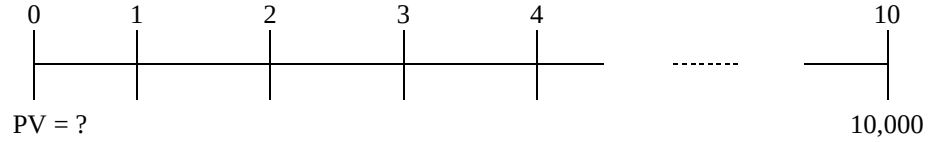
Lecture 3 solutions: The Time Value of Money

4-5. Your brother has offered to give you either $5000 today or $10,000 in 10 years. If the interest rate is 7% per year, which option is preferable?

Timeline:



$$PV = \frac{10,000}{1.07^{10}} = 5,083.49$$

So the 10,000 in 10 years is preferable because it is worth more.

4-7. Suppose you invest $1000 in an account paying 8% interest per year.

a. What is the balance in the account after 3 years? How much of this balance corresponds to "interest on interest"?

b. What is the balance in the account after 25 years? How much of this balance corresponds to interest on interest?

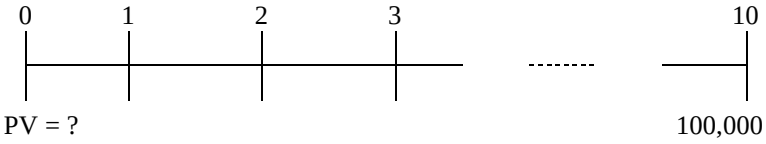
a. The balance after 3 years is $1259.71; interest on interest is $19.71.

b. The balance after 25 years is $6848.48; interest on interest is $3848.48.

rate	8%		
amt	1000		
years	1	3	25
balance	1080	1259.712	6848.47
simple interest	80	240	200
interest on interest	0	19.712	3848.47

4-8. Your daughter is currently eight years old. You anticipate that she will be going to college in 10 years. You would like to have $100,000 in a savings account to fund her education at that time. If the account promises to pay a fixed interest rate of 3% per year, how much money do you need to put into the account today to ensure that you will have $100,000 in 10 years?

Timeline:



$$PV = \frac{100,000}{1.03^{10}} = 74,409.39$$

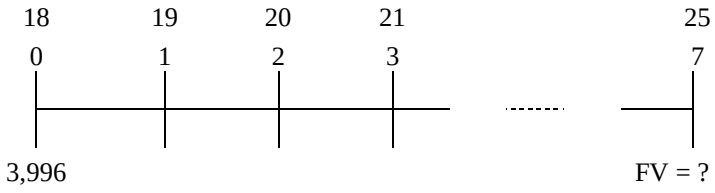
4-10. Your grandfather put some money in an account for you on the day you were born. You are now 18 years old and are allowed to withdraw the money for the first time. The account currently has $3996 in it and pays an 8% interest rate.

a. How much money would be in the account if you left the money there until your 25th birthday?

b. What if you left the money until your 65th birthday?

c. How much money did your grandfather originally put in the account?

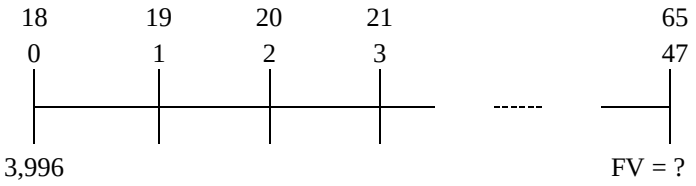
a. Timeline:



$$FV = 3,996(1.08)^{7}$$

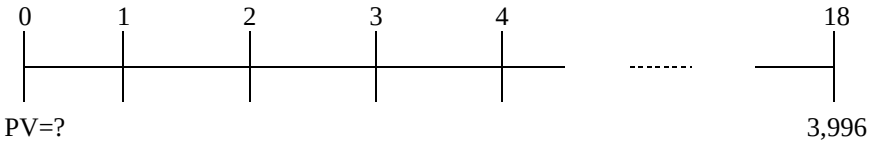
$$= 6,848.44$$

b. Timeline:



$$FV = 3,996(1.08)^{47} = 148,779$$

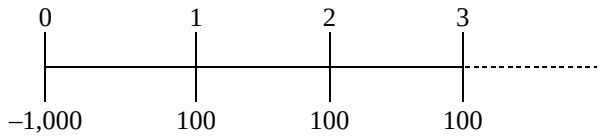
c. Timeline:



$$PV = \frac{3,996}{1.08^{18}} = 1,000$$

4-16. Your buddy in mechanical engineering has invented a money machine. The main drawback of the machine is that it is slow. It takes one year to manufacture $100. However, once built, the machine will last forever and will require no maintenance. The machine can be built immediately, but it will cost $1000 to build. Your buddy wants to know if he should invest the money to construct it. If the interest rate is 9.5% per year, what should your buddy do?

