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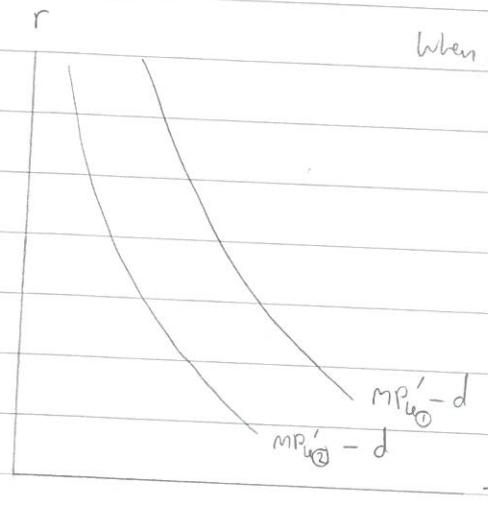
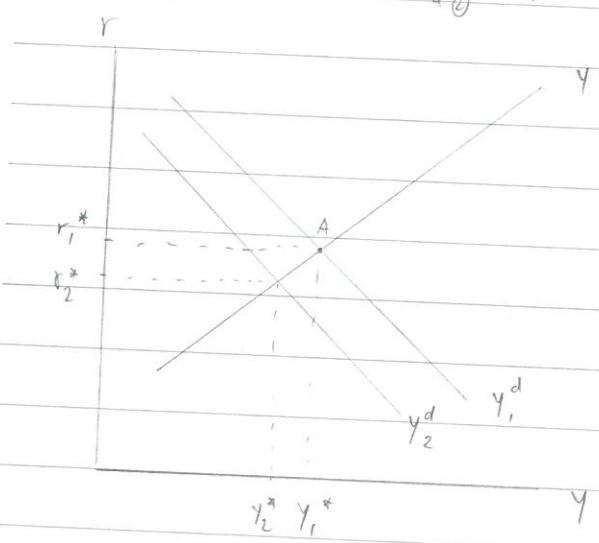
Assignment 6 : Real Intertemporal Model

1)

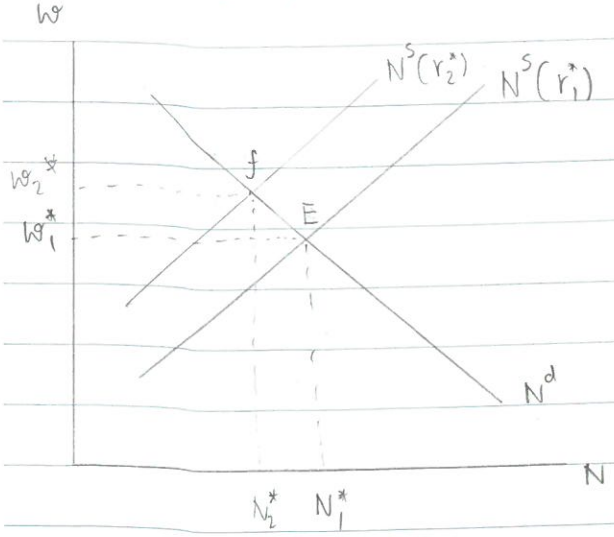


When the future productivity decrease, it will affect investments decision for firms. Firms will optimize by satisfying $MP'_k(k') = r + d$. Lower future productivity will cause firm to choose lower k' in order to maintain level of productivity equal to the required rate of return $(r+d)$. By deciding to lower k', given the same current capital (k), Firm's level of investments will have to drop. Thus, Investments curve will shift to the left from $MP'_{k①} - d$ to $MP'_{k②} - d$. Since $y^d = C + I(r) + \bar{G}$, lower investments will cause aggregate demand to decrease for any level of interest rates. Thus, the aggregate demand will shift to the left from Y_1^d to Y_2^d.



With this shock, the aggregate supply curve will not be affected. Because current labor productivity is unchanged, the labor demand will not be affected. As to labor supply, The effect of investment have no impact to wealth and income of consumer. The labor supply will not shift. Thus, there's no impact to the aggregate supply caused by lower investment.

Before the shock hit the economy, equilibrium is at point A (r_1^*, Y_1^*). When the effect of lower future productivity take place, the aggregate demand will be lower. At the original interest rate r_1^*, there will excess supply in the economy. The market can't sustained at this point. Thus, interest rate will be lower. The new equilibrium is at (r_2^*, Y_2^*) where there's no excess demand.



