Univariate Time Series Model

Stationary Process Models

- Moving Average (MA(q))
- Autoregressive (AR(p))
- Autoregressive Moving Average (ARMA(p,q))

Nonstationary Process Models

- Autoregressive Integrated Moving Average (ARIMA(p,d,q))

Moving Average (MA(q))

Relationship between mean value of dependent variable with weighted average of random disturbances up to q period.

Moving Average process of order q or MA(q)

$$y_t = \delta + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_q\varepsilon_{t-q}$$

Autoregressive (AR(p))

Relationship between mean value of dependent variable with weighted average of its previous values up to p period.

Autoregressive process of order p or AR(p)

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t$$

Autoregressive Moving Average (ARMA(p,q))

Combination of Autoregressive and Moving Average of order p and q or ARMA(p,q)

$$\begin{aligned} y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} \\ + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \end{aligned}$$

Autoregressive Integrated Moving Average (ARIMA(p,d,q))

Model for integrated nonstationary series of order d or $(I(d))$ that already transformed using combination of Autoregressive and Moving Average process of order p and q or ARIMA(p,d,q)

$$\Delta^d y_t = \delta + \phi_1 \Delta^d y_{t-1} + \phi_2 \Delta^d y_{t-2} + \cdots + \phi_p \Delta^d y_{t-p}$$
$$+\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}$$

Seasonality

Time Series Component

Seasonal Adjustment – Census X-12

Seasonal ARIMA (SARIMA)

Box-Jenkins Methodology

1. Identification

Identify appropriated value of p, d, &q

- use ACF, PACF, and LB Test
- use AIC or SIC – (use the model with lowest value)

$$AIC = e^{2(p+q+1)/T} \frac{\sum \hat{\varepsilon}_t^2}{T}$$

$$SIC = T^{(p+q+1)/T} \frac{\sum \hat{\varepsilon}_t^2}{T}$$

Box-Jenkins Methodology

2. Estimation

Estimate ARIMA(p,d,q) using appropriated method (e.g. NLS, MLE, or GMM)

3. Diagnostic Checking

Check whether the series follow properties of the models. e.g. check residuals from the estimated ARIMA model whether they are normally distributed.

Box-Jenkins Methodology

4. Forecasting

Choose the model that best fit the series based on your objective (e.g. accurate prediction—check forecasting error (RMSE, or Theil's Inequality Coefficient)

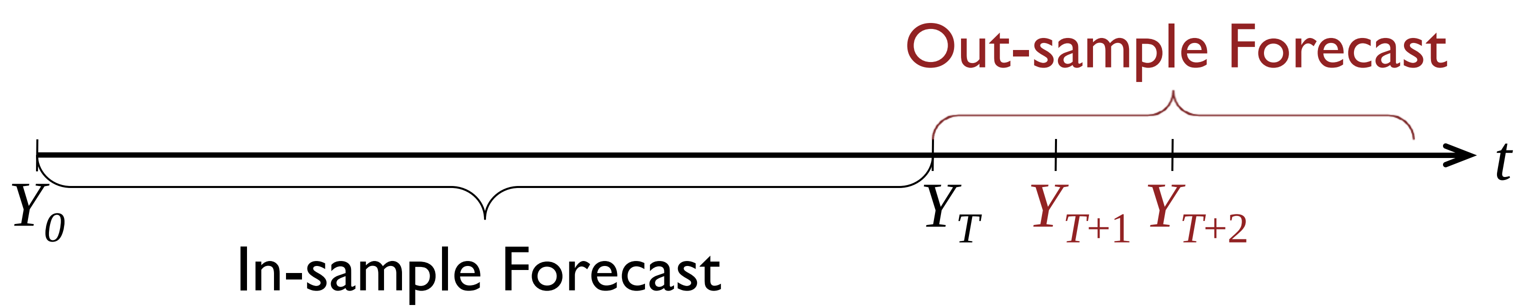
Forecasting

In-sample Forecast

Forecast that use data within the estimation sample.

Out-sample Forecast

Forecast that use data outside the estimation sample.



Forecasting

Static Forecast

Forecast that use actual value as predictors.

Dynamic Forecast

Forecast that use forecast value as predictors.

t	Y_t^A	Y_t^S	Y_t^D
1	Y_1^A		
2	Y_2^A	Y_2^S	Y_2^D
3	Y_3^A	Y_3^S	Y_3^D
4	Y_4^A	Y_4^S	Y_4^D
$\vdots$	$\vdots$	$\vdots$	$\vdots$
T-1	Y_{T-1}^A	Y_{T-1}^S	Y_{T-1}^D
T	Y_T^A	Y_T^S	Y_T^D

where: Y_t^A = Actual Value

Y_t^S = Static Forecast Value

Y_t^D = Dynamic Forecast Value

Index for Forecasting Error

Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{\sum_{t=1}^{T}(Y_t^A - Y_t^D)^2}{T}}$$

Theil's Inequality Coefficient

$$U = \sqrt{\frac{\sum_{t=1}^{T}(Y_t^A - Y_t^D)^2}{\left(\sum_{t=1}^{T} Y_t^A\right)^2 + \left(\sum_{t=1}^{T} Y_t^D\right)^2}}$$

Forecasting

One-step-ahead and Multi-step-ahead forecasts.

One-step-ahead $\hat{y}_{t+1} = f(Y_t)$

Multi-step-ahead $\hat{y}_{t+h} = E[y_{t+h} \mid Y_t]$

Forecasting and AR Process

One-step-ahead $\hat{y}_{t+1} = \alpha + \phi_1 y_t + \cdots + \phi_p y_{t+1-p}$

Two-step-ahead

$$\hat{y}_{t+2} = \alpha + \phi_1 \hat{y}_{t+1} + \phi_2 y_t + \cdots + \phi_p y_{t+2-p}$$

Forecasting

One-step-ahead Forecasting Error

$$y_{t+1} - \hat{y}_{t+1} = \varepsilon_{t+1}$$

Forecasting Error Variance σ^2

Two-step-ahead Forecasting Error

$$y_{t+2} - \hat{y}_{t+2} = \varepsilon_{t+2} + \phi_1 \varepsilon_{t+1}$$

Forecasting Error Variance $\sigma^2 (1 + \phi_1^2)$

Forecasting