Quiz 4

1. Find

$$\int \left[\frac{1}{\sqrt{3-2x}} + \frac{\sqrt{1-e^{-2x}}}{e^{2x}} \right] dx.$$

Solution

Consider

$$\int \left[\frac{1}{\sqrt{3-2x}} + \frac{\sqrt{1-e^{-2x}}}{e^{2x}} \right] dx = \int \frac{1}{\sqrt{3-2x}}\, dx + \int \frac{\sqrt{1-e^{-2x}}}{e^{2x}}\, dx.$$

We can apply substitution technique for each of the terms $\int \frac{1}{\sqrt{3-2x}}\, dx$ and $\int \frac{\sqrt{1-e^{-2x}}}{e^{2x}}\, dx$ separately.

(I) Let $u = 3-2x$. Then $du = -2dx$ or $dx = \frac{1}{-2}du$.

$$\begin{aligned}
\int \frac{1}{\sqrt{3-2x}}\, dx &= \int \frac{1}{\sqrt{u}} \left[\frac{1}{-2}\right] du \\
&= -\frac{1}{2}\int u^{-1/2}\, du \\
&= -\frac{1}{2}\left[\frac{u^{1/2}}{1/2}\right] + C \\
&= -(3-2x)^{1/2} + C
\end{aligned}$$

(II) Let $u = 1-e^{-2x}$. Then $du = -(-2e^{-2x}dx)$ or $dx = \frac{1}{2e^{-2x}}du$.

$$\begin{aligned}
\int \frac{\sqrt{1-e^{-2x}}}{e^{2x}}\, dx &= \int \frac{\sqrt{u}}{e^{2x}} \left[\frac{1}{2e^{-2x}}\right] du \\
&= \frac{1}{2}\int u^{1/2}\, du \\
&= \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right] + C \\
&= \frac{1}{3}(1-e^{-2x})^{3/2} + C
\end{aligned}$$

From (I) and (II),

$$\int \left[\frac{1}{\sqrt{3-2x}} + \frac{\sqrt{1-e^{-2x}}}{e^{2x}} \right] dx = -(3-2x)^{1/2} + \frac{1}{3}(1-e^{-2x})^{3/2} + C$$

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