In labor market, the effect of interest rate can be shown as follow. When interest rate decline, the opportunity cost of having leisure is lower. Household will choose to work less according to substitution effect. There's also income effect. Lower interest rate will cause consumers to feel that they must save more in order to maintain the same level of future consumptions.

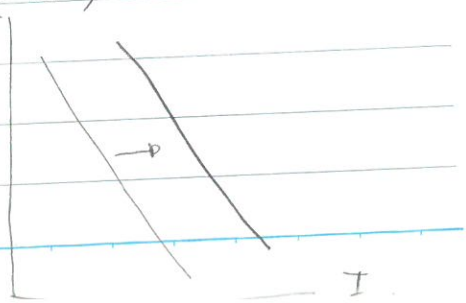
If substitution effect dominates, the consumers will choose to work less and thus, labor supply decrease. Labor supply curve will shift to the left from $N^S(r_1^*)$ to $N^S(r_2^*)$. The equilibrium will move from the original point $E(w_1^*, N_1^*)$ to point $f(w_2^*, N_2^*)$.

Conclusion The effects of lower future productivity on economic variables are as follow.

① lower investment
② lower output (Y_1^* to Y_2^*)
③ lower interest rate (r_1^* to r_2^*)
④ Higher wage (w_1^* to w_2^*)
⑤ lower employment (N_1^* to N_2^*)

2.1) To analyse the effect of depreciation, we can seperate into three case.

1st current period d increases, future period d remain the same
- Higher current depreciation will cause the value of current capital stock to be lower going to the next period. Given that the required rate of return is fixed at r+d, the optimal level of k' will also be fixed. This means that with the lower value of current capital, to compensate this loss, firm must increase level of investment in order to maintain the same amount of future capital. Thus, in this case, Investments will be higher. Investment curve will shift to the right.



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2.2) Let's consider the first policy.

(1st) firm receive a subsidy in the current period t per unit output.

current profit (π); $\pi = (1+t) z f(k,N) - wN - I$

after subsidy

Future profit (π'); $\pi' = z' f(k', N') - w'N' + k \cdot (1-d')$

Value of the firm; $V = \pi + \frac{\pi'}{1+r}$

$$= (1+t) z f(k,N) - wN - I + \frac{z' f(k', N) - w'N' + k(1-d')}{1+r}$$

First order condition

$$\frac{\partial V}{\partial N} = (1+t) MP_N - w = 0 \Rightarrow (1+t) MP_N = w \quad -①$$

$$\frac{\partial V}{\partial N'} = MP_N' - w' = 0 \Rightarrow MP_N' = w' \quad -②$$

$$\frac{\partial V}{\partial I} = \frac{MP_k' + (1-d)}{1+r} - 1 = 0 \Rightarrow MP_k' = r + d \quad -③$$

From ①; we can see that for any level of employment demand (N^d), wage that firms willing to pay is higher. That is, labor demand will be higher. From ③, the optimality condition for investments is still the same as the case without output subsidy. Thus, the level of investment is still the same.

(2nd) subsidy s per unit of investment in current period.

- current profit (π); $\pi = z f(k,N) - wN - I + sI = z f(k,N) - wN - (1-s) I$

  after subsidy
- Future profit (π); $\pi' = z' f(k', N') - w'N' + k'(1-d)$

Value of the firm; $V = \pi + \frac{\pi'}{1+r}$

$$= z f(k,N) - wN - (1-s) I + \frac{z' f(k', N') - w'N' + k'(1-d)}{1+r}$$

First order condition

$$\frac{\partial V}{\partial N} = MP_N - w = 0 \Rightarrow MP_N = w \quad -④$$

$$\frac{\partial V}{\partial N'} = MP_N' - w' = 0 \Rightarrow MP_N' = w' \quad -⑤$$

$$\frac{\partial V}{\partial I} = \frac{MP_k' + (1-d)}{1+r} - (1-s) = 0 \Rightarrow MP_k' + 1 - d = (1+r) - s(1+r)$$

$$MP_k' = r + d - s(1+r) \quad -⑥$$

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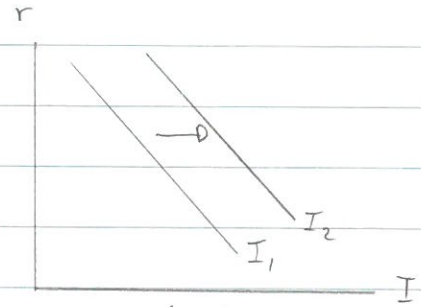
2nd case : current period d increase, future period d increase

- The effect of higher current period d is the same as the previous case. Investments will be higher.

