increase in the inflation rate and therefore an acceleration of the price level, hence the name accelerationist Phillips curve.

- As expected, the relation between the change in inflation rate and the unemployment rate is negative – a low unemployment rate leads to an increase in the inflation rate and therefore an acceleration of the price level, hence the name accelerationist Phillips curve

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$
$$Y_i = \beta_1 + \beta_2\left(\frac{1}{X_i}\right) + u_i$$

Model 1

Linear model:

$$\widehat{(\pi_t - \pi_{t-1})} = 3.7844 - 0.6385UN_t$$
$$t = (4.1912) \quad (-4.2756)$$
$$r^2 = 0.2935$$

Model 2

Reciprocal model:

$$\widehat{(\pi_t - \pi_{t-1})} = -3.0684 + 17.2077\left(\frac{1}{UN_t}\right)$$
$$t = (-3.1635) \quad (3.2886)$$
$$r^2 = 0.1973$$

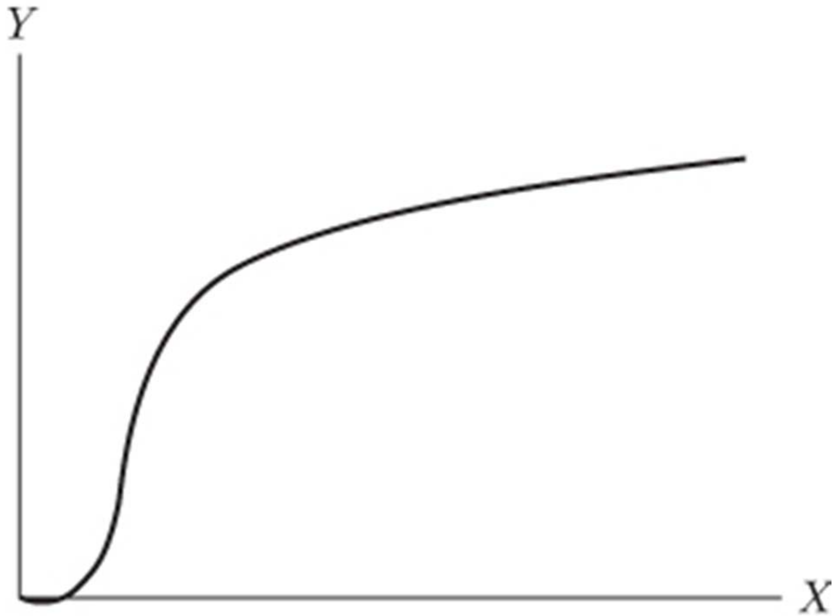
Model 1 shows that if the unemployment rate goes down by 1 percentage point, on average, the change in the inflation rate goes up by about 0.64 percentage points

Model 2 shows that even if the unemployment rate increases indefinitely, the most the change in the inflation rate will go down will be about 3.07 percentage points We can compute the underlying natural rate of unemployment as: (Model 1)

$$UN_t = \frac{\widehat{\beta_1}}{-\widehat{\beta_2}} = \frac{3.7844}{0.6385} = 5.9270$$

The natural rate of unemployment is about 5.93%

# The logarithmic reciprocal model




$$\ln Y_i = \beta_1 - \beta_2 \left( \frac{1}{X_i} \right) + u_i$$

## The logarithmic reciprocal model

- Initially Y increases at an increasing rate and then it increases at a decreasing rate

- E.g. short run production function

Microeconomics – if labor and capital are the inputs in a production function and if we keep the capital input constant but increase the labor input, the short-run output-labor relationship will resemble in the logarithmic reciprocal model

# Choice of Functional Form

**TABLE 6.6**

| Model | Equation | Slope $\left(=\frac{dY}{dX}\right)$ | Elasticity $\left(=\frac{dY}{dX}\frac{X}{Y}\right)$ |
|---|---|---|---|
| Linear | $Y = \beta_1 + \beta_2 X$ | $\beta_2$ | $\beta_2\left(\frac{X}{Y}\right)^*$ |
| Log–linear | $\ln Y = \beta_1 + \beta_2 \ln X$ | $\beta_2\left(\frac{Y}{X}\right)$ | $\beta_2$ |
| Log–lin | $\ln Y = \beta_1 + \beta_2 X$ | $\beta_2 (Y)$ | $\beta_2 (X)^*$ |
| Lin–log | $Y = \beta_1 + \beta_2 \ln X$ | $\beta_2\left(\frac{1}{X}\right)$ | $\beta_2\left(\frac{1}{Y}\right)^*$ |
| Reciprocal | $Y = \beta_1 + \beta_2\left(\frac{1}{X}\right)$ | $-\beta_2\left(\frac{1}{X^2}\right)$ | $-\beta_2\left(\frac{1}{XY}\right)^*$ |
| Log reciprocal | $\ln Y = \beta_1 - \beta_2\left(\frac{1}{X}\right)$ | $\beta_2\left(\frac{Y}{X^2}\right)$ | $\beta_2\left(\frac{1}{X}\right)^*$ |

*Note:* * indicates that the elasticity is variable, depending on the value taken by $X$ or $Y$ or both. When no $X$ and $Y$ values are specified, in practice, very often these elasticities are measured at the mean values of these variables, namely, $\bar{X}$ and $\bar{Y}$.

# Source

Gujarati, D.N. (2009) Basic Econometrics. 5th ed. Singapore, McGraw-Hill.

Midterm exam (2 hrs)

1 Calculator allowed (Any types)

1 A-4 paper (Both sides) - Own handwriting

↓

can be printed out.

or written on paper.

Pens (Black or Blue)

35 % of Total score

35 pts (Total Midterm Score)

Good Luck :)