Timeline:



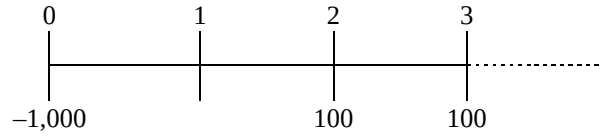
To decide whether to build the machine you need to calculate the NPV. The cash flows the machine generates are a perpetuity, so by the PV of a perpetuity formula:

$$PV = \frac{100}{0.095} = 1,052.63.$$

So the $NPV = 1,052.63 - 1,000 = 52.63$. He should build it.

4-17. How would your answer to Problem 16 change if the machine takes one year to build?

Timeline:



To decide whether to build the machine, you need to calculate the NPV: The cash flows the machine generates are a perpetuity with first payment at date 2. Computing the PV at *date 1* gives:

$$\text{PV}_1 = \frac{100}{0.095} = 1,052.63.$$

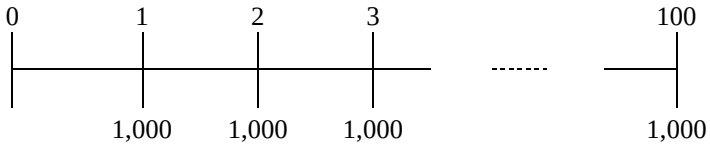
So the value today is

$$\text{PV}_0 = \frac{1,052.63}{1.095} = 961.31. \text{ So the } \text{NPV} = 961.31 - 1,000 = -38.69.$$

He should not build the machine.

4-19. What is the present value of $1000 paid at the end of each of the next 100 years if the interest rate is 7% per year?

Timeline:

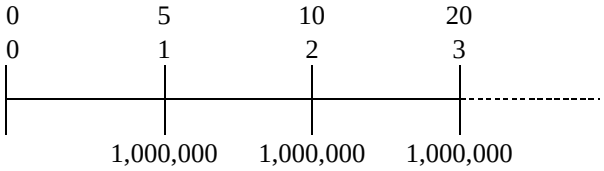


The cash flows are a 100-year annuity, so by the annuity formula:

$$\text{PV} = \frac{1,000}{0.07}\left(1 - \frac{1}{1.07^{100}}\right) = 14,269.25.$$

4-20. You are head of the Schwartz Family Endowment for the Arts. You have decided to fund an arts school in the San Francisco Bay area in perpetuity. Every five years, you will give the school $1 million. The first payment will occur five years from today. If the interest rate is 8% per year, what is the present value of your gift?

Timeline:



First we need the five-year interest rate. If the annual interest rate is 8% per year and you invest $1 million for five years you will have, by the 2nd rule of time travel, $(1.08)^5 = 1.46932808$. So the five-year interest rate is 46.93%. The cash flows are a perpetuity, so:

$$\text{PV} = \frac{1,000,000}{0.46932808} = 2,130,833.$$

4-21. When you purchased your house, you took out a 30-year annual-payment mortgage with an interest rate of 6% per year. The annual payment on the mortgage is $12,000. You have just

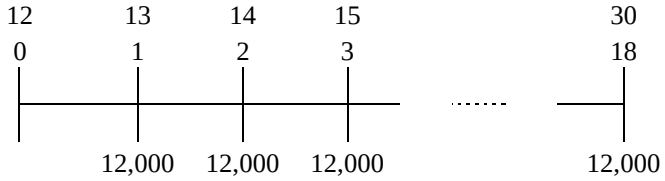
made a payment and have now decided to pay the mortgage off by repaying the outstanding balance. What is the payoff amount if:

a. You have lived in the house for 12 years (so there are 18 years left on the mortgage)?

b. You have lived in the house for 20 years (so there are 10 years left on the mortgage)?

c. You have lived in the house for 12 years (so there are 18 years left on the mortgage) and you decide to pay off the mortgage immediately *before* the twelfth payment is due?

