

A hand is shown placing a smooth, dark stone onto a stack of four other smooth stones on a pebbly beach. The background is a blurred view of the ocean with gentle waves. The text 'Risk Preferences' is overlaid on the image, centered horizontally and partially over the stones.

Risk Preferences

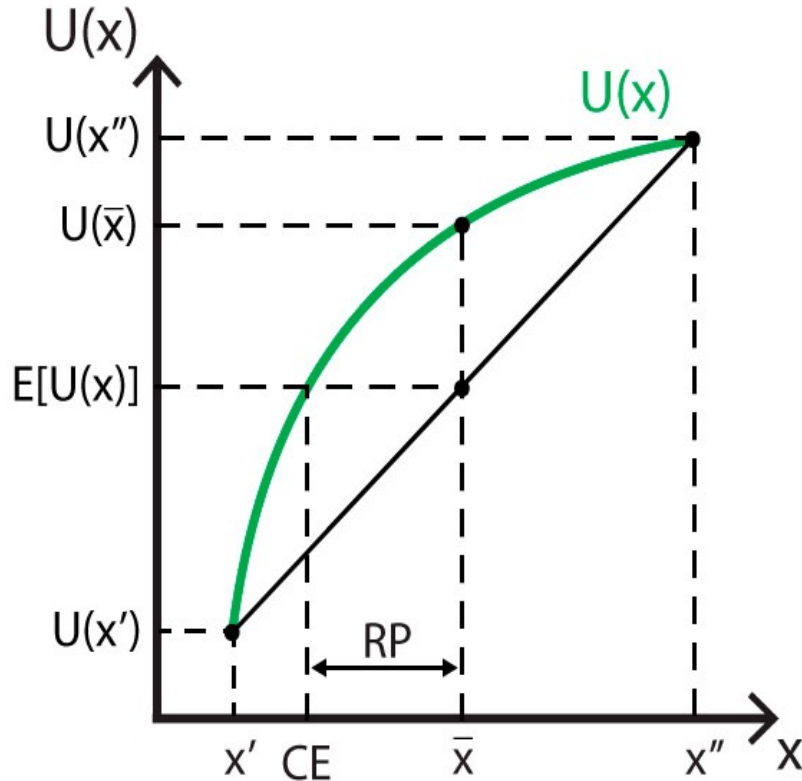
PROSPECT THEORY PART2:
EDITING STAGE & EVALUATION STAGE'S VALUE FUNCTION

Recap: Risk aversion & Certainty equivalent

$$U(CE) = E[U(x)]$$

The **certainty equivalent** is an amount of money that provides equal utility to the random payoff of the gamble.

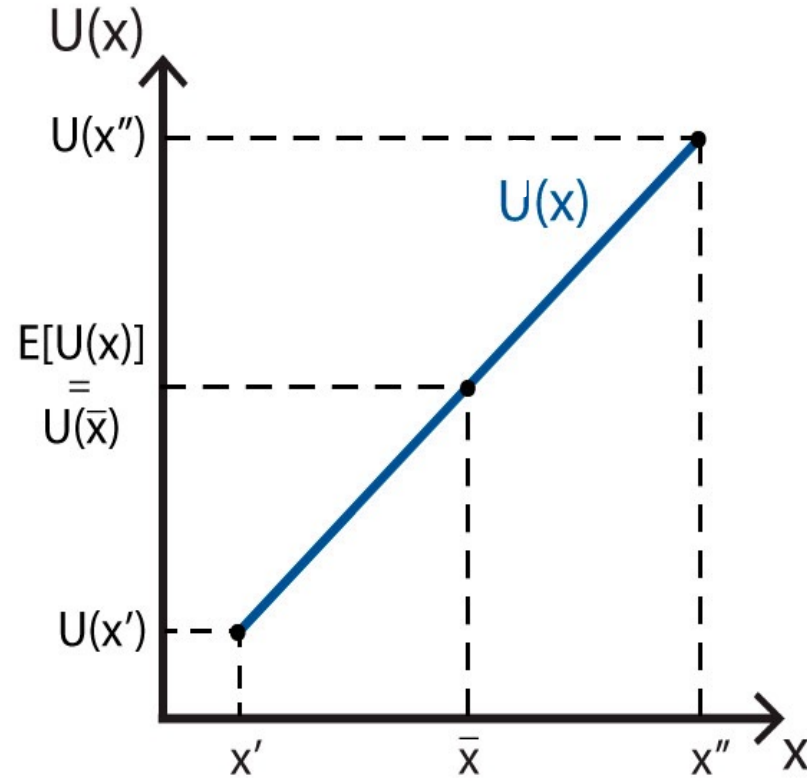
The certainty equivalent is labeled CE in the figure. Note that CE is less than the expected outcome, if the person is risk averse. This is because risk-averse individuals prefer the expected outcome to the risky outcome.



