6104640252
Jivathorn Chanthaweekul

EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$$\hat{\beta}_1 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$\bar{x} = \frac{63+72+\dots+90}{8} = \frac{621}{8} = 77.625$, $\bar{y} = \frac{2.8+3.4+\dots+3.7}{8} = \frac{25.7}{8} = 3.2125$

$$\hat{\beta}_1 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} = \frac{2012.4 - (8)(77.625)(3.2125)}{48,717 - (8)(77.625)^2} = \frac{17.4375}{511.875} = 0.034066$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 3.2125 - (0.034066)(77.625) = 0.5681$$

1.2 For each observation i, find $\widehat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$ $\quad$ $\hat{u}_i = Y_i - \hat{Y}_i$

$\hat{Y}_1 = 0.5681 + (0.034066)(63) = 2.714$
$\hat{Y}_2 = 0.5681 + (0.034066)(72) = 3.021$
$\hat{Y}_3 = 0.5681 + (0.034066)(78) = 3.225$
$\hat{Y}_4 = 0.5681 + (0.034066)(81) = 3.329$
$\hat{Y}_5 = 0.5681 + (0.034066)(87) = 3.532$
$\hat{Y}_6 = 0.5681 + (0.034066)(75) = 3.123$
$\hat{Y}_7 = 0.5681 + (0.034066)(75) = 3.123$
$\hat{Y}_8 = 0.5681 + (0.034066)(90) = 3.634$

$\hat{u}_1 = Y_1 - \hat{Y}_1 = 2.8 - 2.714 = 0.086$
$\hat{u}_2 = Y_2 - \hat{Y}_2 = 3.4 - 3.021 = 0.379$
$\hat{u}_3 = Y_3 - \hat{Y}_3 = 3 - 3.225 = -0.225$
$\hat{u}_4 = Y_4 - \hat{Y}_4 = 3.5 - 3.327 = 0.173$
$\hat{u}_5 = Y_5 - \hat{Y}_5 = 3.6 - 3.532 = 0.068$
$\hat{u}_6 = Y_6 - \hat{Y}_6 = 3 - 3.123 = -0.123$
$\hat{u}_7 = Y_7 - \hat{Y}_7 = 2.7 - 3.123 = -0.432$
$\hat{u}_8 = Y_8 - \hat{Y}_8 = 3.7 - 3.634 = 0.066$

$\sum_{i=0}^{N} \hat{u}_i = 0.086 + 0.379 - 0.225 + 0.173 + 0.068 - 0.123 - 0.432 + 0.066$
$= 0$

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1.3 Find $var(\hat{u}_i), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$Var(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{(0.086)^2 + ... + (0.066)^2}{6} = \frac{0.4349}{6} = 0.0725 = \sigma^2$

$Var(\hat{\beta}_0) = \frac{\sigma^2}{\sum x_i^2} = \frac{0.0725}{63+72+...+90} = 0.0001416$

$Var(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum x_i^2} \cdot \sigma^2 = \frac{(48717)(0.0725)}{(8)(511.875)} = 0.8625$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning. $\bar{X} = 20$, $\bar{Y} = 9.1$

$\hat{\beta}_1 = \frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2} = \frac{2214 - (10)(20)(9.1)}{4440 - (10)(20)^2} = \frac{394}{440} = 0.895$

$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = 9.1 - (0.895)(20) = -8.809$

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$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$ $\hat{u}_i = y_i - \hat{y}_i$ $\hat{u}_i = y_i - \hat{y}_i$