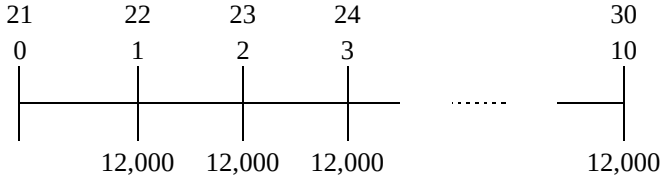
a. Timeline:



To pay off the mortgage you must repay the remaining balance. The remaining balance is equal to the present value of the remaining payments. The remaining payments are an 18-year annuity, so:

$$\text{PV} = \frac{12,000}{0.06}\left(1 - \frac{1}{1.06^{18}}\right)$$
$$= 129,931.24.$$

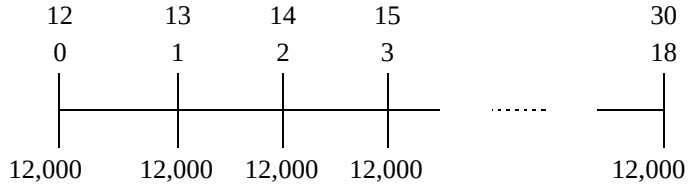
b. Timeline:



To pay off the mortgage you must repay the remaining balance. The remaining balance is equal to the present value of the remaining payments. The remaining payments are a 10-year annuity, so:

$$\text{PV} = \frac{12,000}{0.06}\left(1 - \frac{1}{1.06^{10}}\right) = 88,321.04.$$

c. Timeline:



If you decide to pay off the mortgage immediately before the twelfth payment, you will have to pay exactly what you paid in part (a) as well as the twelfth payment itself:

$$129,931.24 + 12,000 = 141,931.24.$$

4-22. You are 25 years old and decide to start saving for your retirement. You plan to save $5000 at the end of each year (so the first deposit will be one year from now), and will make the last deposit when you retire at age 65. Suppose you earn 8% per year on your retirement savings.

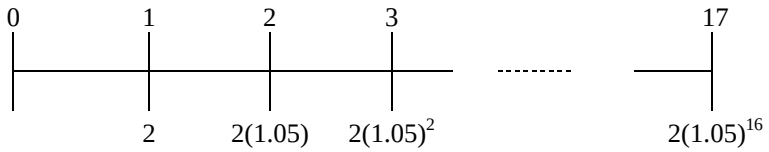
a. How much will you have saved for retirement?

b. How much will you have saved if you wait until age 35 to start saving (again, with your first deposit at the end of the year)?

amount	$5,000	
rate	8%	
retirement age	65	
start age	25	35
Savings	1,295,282.5	9566,416.0

4-26. You work for a pharmaceutical company that has developed a new drug. The patent on the drug will last 17 years. You expect that the drug's profits will be $2 million in its first year and that this amount will grow at a rate of 5% per year for the next 17 years. Once the patent expires, other pharmaceutical companies will be able to produce the same drug and competition will likely drive profits to zero. What is the present value of the new drug if the interest rate is 10% per year?

Timeline:



This is a 17-year growing annuity. By the growing annuity formula we have

$$\text{PV} = \frac{2,000,000}{0.1-0.05}\left(1-\left(\frac{1.05}{1.1}\right)^{17}\right) = 21,861,455.80$$

4-31. Suppose you currently have $5000 in your savings account, and your bank pays interest at a rate of 0.5% per month. If you make no further deposits or withdrawals, how much will you have in the account in 5 years?

We calculate the future value as $FV = C \times (1+r)^n$. The initial amount C = $5000 and the interest rate r = 0.5% per month. Because we have a monthly interest rate, we also need to express the number of periods, n, in months, so $n = 5 \times 12 = 60$. Thus,

$\text{FV} = \$5000 \times 1.005^{60} = \$6744.25$

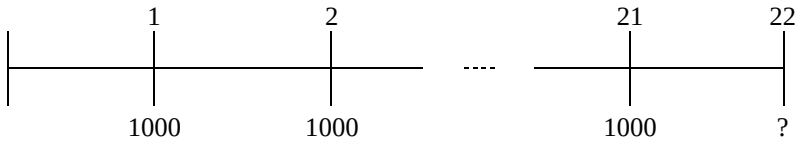
We will have $6744.25 in the account in 5 years time.