Risk averse individual

$$E[U(x)] < U(\bar{x})$$

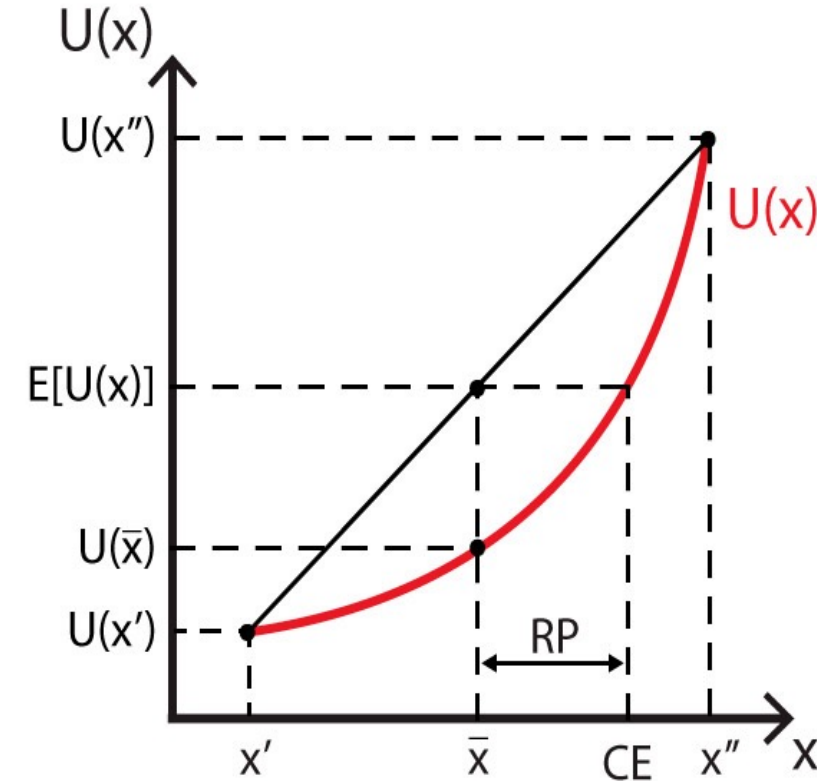
$$CE < \bar{x}$$



Risk neutral individual

$$E[U(x)] = U(\bar{x})$$

$$CE = \bar{x}$$



Risk loving individual

$$E[U(x)] > U(\bar{x})$$

$$CE > \bar{x}$$

Recap

Suppose you have initial wealth w ,

Consider lotteries $X = (x, p; 0, 1 - p)$, i.e. $X = (x, p)$

How will you evaluate lottery X ?

Expected Value Theory

$$EV = px + w$$

Expected Utility Theory

$$EU = pU(w + x) + (1 - p)U(w)$$

Prospect theory

$V(X, p) = \pi(p)v(x)$, $v(\cdot)$ at the reference point is usually normalized to 0.

Recap

Suppose you have initial wealth w ,

Consider two lotteries $X = (x, p; 0, 1 - p)$ and $Y = (y, q; 0, 1 - q)$

You will prefer lottery X over lottery Y IF....

