FN211 Basic Matrices
Part 1

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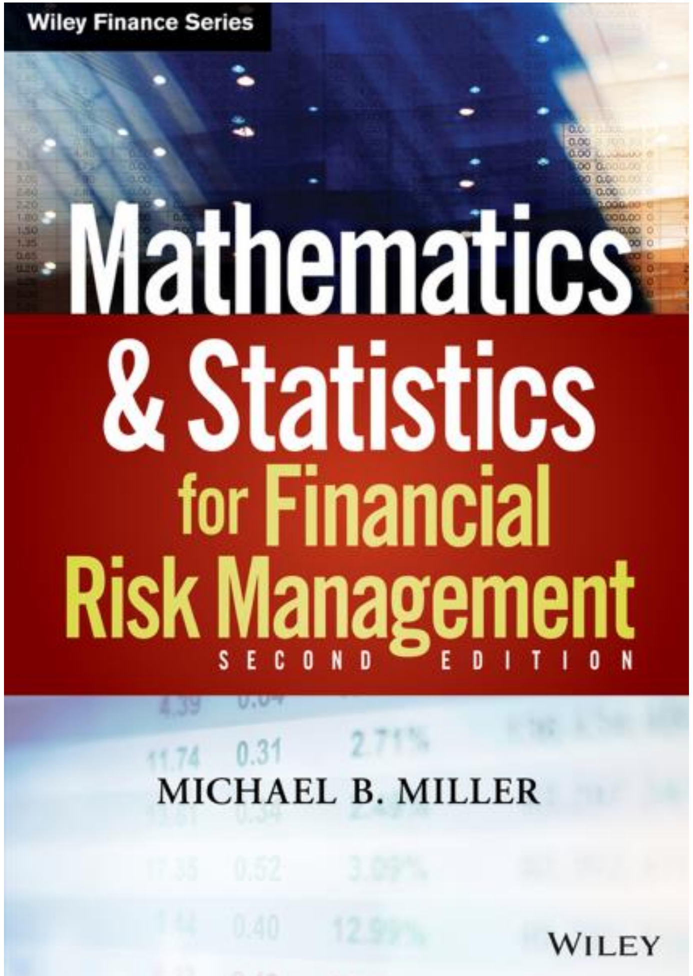

Outline

- Introduction and Types of Matrices
- Matrix Operations
- Inverse of a Matrix
- *Application**

Reading:

Chapter 8

Introduction and Types of Matrices

(1) Column matrix or vector

(2) Row matrix or vector

(3) Rectangular matrix

(4) Square matrix

(5) Diagonal matrix

(6) Unit or Identity matrix - I

(7) Null (zero) matrix – 0

(8) Triangular matrix

(9) Scalar matrix

Matrices - Introduction

- Matrix algebra has at least two advantages:
- Reduces complicated systems of equations to simple expressions
- Adaptable to systematic method of mathematical treatment and well suited to computers

Definition:

A matrix is a set or group of numbers arranged in a square or rectangular array enclosed by two brackets

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 4 & 2 \\ -3 & 0 \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Matrices - Introduction

- **Properties:**
  - A specified number of rows and a specified number of columns
  - Two numbers (rows x columns) describe the dimensions or size of the matrix.

Examples:

3x3 matrix

2x4 matrix

1x2 matrix

$$\begin{bmatrix} 1 & 2 & 4 \\ 4 & -1 & 5 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & -3 \\ 0 & 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix}$$

Matrices - Introduction

- A matrix is denoted by a bold capital letter and the elements within the matrix are denoted by lower case letters

- e.g. matrix [**A**] with elements a_{ij}

$$\mathbf{A}_{mxn} = \quad {}_{m}\mathbf{A}^{n} \qquad \begin{bmatrix} a_{11} & a_{12}\dots & a_{ij} & a_{in} \\ a_{21} & a_{22}\dots & a_{ij} & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{ij} & a_{mn} \end{bmatrix}$$

i goes from 1 to m

j goes from 1 to n

Types of Matrices

1.Column matrix or vector

- The number of rows may be any integer but the number of columns is always 1

$$\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -3 \end{bmatrix} \quad \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$$

