1. firm 1 : $\Pi_1 = TR_1 - TC_1$

$$= (2 - bQ_1 - bQ_2 - bQ_3)Q_1 - C_1$$

$$\frac{d\Pi_1}{dQ_1} = 2 - 2bQ_1 - bQ_2 - bQ_3 = 0$$

$$2 - bQ_2 - bQ_3 = 2bQ_1$$

$$Q_1 = \frac{2 - bQ_2 - bQ_3}{2b}$$

firm 2 : $TR_2 - TC_2$

$$= (2 - 6Q_1 - 6Q_2 - 6Q_3)Q_2 - C_2$$

$$\frac{d\Pi_2}{dQ_2} = 2 - bQ_3 - 2bQ_2 - 6Q_3 = 0$$

$$2 - bQ_1 - bQ_3 = 2bQ_2$$

substitute Q_1

$$2bQ_2 = 2 - b\left(\frac{a - bQ_2 - bQ_3}{2b}\right) - bQ_3$$

$$2bQ_2 = \frac{2a - a + bQ_2 + bQ_3 - 2bQ_3}{2}$$

$$4bQ_2 = a + bQ_2 - bQ_3$$

$$3bQ_2 = a - bQ_3$$

$$Q_2 = \frac{a - bQ_3}{3b} ※$$

substitute Q_2

$$Q_1 = \frac{a - b\left(\frac{2 - bQ_3}{3b}\right) - bQ_3}{2b}$$

$$= \frac{3a - 2 + 6Q_3 - 3bQ_3}{6b}$$

$$Q_1 = \frac{2 - 6Q_3}{3b} \#$$

firm 3 : $\Pi_3 = TR_3 - TC_3$

$$\Pi_3 = (a - bQ_1 - bQ_2 - bQ_3)Q_3 - C_3$$

$$\frac{d\Pi_3}{dQ_3} = 2 - bQ_1 - bQ_2 - 2bQ_3 = 0$$

$$2 - bQ_1 - bQ_2 = 2bQ_3$$

$$Q_3 = 2 - bQ_1 - bQ_2$$

$$= \frac{a - b\left(\frac{a - bQ_3}{36}\right) - b\left(\frac{a - bQ_3}{36}\right)}{2b}$$

$$= \frac{3a - a - bQ_3 - a + 6Q_3}{6b}$$

$$= \frac{a + 2bQ_3}{6b}$$

$$6bQ_3 = a + 2bQ_3$$

$$4bQ_3 = a + 2bQ_3$$

$$4bQ_3 = a$$

$$Q_3 = \frac{a}{4b}$$

sub Q_3

$$Q_1 = \frac{2 - \left(\frac{2}{4b}\right)b}{3b} = \frac{a}{4b}$$

$$Q_2 = \frac{2 - \left(\frac{a}{4b}\right)b}{36} = \frac{4a - 2}{12b} = \frac{a}{4b}$$

equilibrium price ; $P = a - bQ$

$$= a - b(Q_1 + Q_2 + Q_3)$$

$$= a - \frac{3a}{4} = \frac{a}{4} = 0.25a$$

firm$_1$; $\Pi_1 = P \cdot Q - C_1$

$= 0.25a \cdot \frac{2}{4b} - C_1$

$\Pi_1 = \frac{a^2}{16b} - C_1$

firm 2 ; $\Pi_2 = P \cdot Q_2 - C_2$

$= 0.25a \cdot \frac{a}{4b} - C_2$

$= \frac{a^2}{16b} - C_2$

firm 3 ; $\Pi_3 = P \cdot Q_3 - C_3$

$= 0.25a \cdot \frac{2}{4b} - C_3$

$= \frac{a^2}{16b} - C_3$

2. If there are N firms

$q_i^* = f(N)$, $P = f(N)$, $\Pi_i = f(N)$

assume $q_1 + q_2 + \dots q_n = A$

$P = q - b(q_1 + q_2 + \dots q_n)$

$P = q - bq_1 - bq_2 + \dots + bq_n$

$\Pi_i = (P-C)q_i$

$\Pi_1 = (a - bq_1 - bq_2 - \dots - bq_n)q_1 - C_1$

$\frac{\partial \Pi_1}{\partial q_1} = 0 \Rightarrow q_1 = \frac{a}{2b} - 0.5(q_2 + q_3 + \dots + q_n)$

$\vdots$

$q_n = \frac{a}{2b} - 0.5(q_2 + q_3 + \dots q_{n-1})$

$\therefore q_1^* = q_2^* = \dots = q_n^* = \frac{a}{b} - A$ — ①

$A = q_1 + q_2 + \dots + q_n$

$A = n\left(\frac{q}{b} - A\right)$

$A = \frac{nq}{b} - nA$

$A + nA = n\left(\frac{a}{b}\right)$

$A(n+1) = n\left(\frac{a}{b}\right) \rightarrow A = \frac{na}{(n+1)b}$

sub A into ① ; $q_1 = \frac{q}{(n+1)b}$

$\therefore q_i = \frac{q}{(n+1)b}$

$p = a - bA$

$= a - b\left(\frac{na}{(n+1)b}\right)$

$= \frac{na + a - na}{n+1}$ $\therefore P = \frac{a}{n+1}$

$\Pi_i = P \cdot q_i + C_i$

$= \frac{a}{n+1} \cdot \frac{a}{(n+1)b} - C_i$ $\therefore \Pi_1 = \frac{a^2}{(n+1)^2 b} - C_i$

3. from question 2, what happen if $N \to \infty$

" $N = 1$

In general

- If $N \to \infty$, it means that the market output (Q) goes to a competitive level and the price (p^*) converges to marginal cost. The market is perfectly competitive market.

- If cournot model, there must be 2 or more firms competing in the market. so, if $n=1$, firm will becomes a monopolist.

if $n \to \infty$;

- $q_i = \frac{a}{(n+1)b}$ will be nearly to 0 and each firm will sell of q nearly 0 unit.
- $A = \frac{nq}{(n+1)b}$ will be nearly to 0, q of every firms combined will be nearly a units.
- $P = \frac{q}{n+1}$ will be neary to 0, supply will increase as p decreases to nearly 0
- $\Pi_i = \frac{q^2}{(n+1)^2 b} - C_i \rightarrow$ each firms will lose their profit.

if $n=1$

- $q_i = \frac{q}{(n+1)b} = \frac{q}{2b} \rightarrow$ since $q = \frac{a}{2b} < q = \frac{na}{(n-1)b}$, monopoly will sell less.
- $A = \frac{nq}{(n+1)b} = q$, it mean then firm will become a monopolist.
- $P = \frac{q}{n+1} = \frac{q}{2} \rightarrow$ since $P_m = \frac{q}{2} > P_c = \frac{q}{n+1}$, monopoly will set higher price.
- $\Pi_i = \frac{q^2}{(n+1)^2 b} - C_i = \frac{q^2}{4b} - C_i \rightarrow$ since $\Pi_m > \Pi_c$, monopolist will each higher profit.