Expected Value Theory

$$w + px > w + qy$$

Expected Utility Theory

$$pU(w + x) + (1 - p)U(w) > qU(w + y) + (1 - q)U(w)$$

Prospect theory

$$\pi(p)v(x) > \pi(q)v(y)$$



Editing Stage



Evaluation Stage

Prospect Theory

Prospect Theory: Editing Stage

organize & reformulate the problem

What is going on in editing stage? Taking an objective prospect $(\hat{x}_1, \hat{p}_1; \dots; \hat{x}_m, \hat{p}_m)$ and transforming it into an object for evaluation $(x_1, p_1; \dots; x_n, p_n)$ by:

- Coding: code outcomes as gains & losses relative to reference point.
- Combination: e.g., $(100, .5; 100, .5)$ replaced with $(100, 1)$.
- Segregation: e.g., $(100, .5; 200, .5)$ replaced with 100 for sure plus $(0, .5; 100, .5)$.
- Cancellation: discard shared components.
- Simplification: rounding of probabilities.
- Eliminating dominated alternatives.

Prospect theory: Evaluation stage

A prospect can be written as $(x, p; y, q)$ with $p + q \leq 1$.

Note: $p + q < 1$ implies prospect yields 0 with probability $1 - p - q$.

A person evaluates a prospect $(x, p; y, q)$ according to the functional

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y)$$

Note: Nowadays, it might be best to take Prospect theory as a theory for simple prospects with at most two non-zero outcomes.

Prospect Theory: Value Function

Three key features of the value function $v(\cdot)$:

- The carriers of value are changes in wealth, instead of final wealth.
 - The value function is defined on deviations from **the reference point**.
 - $v(0) = 0$
- Diminishing sensitivity to the magnitude of changes:
 - Concave in the gain domain: $v''(x) < 0$ for $x > 0$
 - Convex in the loss domain: $v''(x) > 0$ for $x < 0$
 - The marginal value of both gains and losses generally decreases with their magnitude.
- Loss aversion: losses loom larger than gains.

$$\frac{v'(x|x < 0)}{v'(x|x > 0)} > 1, \text{ or } (y, 0.5; -y, 0.5) \succ (x, 0.5; -x, 0.5) \text{ if } 0 \leq y < x$$

The reference point

- The reference point is usually where you are (the status quo), but can be where you would like to be, or think you should be.
- For example, in terms of wealth, reference point can be the previous level, the aspired level, the wealth of other players, etc.
- We do not yet fully understand how reference points are determined, but they should be subject to phenomena such as adaptation and social pressure (social norms), etc.

Value function's diminishing sensitivity

Prospect theory attempts to explain **risk aversion in gains** and **risk loving in losses** by positing that the utility function is fundamentally wrong from biological principles as follows:

Value function's diminishing sensitivity

- The nervous system is set up to primarily **detect differences, not absolute levels.**
- A gain is perceived as a pleasurable change from the status quo (reference point) and the nervous system shows **a decreasing response both to the intensity and duration of pleasurable stimuli.**
- A loss is perceived as a painful change from the status quo (reference point) and the nervous system shows **a decreasing response both to the intensity and duration of painful stimuli.**

Risk aversion over gain & Risk seeking over loss

In a situation where a sure gain is compared to a gamble with (averagely or highly) probable larger gain, diminishing sensitivity cause risk aversion.

In a situation where a sure loss is compared to a gamble with (averagely or highly) probable larger loss, diminishing sensitivity cause risk seeking.

At the same level of wealth, an individual can both be risk averse and risk seeking.