2. Row matrix or vector

- Any number of columns but only one row

$$\begin{bmatrix} 1 & 1 & 6 \end{bmatrix} \quad \begin{bmatrix} 0 & 3 & 5 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \end{bmatrix}$$

Types of Matrices

3. Rectangular matrix

- Contains more than one element and number of rows is not equal to the number of columns

$$\begin{bmatrix} 1 & 1 \\ 3 & 7 \\ 7 & -7 \\ 7 & 6 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 3 & 0 \end{bmatrix}$$

$$m \neq n$$

4. Square matrix

- The number of rows is equal to the number of columns (a square matrix A m x m has an order of m)

$$\begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 9 & 9 & 0 \\ 6 & 6 & 1 \end{bmatrix}$$

The principal or main diagonal of a square matrix is composed of all elements a_{ij} for which $i=j$

Types of Matrices

5. Diagonal matrix

- A square matrix where all the elements are zero except those on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

i.e. a_{ij} =0 for all i not equal j

a_{ij} = 0 for some or all $i = j$

6. Unit or Identity matrix - I

- A diagonal matrix with ones on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} a_{ij} & 0 \\ 0 & a_{ij} \end{bmatrix}$$

i.e. a_{ij} =0 for all i not equal j

a_{ij} = 1 for some or all $i = j$

Types of Matrices

7. Null (zero) matrix - 0

- All elements in the matrix are zero

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$a_{ij} = 0$ For all i,j

8. Triangular matrix

- A square matrix whose elements above or below the main diagonal are all zero

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 8 & 9 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

Types of Matrices

8a. Upper triangular matrix

A square matrix whose elements below the main diagonal are all zero

$$\begin{bmatrix} a_{ij} & a_{ij} & a_{ij} \\ 0 & a_{ij} & a_{ij} \\ 0 & 0 & a_{ij} \end{bmatrix} \quad \begin{bmatrix} 1 & 8 & 7 \\ 0 & 1 & 8 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 7 & 4 & 4 \\ 0 & 1 & 7 & 4 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i > j$

8b. Lower triangular matrix

A square matrix whose elements above the main diagonal are all zero

$$\begin{bmatrix} a_{ij} & 0 & 0 \\ a_{ij} & a_{ij} & 0 \\ a_{ij} & a_{ij} & a_{ij} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i < j$

Types of Matrices

9. Scalar matrix

- A diagonal matrix whose main diagonal elements are equal to the same scalar
- A scalar is defined as a single number or constant

$$\begin{bmatrix} a_{ij} & 0 & 0 \\ 0 & a_{ij} & 0 \\ 0 & 0 & a_{ij} \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all i not equal j

$a_{ij} = a$ for all $i = j$

Matrix Operations

(1) Equality of matrices

(2) Addition and subtraction of matrices

(3) Scalar multiplication of matrices

(4) Multiplication of matrices

(5) Transpose of a Matrix

(1) EQUALITY OF MATRICES

- Two matrices are said to be equal only when all corresponding elements are equal
- Therefore their size or dimensions are equal as well

A = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$ **B** = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$ **A** = **B**

(1) EQUALITY OF MATRICES

- Some properties of equality:
- IIf A = B, then B = A for all A and B
- IIf A = B, and B = C, then A = C for all A, B and C

A = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$ **B =** $\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

If **A** = **B** then $a_{ij} = b_{ij}$

(2) ADDITION AND SUBTRACTION OF MATRICES

- The sum or difference of two matrices, **A** and **B** of the same size yields a matrix **C** of the same size

$$c_{ij} = a_{ij} + b_{ij}$$

- Matrices of different sizes cannot be added or subtracted
- Commutative Law: **A** + **B** = **B** + **A**
- Associative Law: **A** + (**B** + **C**) = (**A** + **B**) + **C** = **A** + **B** + **C**