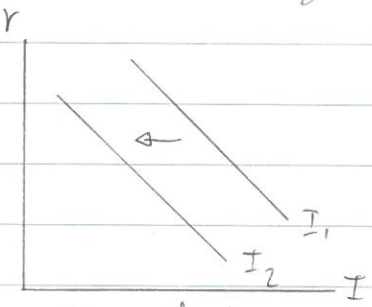
When future period d increase, the required rate of return increase. Firms have to choose lower k' in order to satisfy $MP_{k'} = r + d$. With lower optimal k', Investments will be lower.

The overall effect on investments depend on which effect dominates. If the effect of current d dominates, Investments will be higher. On the contrary, if the effect of future d dominates, Investments will be lower. If the effect of both cancel out, the level of investments will be unchanged.



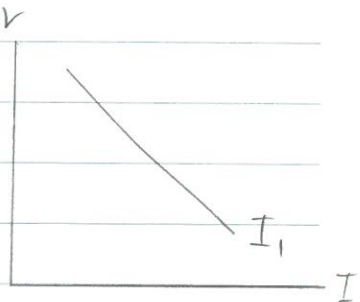
Case : current d dominates

Investment curve will shift to the right



case : future d dominates

Investment curve will shift to the left

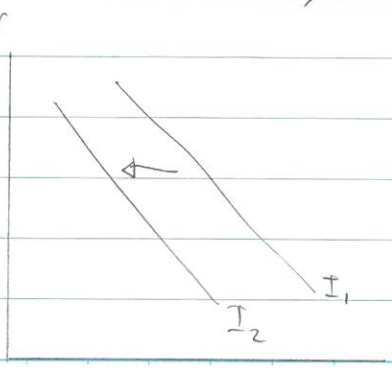


case : cancel out

Investment curve will stay the same.

3rd case : current d stay the same, future d increase

- As I mentioned in the previous case, higher future depreciation rate will cause the required rate of return to increase. In order to satisfy the optimality condition, $MP_{k'} = r + d$, firms must increase productivity by lowering k'. By doing so, firms will invest less. Thus, Investments will be lower.



Investment will be lower for any given interest rate. Investment curve will shift to the left from I_1 to I_2

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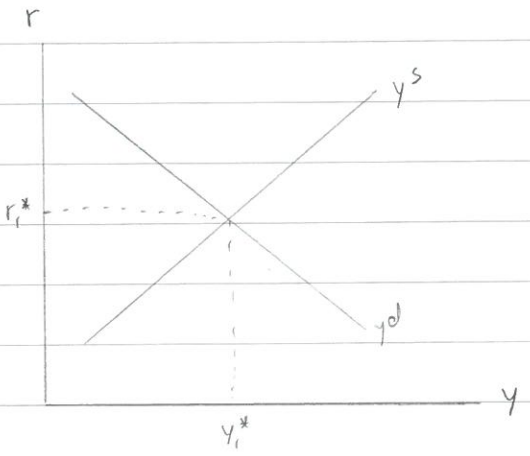
From the (2nd) case, we can see that the optimality condition has changed to $MP_0' = r + d - s(1+r)$. Since $s(1+r)$ is positive, the required rate of return for firms become lower due to subsidy from the government. This means that for a given level of future productivity, firms will choose higher ω'. Given a constant current capital, Investments will be higher.

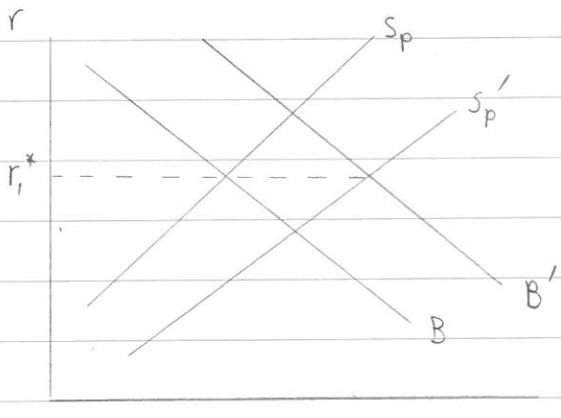
Thus, the second policy is more effective on increasing the level of investment of the firm.