Prospect Theory: Value Function

Two common functional forms for the value function:

Tversky & Kahneman (1992)

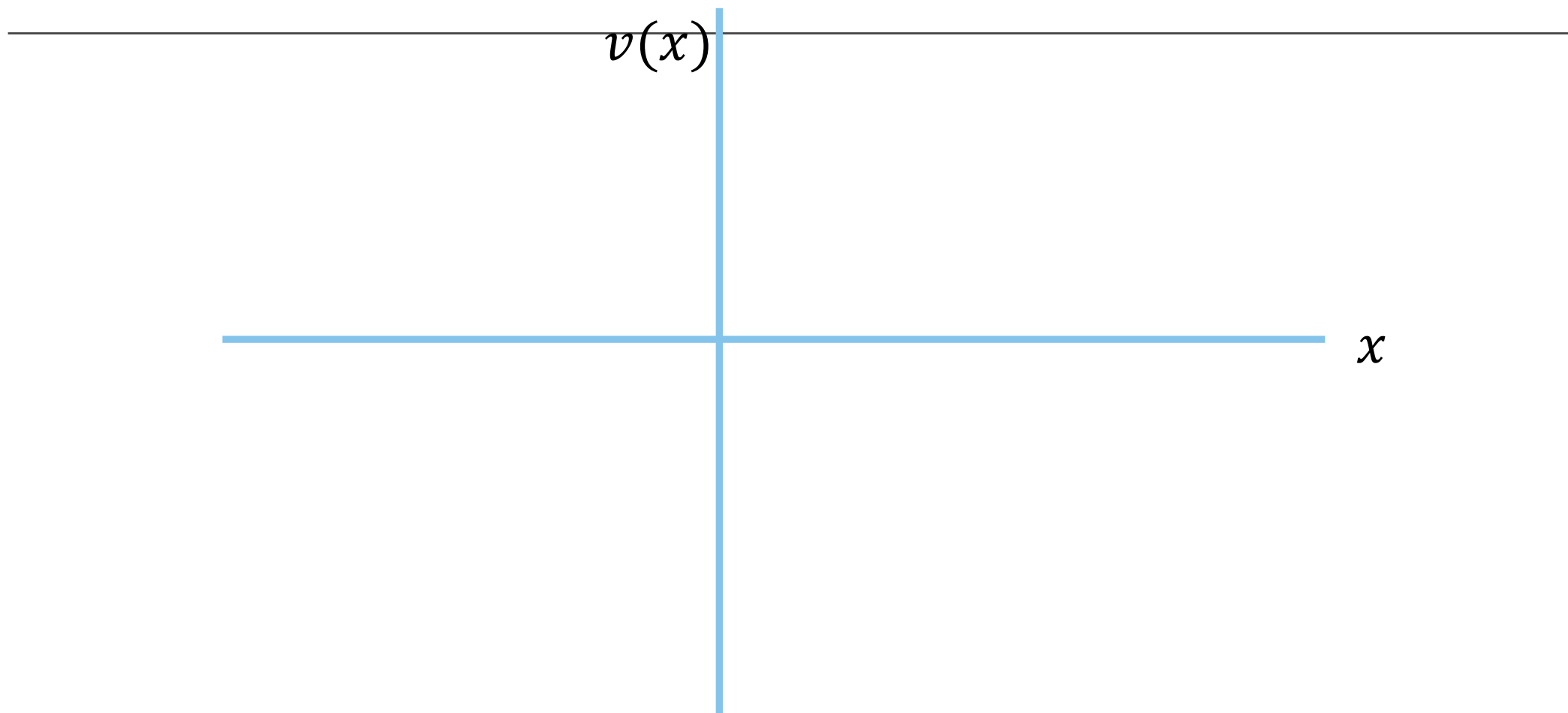
$$v(x) = \begin{cases} x^\alpha & \text{if } x > 0 \\ -\lambda(-x)^\beta & \text{if } x < 0 \end{cases} \quad , \text{where } \alpha, \beta \in (0,1] \text{ and } \lambda \geq 1$$

Two-part linear

$$v(x) = \begin{cases} x & \text{if } x > 0 \\ \lambda x & \text{if } x < 0 \end{cases} \quad , \text{where } \lambda \geq 1$$

λ is the coefficient of loss aversion.

Value function



Utility function vs. Value function

Utility function

Evaluation of choices is made over final states of wealth.

x-axis: final wealth

y-axis: utility

Value function

Evaluation of choices is made relative to a reference point.

x-axis: gain, loss

y-axis: value

Utility function vs. Value function