(2) ADDITION AND SUBTRACTION OF MATRICES

Example

$$\begin{bmatrix} 7 & 3 & -1 \\ 2 & -5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 6 \\ -4 & -2 & 3 \end{bmatrix} =$$

A 2x3 **B** 2x3 **C** 2x3

Example

$$\begin{bmatrix} 6 & 4 & 2 \\ 3 & 2 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 8 \end{bmatrix} =$$

(2) ADDITION AND SUBTRACTION OF MATRICES

- $\mathbf{A} + \mathbf{0} = \mathbf{0} + \mathbf{A} = \mathbf{A}$
- $\mathbf{A} + (-\mathbf{A}) = \mathbf{0}$ (where $-\mathbf{A}$ is the matrix composed of $-a_{ij}$ as elements)

(3) SCALAR MULTIPLICATION OF MATRICES

- Matrices can be multiplied by a scalar (constant or single element)
- Let k be a scalar quantity; then kA = Ak

Ex. If k=4 and $A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix}$, then

(3) SCALAR MULTIPLICATION OF MATRICES

- Properties:

- $k(\mathbf{A} + \mathbf{B}) = k\mathbf{A} + k\mathbf{B}$
- $(k + g)\mathbf{A} = k\mathbf{A} + g\mathbf{A}$
- $k(\mathbf{AB}) = (k\mathbf{A})\mathbf{B} = \mathbf{A}(k)\mathbf{B}$
- $k(g\mathbf{A}) = (kg)\mathbf{A}$

(4) MULTIPLICATION OF MATRICES

- The product of two matrices is another matrix
- Two matrices **A** and **B** must be conformable for multiplication to be possible i.e. the number of columns of **A** must equal the number of rows of **B**

Example:

A x B = C

(1x3) (3x1) (1x1)

B x A =

(2x1) (4x2)

A x B =

(6x2) (6x3)

A x B =

(2x3) (3x2)

(4) MULTIPLICATION OF MATRICES

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$(a_{11} \times b_{11}) + (a_{12} \times b_{21}) + (a_{13} \times b_{31}) = c_{11}$$
$$(a_{11} \times b_{12}) + (a_{12} \times b_{22}) + (a_{13} \times b_{32}) = c_{12}$$
$$(a_{21} \times b_{11}) + (a_{22} \times b_{21}) + (a_{23} \times b_{31}) = c_{21}$$
$$(a_{21} \times b_{12}) + (a_{22} \times b_{22}) + (a_{23} \times b_{32}) = c_{22}$$

- Successive multiplication of row *i* of **A** with column *j* of **B** – row by column multiplication

(4) MULTIPLICATION OF MATRICES

- Example $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 7 \end{bmatrix} \begin{bmatrix} 4 & 8 \\ 6 & 2 \\ 5 & 3 \end{bmatrix} =$

Remember also: **IA** = **A**

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix}$$

(4) MULTIPLICATION OF MATRICES

- Assuming that matrices **A**, **B** and **C** are conformable for the operations indicated, the following are true:

  1. **AI** = **IA** = **A**
  2. **A**(**BC**) = (**AB**)**C** = **ABC** - (associative law)
  3. **A**(**B**+**C**) = **AB** + **AC** - (first distributive law)
  4. **(A+B)C = AC + BC** - (second distributive law)

Caution!

