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Techatanates
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Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

1. Given this information

$\sum u_i^2 = \sum (y-\bar{y})^2 - \frac{(\sum(x-\bar{x})(y-\bar{y}))^2}{\sum(x-\bar{x})^2} = 882.97 - \frac{(-174.20)^2}{1098.8} = 855.3529$

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$		$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

$$\hat{\beta}_1 = \overline{Y} - \hat{\beta}_2 \overline{X} = 21.03 - (-0.1587)(12.20) = 21.03 + 1.9341 = \underline{22.9641}$$

$$\hat{\beta}_2 = \frac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{\sum (X_i - \overline{X})^2} = \frac{-174.20}{1098.8} = \underline{-0.1587}$$

when x does not change we have $y = \hat{\beta}_1$, when x change 1 unit then y change $\hat{\beta}_2$ units.

b) Find r^2 and explain its meaning.

$$r^2 = \frac{\sum\left((x-\bar{x})(y-\bar{y})\right)^2}{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}$$

$$\frac{\sum(\hat{y}_i-\bar{y})^2}{\sum(y_i-\bar{y})^2} = \frac{\sum \hat{u}_i^2}{\sum(y_i-\bar{y})^2} = 1 - \frac{873.14}{882.97} = 0.0111$$

$$r^2 = \frac{(-179.20)^2}{(1098.8)(882.97)}$$

$$= 0.0313$$

the 0.0111 means that the estimator is very close to the actual regression line.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

$$\hat{y}_i = \beta_1 + \beta_2 x$$

$$\hat{y}_i = \beta_1 + \beta_2(5)$$

$$\hat{y}_i = 22.9641 - 0.1585(5)$$

$$\hat{y}_i = 22.1716$$

this means that when $X_i = 5$ the expected value of y_i is 22.1716

d) Find the estimators of $\text{var}(u_i)$, $\text{var}(\hat{\beta}_1)$ and $\text{var}(\hat{\beta}_2)$

$$Var(u_i) = \sigma^2$$

$$= \frac{\sum \hat{u}_i^2}{n-k}$$

$$= \frac{\sum(y_i-\hat{y})^2}{n-k}$$

$$= \frac{882.97}{30-2}$$

$$= 31.5346$$

$$Var(\hat{\beta}_1) = \sigma^2_{\hat{\beta}_1}$$

$$= \frac{\sum x_i^2}{n\sum x_i^2}\sigma^2$$

$$= \frac{\sum x_i^2 \sigma^2}{n\sum(x_i-\bar{x})^2}$$

$$= \frac{(5,564)(31.5346)}{(30)(1098.8)}$$

$$= 5.3227$$

$$Var(\hat{\beta}_2) = \sigma^2_{\hat{\beta}_2}$$

$$= \frac{\sigma^2}{\sum(x-\bar{x})^2}$$

$$= \frac{31.5346}{1098.8}$$

$$= 0.0287$$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

e. step 1: $H_0 = 0$

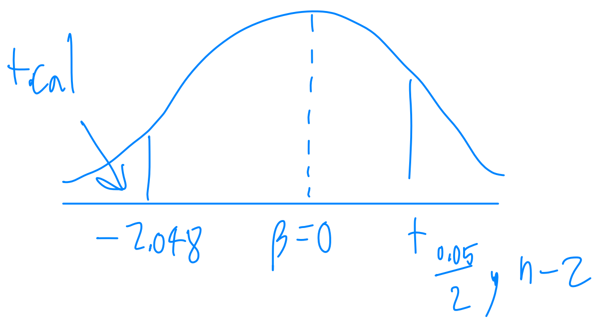
$H_a \neq 0$

step 2: $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}}$

$= \dfrac{-0.1587 - 0}{0.0287}$

$= -5.530$

step 3:



reject null hypothesis. $t_{0.025, 28}$

2.048

there is evidence to claim that $\hat{\beta}_2$ is not zero with 0.05 sig.

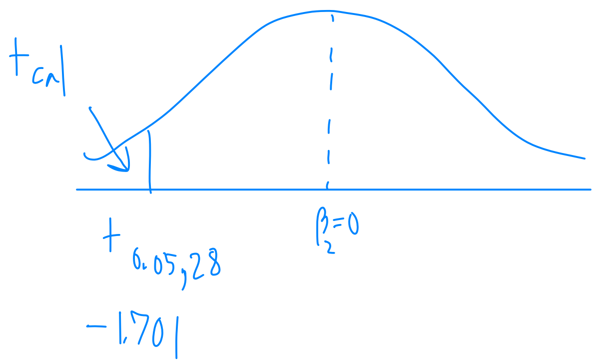
e. $H_0 : \beta_2 \leq 0$

$H_a : \beta_2 > 0$

step 2: $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}}$

$$= \frac{-0.1587 - 0}{0.0287}$$

$$= -5.530$$



Cannot reject null hypothesis

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45,$

$\hat{\sigma}^2 = 212{,}877,$

$\sum(X_i - \bar{X})^2 = 78.73,$

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

From $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$ $\hat{\beta}_1$ is the initial price of the car while $\hat{\beta}_2$ is the depreciation of the car and X_i is the number of years. As the years go by it makes sense for the value of the car to drop as the car age and is used,

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

$a = 0.05$

formula for finding $\hat{Y}_i$

$$E(Y|X=5) = \beta_1 + \beta_2 X_i$$
$$= 7836 - 502.4(5)$$
$$= 5324$$

Given $a = 0.05$, $\frac{a}{2} = 0.025$, $d.f. = n - k = 11 - 2 = 9$

Confidence interval

1. Find $var(\hat{Y}_0) = \hat{\sigma}^2 \left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum (X_i - \bar{X})^2}\right]$

$$= 212{,}877\left(\frac{1}{11} + \frac{(5 - 7.45)^2}{78.73}\right)$$
$$= 35582.5345$$

2. $\hat{\sigma}_{\hat{Y}_0} = \sqrt{35{,}582.5345} = 188.6333$

3. find 95% of confidence interval from $E(Y|X_0=5)$

lower bound : $\hat{Y}_0 - (t_{\frac{a}{2}} \cdot \hat{\sigma}_{\hat{Y}_0}) = 5324 - (2.262 \times 188.6333)$

$= 4897.3115$ usd

upper bound : $\hat{Y}_0 + (t_{\frac{a}{2}} \cdot \hat{\sigma}_{\hat{Y}_0}) = 5{,}324 + (2.262 \cdot 768.6333)$

$= 5750.6885$

$\therefore$ the market price range where we can be 95% sure that $E(Y_0|X_0=3)$ is within 4897.3115 to 5750.6885 usd

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

$\hat{Y}_i = 7836 - 502,4X_i$

New SRF: $\hat{Y}_i = 7836 - 5024X_i$

the interpretation should be the same so the new standard error is 4118

d) Calculate the elasticity of market price when a car is 10 years old.

$\hat{Y}_i = 7836 - 502,4(10)$

$= 2812$

elasticity $= \beta_2 \left(\frac{X}{Y}\right)$

$= 7836 \left(\frac{10}{2812}\right)$

$= 27,866$