2.3) For this problem, this is the case of Ricardian Equivalence.



The wealth of consumers doesn't change because the effect of lower current taxes is ruled out by an increase in future taxes by $\Delta t(1+r)$. This means that the level of consumption will not change. Both aggregate demand and aggregate supply will not be affected. Thus, the equilibrium is still the same at (r_1^*, Y_1^*).

The only effect is in credit market. Since the level of government spending is fixed, Demand for borrowing will



increase for given interest rate ($G-T=B$). Although, there's no effect on consumption, current disposable income will increase due to lower taxes. Fixed consumption will cause savings to be higher. Thus, private saving will increase for any interest rate. The equilibrium in credit market is the condition $G - T = Y - T - C$. Since Y, G, C are unchanged, the equilibrium interest rate will not changed, only the quantity of equilibrium Funds, which is higher. Since, the aggregate demand doesn't change, there's no multiplier.

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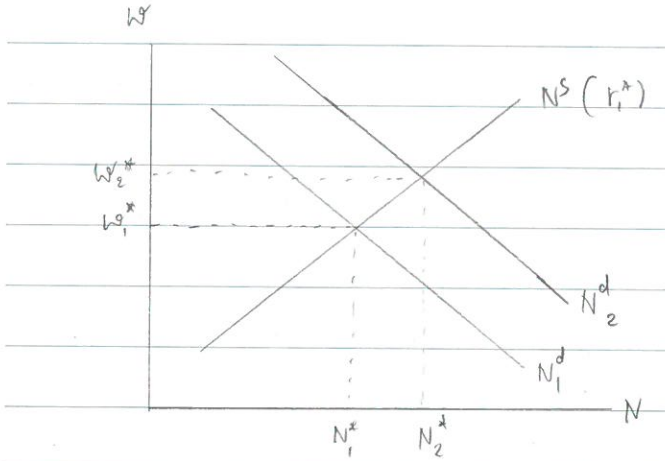
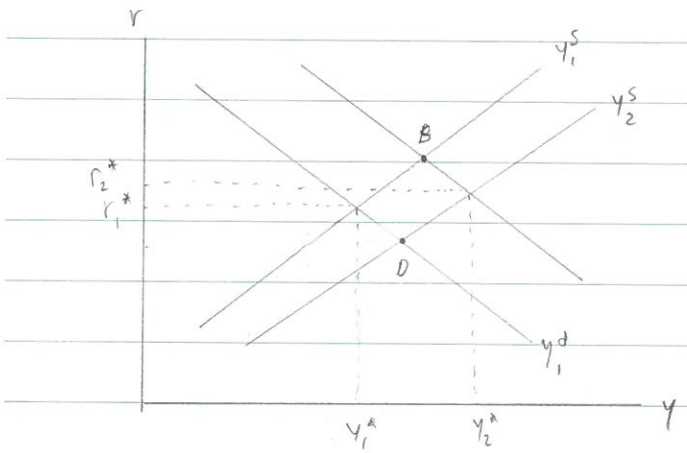
2.4) Suppose that there's a permanent increase in total factor productivity.



An increase in current Z will cause current labor productivity to be higher. At any level of employment, firms are willing to demand at higher wage. Thus, labor demand increase following the optimality condition $MP_N = W$. Labor demand will increase from N_1^d to N_2^d. This will affect the overall supply level of the economy. At any interest rate, aggregate supply will increase. Y^S will shift from Y_1^S to Y_2^S. Secondly, An increase in future productivity will cause the firm to increase future capital, in order to satisfy $MP_K' = r + d$. Investment will be higher. Aggregate demand will increase and shift from Y_1^d to Y_2^d. The new equilibrium will be at (r_2^*, Y_2^*).



Compare with the case that only current factor productivity

- The equilibrium in this case is at point D.

Final Output is less than the case of permanent increase. The interest rate is lower.

Compare with the case that only future factor productivity

- The equilibrium in this case is at point B.

Final Output is also less than the case of permanet increase. The interest rate is higher.