1. **AB** not generally equal to **BA**, **BA** may not be conformable
2. If **AB** = **0**, neither **A** nor **B** necessarily = **0**
3. If **AB** = **AC**, **B** not necessarily = **C**

(4) MULTIPLICATION OF MATRICES

- **AB** not generally equal to **BA**, **BA** may not be conformable

$$T = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix}$$

$$TS =$$

$$ST =$$

(4) MULTIPLICATION OF MATRICES

- If **AB** = **0**, neither **A** nor **B** necessarily = **0**

Example: $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix} =$

(5) TRANSPOSE OF A MATRIX

- If : $\underset{2x3}{A} = {}_2A^3 = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 3 & 1 \end{bmatrix}$

- Then transpose of A, denoted A^T is: $A^T = {}_2A^{3^T} = \begin{bmatrix} 2 & 5 \\ 4 & 3 \\ 7 & 1 \end{bmatrix}$

$$a_{ij} = a_{ji}^T$$ For all *i* and *j*

(5) TRANSPOSE OF A MATRIX

- To transpose:
- Interchange rows and columns
- The dimensions of A^T are the reverse of the dimensions of A

$$A = {}_2A^3 = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 3 & 1 \end{bmatrix}$$

2 x 3

$$A^T = {}_3A^{T\,2} = \begin{bmatrix} 2 & 5 \\ 4 & 3 \\ 7 & 1 \end{bmatrix}$$

3 x 2

(5) TRANSPOSE OF A MATRIX

- Properties of transposed matrices:

1. $(A+B)^T = A^T + B^T$
2. $(AB)^T = B^T A^T$
3. $(kA)^T = kA^T$
4. $(A^T)^T = A$

(5) TRANSPOSE OF A MATRIX

- Properties of transposed matrices:

1. $(A+B)^T = A^T + B^T$

$$\begin{bmatrix} 7 & 3 & -1 \\ 2 & -5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 6 \\ -4 & -2 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} 7 & 2 \\ 3 & -5 \\ -1 & 6 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ 5 & -2 \\ 6 & 3 \end{bmatrix} =$$

(5) TRANSPOSE OF A MATRIX

- Properties of transposed matrices:

2. $(AB)^T = B^T A^T$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 0 & 3 \end{bmatrix} =$$

Inverse of a Matrix

INVERSE OF A MATRIX

- Consider a scalar k. The inverse is the reciprocal or division of 1 by the scalar.

Example: $k=7$ the inverse of k or $k^{-1} = 1/k = 1/7$

- **Division of matrices is not defined** since there may be $\mathbf{AB} = \mathbf{AC}$ while $\mathbf{B}$ is not equal to $\mathbf{C}$.
- Instead matrix inversion is used.
- The inverse of a square matrix, $\mathbf{A}$, if it exists, is the unique matrix $\mathbf{A}^{-1}$ where:

$$\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

INVERSE OF A MATRIX

- The inverse of a square matrix, **A**, if it exists, is the unique matrix $\mathbf{A}^{-1}$ where:

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

Example:

$$A{=}_2A^2 = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

Because:

$$\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} =$$

INVERSE OF A MATRIX

- Properties of the inverse:

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(A^{-1})^{-1} = A$$

$$(A^{T})^{-1} = (A^{-1})^{T}$$

$$(kA)^{-1} = \frac{1}{k}A^{-1}$$

Note:

- A square matrix that has an inverse is called a nonsingular matrix
- A matrix that does not have an inverse is called a singular matrix
- Square matrices have inverses except when the determinant is zero
- When the determinant of a matrix is zero, the matrix is singular

INVERSE OF A MATRIX

- The inverse of a square matrix: $$A^{-1} = \frac{adjA}{|A|}$$

  (A) Adjoint of matrix

  (B) Determination of matrix

TO BE CONTINUED …

(A) ADJOINT MATRIX

- The adjoint matrix of **A**, denoted by adj **A**, is the transpose of its cofactor matrix

$$adjA = C^T$$

- The cofactor C_{ij} of an element a_{ij} is defined as:

$$C_{ij} = (-1)^{i+j} m_{ij}$$

- A minor of **A**, m_{ij} , is the determinant of the submatrix when deleting row and column (*i* and *j*), respectively.
- Then, m_{ij} is the minor of the element a_{ij} in **A**.

