Chapter 7 Curve Sketching

In this chapter we will learn how to sketch the curve of relation by considering its properties such as the intercepts, symmetrical, domain, range, and asymptotes.

Guidelines for Curve Sketching

1. **Domain (D) and Range (R)**

   Domain is the set of values of x for which $f(x)$ is defined.

   Range is the set of values of $f(x)$ correspond to the domain.

2. **Intercepts**

   To find y-intercept, we set $x = 0$ and solve for y.

   To find x-intercept, we set $y = 0$ and solve for x.

3. **Symmetry**

   (i) If $f(-x) = f(x)$ for all x in D, then f is an even function and the curve is symmetric about the y-axis (reflectional symmetry).

   (ii) Replace y with $-y$ and get the same function then the curve is symmetry about the x- axis.

   (iii) If $f(-x) = -f(x)$ for all x in D, then f is an odd function and the curve is symmetric about the origin (rotational symmetry).

4. **Asymptotes**

   (i) *Horizontal Asymptotes*.

   If either $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$, then the line $y = L$ is a horizontal asymptote of the curve $y = f(x)$.

   (ii) *Vertical Asymptotes*.

   The line $x = a$ is a vertical asymptote if at least one of the following statements is true:

$$\lim_{x\to a^+} f(x) = \infty \qquad \lim_{x\to a^-} f(x) = \infty$$

$$\lim_{x\to a^+} f(x) = -\infty \qquad \lim_{x\to a^-} f(x) = -\infty$$

   Another method to find asymptotes:

   (Arranging the relation in the form of a product of linear factors)

   We can use method to find not only the horizontal and vertical asymptotes but also different type of asymptotes as well. To find the asymptotes, we arrange the relation in the form of the product of linear factors on one side and a constant on the other side, then the equation of the asymptote can be found by setting each linear factor equal to zero.

5. Intervals of Increase or Decrease

Compute $f'(x)$ and use the Increasing/Decreasing Test.

- If $f'(x) > 0$, then f is increasing.
- If $f'(x) < 0$, then f is decreasing.

Step 1) Find $f'(x)$

Step 2) Find critical numbers

Step 2.1) Find the values of x that make $f'(x)$ does not exist.

Step 2.2) Set $f'(x) = 0$, then solve for *x*.

Step 3) Find the sign of $f'(x)$

6. Local Maximum and Minimum Values

- If f' changes from positive to negative at critical number c, then $f(c)$ is a local maximum.
- If f' changes from negative to positive at critical number c, then $f(c)$ is a local minimum.

7. Concavity and Points of Inflection

Compute $f''(x)$ and use the Concavity Test.

- If $f''(x) > 0$, then the curve is concave upward.
- If $f''(x) < 0$, then the curve is concave downward.
- There exist the inflection points when $f''(x)$ changes concave upward to concave downward and vice versa.

Step 1) Find $f''(x)$

Step 2) Find critical numbers

Step 2.1) Find the values of x that make $f''(x)$ does not exist.

Step 2.2) Set $f''(x) = 0$, then solve for *x*.

Step 3) Find the sign of $f''(x)$.

8. **Sketch the Curve** by using the information from step 1 – step 7.

Example 7.1: Consider the relation $x^2y - x^2 - y + 4 = 0$. Find the domain, range, intercepts, symmetry, asymptotes, and sketch the curve of this relation.

Example 7.2: Consider the relation $x^2y + 2y = 6$. Find the domain, range, intercepts, symmetry, asymptotes, and sketch the curve of this relation.

Example 7.3: Consider the relation $x^2 - xy - x + y - 2 = 0$. Find the domain, range, intercepts, symmetry, asymptotes, and sketch the curve of this relation.