4-32. Your firm spends $5000 every month on printing and mailing costs, sending statements to customers. If the interest rate is 0.5% per month, what is the present value of eliminating this cost by sending the statements electronically?

The $5000 cost is a monthly perpetuity. Using the perpetuity formula with monthly cash flows and the monthly interest rate, this cost has a present value of $5000/.005 = $1 million.

4-33. You have just entered an MBA program and have decided to pay for your living expenses using a credit card that has no minimum monthly payment. You intend to charge $1000 per month on the card for the next 21 months. The card carries a monthly interest rate of 1%. How much money will you owe on the card 22 months from now, when you receive your first statement post-graduation?

We want to compute the future value of our account balance. Let's begin with the timeline over the next 12 months:



Our charges correspond to a 21-month annuity. Therefore, using the FV of an annuity formula, the future value at the end of 21 months is:

$$\text{FV(Annuity)} = \$1000 \times \frac{1}{0.01}\left(1.01^{21} - 1\right) \qquad = \$23{,}239.19$$

Or using the annuity calculator:

(c) Berk & DeMarzo 2006	NPER	RATE	PV	PMT	FV	Excel Formula
Given:	21	1.00%	0	1000		
Solve For FV:					(23,239)	=FV(0.01,21,1000,0)

Of course, we are not quite done. When we receive our statement in the 22nd month, there will be one more month's worth of interest charged. Therefore, we will have a final balance of \$23,239.19 × 1.01 = \$23.471.58.

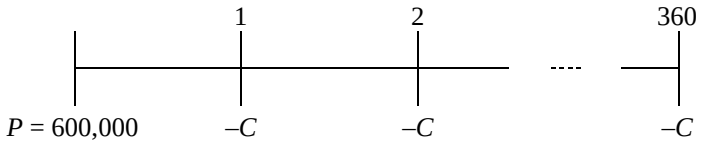
Note that the future value formula for an annuity computes the future value as of the date of the last payment. In this question we need to compute the future value one month after the final payment, which requires an additional calculation. (We could have alternatively computed the PV of the annuity, and then computed its future value 22 months in the future.)

4-38. **You have just made an offer on a new home and are seeking a mortgage. You need to borrow \$600,000.**

a. **The bank offers a 30-year mortgage with fixed monthly payments and an interest rate of 0.5% per month. What is the amount of your monthly payment if you take this loan?**

b. **Alternatively, you can get a 15-year mortgage with fixed monthly payments and an interest rate of 0.4% per month. How much would your monthly payments be if you take this loan instead?**

a. Note that a 30-year loan has 30 × 12 = 360 monthly payments. Here is the timeline:



We can solve for the loan payment using the formula:

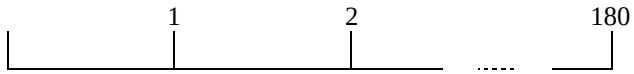
$$C = \frac{P}{\frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)} = \frac{600{,}000}{\frac{1}{0.005}\left(1 - \frac{1}{(1.005)^{360}}\right)}$$

$$= \$3597.30$$

Or, using the annuity calculator:

(c) Berk & DeMarzo 2006	NPER	RATE	PV	PMT	FV	Excel Formula
Given:	360	0.50%	600,000		0	
Solve For PMT:				(3,597.30)		=PMT(0.005,360,600000,0)

b. Note that a 15-year loan has 15 × 12 = 180 monthly payments. Here is the timeline:



We can solve for the loan payment using the formula

$$C = \frac{P}{\frac{1}{r}\left(1 - \frac{1}{(1+r)^N}\right)} = \frac{600,000}{\frac{1}{0.004}\left(1 - \frac{1}{(1.004)^{360}}\right)}$$

$$= \$4682.49$$

Or, using the annuity calculator:

(c) Berk & DeMarzo 2006	NPER	RATE	PV	PMT	FV	Excel Formula
Given:	180	0.40%	600,000		0	
Solve For PMT:				(4,682.49)		=PMT(0.004,180,600000,0)