(A) ADJOINT MATRIX

MINORS

- If **A** is an n x n matrix and one row and one column are deleted, the resulting matrix is an (n-1) x (n-1) submatrix of **A**.
- The determinant of such a submatrix is called a minor of **A.**

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Each element in **A** has a minor

Delete first row and column from **A** .

The determinant of the remaining 2 x 2 submatrix is the minor of a_{11}

$$m_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

(A) ADJOINT MATRIX

MINORS

- Therefore the minor of a_{12} is: $$m_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

- And the minor for a_{13} is: $$m_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(A) ADJOINT MATRIX

- ***Example:*** Given $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$, compute its minors.

(A) ADJOINT MATRIX

COFACTORS

The cofactor C_{ij} of an element a_{ij} is defined as: $C_{ij} = (-1)^{i+j} m_{ij}$

- When the sum of a row number *i* and column *j* is even, $c_{ij} = m_{ij}$ and when *i*+*j* is odd, $c_{ij} = -m_{ij}$

$$c_{11}(i=1,\ j=1) = (-1)^{1+1} m_{11} = +m_{11}$$

$$c_{12}(i=1,\ j=2) = (-1)^{1+2} m_{12} = -m_{12}$$

$$c_{13}(i=1,\ j=3) = (-1)^{1+3} m_{13} = +m_{13}$$

(A) ADJOINT MATRIX

COFACTORS

Example: Given $A=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$, compute its cofactors.

(A) ADJOINT MATRIX

- The adjoint matrix of **A**, denoted by adj **A**, is the transpose of its cofactor matrix

$$adjA = C^T$$

Example: Given $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$, compute the adjoint matrix.

(B) DETERMINANT OF A MATRIX

- To compute the inverse of a matrix, the determinant is required
- Each square matrix **A** has a unit scalar value called the determinant of **A**, denoted by det **A** or **|A|**

If $A = \begin{bmatrix} 1 & 2 \\ 6 & 5 \end{bmatrix}$

then $|A| = \begin{vmatrix} 1 & 2 \\ 6 & 5 \end{vmatrix}$

(B) DETERMINANT OF A MATRIX

- If **A** = [**A**] is a single element (1x1), then the determinant is defined as the value of the element
- Then |**A**| =det **A** = a_{11}
- If **A** is (n x n), its determinant may be defined in terms of order (n-1) or less.

(B) DETERMINANT OF A MATRIX

- The determinant of an n x n matrix **A** can now be defined as

$$|A| = \det A = a_{11}c_{11} + a_{12}c_{12} + \ldots + a_{1n}c_{1n}$$

- The determinant of **A** is therefore the sum of the products of the elements of the first row of **A** and their corresponding cofactors.

- (It is possible to define |**A**| in terms of any other row or column but for simplicity, the first row only is used)

(B) DETERMINANT OF A MATRIX

Example: For matrix 2x2:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

What if $A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ *?*

Has cofactors : $c_{11} = m_{11} = |a_{22}| = a_{22}$

$$c_{12} = -m_{12} = -|a_{21}| = -a_{21}$$

And the determinant of A is: $|A| = a_{11}c_{11} + a_{12}c_{12} = a_{11}a_{22} - a_{12}a_{21}$

(B) DETERMINANT OF A MATRIX

Example: Given $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$, compute its determinant.

Note: ADJOINT MATRIX AND DETERMINANT

- It can be shown that: A(adj A) = (adjA) A = |A| I

Example: $A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ *... show the above expression is true.*

INVERSE OF A MATRIX - USING THE ADJOINT MATRIX

- From $AA^{-1} = A^{-1}A = I$,

  and $A(\text{adj } A) = (\text{adj}A)\,A = |A|\,I$,

  then, $$A^{-1} = \frac{adjA}{|A|}$$

INVERSE OF A MATRIX - USING THE ADJOINT MATRIX

Example: Find the inverse of the following matrices:

(1) $A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

(2) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$

Question?