2.2 Find the value of $\widehat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y}_1 = -8.809 + 0.895(10) = 0.141$

$\hat{y}_2 = -8.809 + 0.895(12) = 1.931$

$\hat{y}_3 = -8.809 + 0.895(14) = 3.721$

$\hat{y}_4 = -8.809 + 0.895(16) = 5.511$

$\hat{y}_5 = -8.809 + 0.895(18) = 7.301$

$\hat{y}_6 = -8.809 + 0.895(22) = 10.881$

$\hat{y}_7 = -8.809 + 0.895(24) = 12.671$

$\hat{y}_8 = -8.809 + 0.895(26) = 14.461$

$\hat{y}_9 = -8.809 + 0.895(28) = 16.251$

$\hat{y}_{10} = -8.809 + 0.895(30) = 18.041$

$\hat{u}_1 = 0 - 0.141 = -0.141$

$\hat{u}_2 = 2 - 1.931 = 0.069$

$\hat{u}_3 = 5 - 3.721 = 1.279$

$\hat{u}_4 = 6 - 5.511 = 0.489$

$\hat{u}_5 = 7 - 7.301 = -0.301$

$\hat{u}_6 = -0.881$

$\hat{u}_7 = -2.671$

$\hat{u}_8 = 0.539$

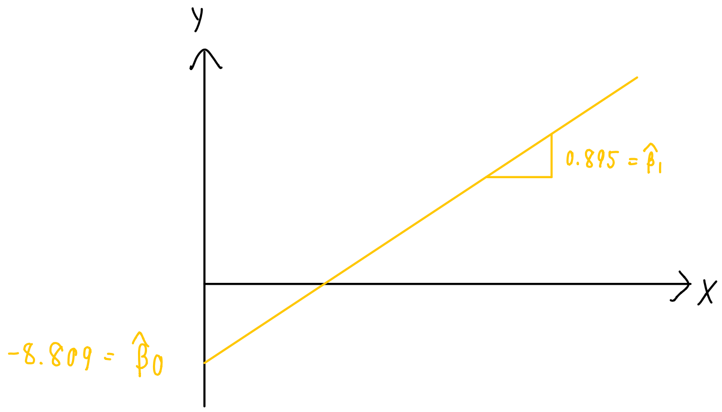
$\hat{u}_9 = -0.251$

$\hat{u}_{10} = 1.959$

$\sum_{i=0}^{N} \hat{u}_i = -0.141 + 0.069 + \ldots + 1.959 = 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

$\bar{X} \to 20$, $\bar{Y} \to 9.1$



$y = \beta_0 + \beta_1 X$

$y = -8.809 + 0.895(20)$

$Y = -8.809 + 17.9$

$Y = 9.091 \approx 9.1$

$\therefore$ The line passes $(\bar{X}, \bar{Y})$

2.4 If X_i =16, what is the predicted Y?

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

$= -8.809 + 0.895(16) = 5.511$

2.5 Find $var(\hat{u}_i)$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$Var(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.09181}{8} = 1.7615$$

$$Var(\hat{\beta}_0) = \frac{\sigma^2}{\sum x_i^2} = \frac{1.7615}{4440} = 0.0003967$$

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{\sum_i (x_i - \bar{x})^2} = \frac{1.7615}{440} = 0.040034$$

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3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$x_i = X_i - \bar{X}$$

$$k = \frac{x_i}{\sum_i x_i^2}$$

$$\hat{\beta}_1 = \sum_{i=1}^{n} (Y_i - \bar{Y}) k_i$$

$$= \sum_{i=1}^{n} (\beta_0 + \beta_1 X_i + u_i - \beta_0 - \beta_1 \bar{X}) k_i$$

$$= \sum_i \beta_1 (X_i - \bar{X}) k_i + \sum_i u_i k_i$$

$$= \beta_1 \sum_i x_i k_i + \sum_i u_i k_i$$

$$= \beta_1 \sum_i x_i \frac{x_i}{\sum_i x_i^2} + \sum_i u_i k_i$$

$$= \beta_1 \frac{\cancel{\sum_i x_i x_i}}{\cancel{\sum_i x_i^2}} + \sum_i u_i k_i$$

what assumption is required for this? SLR4

$$E(\hat{\beta}_1) = E[\beta_1 + \sum_i u_i k_i] = \beta_1 + E[\sum_i u_i k_i]$$

SLR4 : $E(u_i \mid x_i) = 0$

This assumption take the value of X as given (fixed)
So, we can treat X as a constant

$$E(\hat{\beta}_1) = \beta_1 + \sum_i k_i E(u_i)$$

function of X expected term of error is 0

$$E(\hat{\beta}_1) = \beta_1$$