

EMPIRICAL APPLICATIONS OF SOLOW MODEL: SOLOW MODEL WITH HUMAN CAPITAL

EE 462 Development Macroeconomics

Reference: Mankiw, N. G., Romer, D., & Weil, D. N. (1992). A contribution to the empirics of economic growth. *The Quarterly Journal of Economics*, 107(2), 407-437.

MRW (1992)

- MRW examines whether the “textbook” Solow model is consistent with the empirical evidence.
- It augments the Solow model by including *human capital accumulation* and capital accumulation, and tests this **augmented Solow model**.
- It also tests for both unconditional and conditional convergence in per capita income.
- Findings:
 - The prediction of the Solow model are consistent with the data.
 - The cross-country variation in per capita income can best be explained by an augmented Solow model.
 - There is convergence once the differences in saving and population growth rates are controlled for.

“Textbook” Solow Model

- Production function: $Y(t) = K(t)^\alpha (A(t)L(t))^{1-\alpha}$, $0 < \alpha < 1$, where A = Technology.

- L and A are assumed to grow exogenously at rates n and g :

$$L(t) = L(0)e^{nt} \quad \text{and} \quad A(t) = A(0)e^{gt}$$

- Define $k = \frac{K}{AL}$ and $y = \frac{Y}{AL}$, where AL is effective units of labor.
- The change in k is given by:

$$\dot{k} = sy(t) - (n + g + \delta)k(t)$$

- Hence, the steady-state value k^* is:

$$k^* = [s/(n+g+\delta)]^{1/(1-\alpha)}$$

“Textbook” Solow Model (cont’d)

- Substituting k^* in the production function and taking log yields:

$$\ln \left(\frac{Y(t)}{L(t)} \right) = \ln A(0) + gt + \frac{\alpha}{1-\alpha} \ln(s) - \frac{\alpha}{1-\alpha} \ln(n + g + \delta)$$

- Assume that g and δ are constant across countries. (In this paper, it's assume that $g + \delta = 0.05$.)
- Also, assume that $\ln A(0) = a + \epsilon$, where a is a constant, and ϵ is a country-specific shock.
- Hence, log income per capita at a given point time (say $t=0$) is:

$$\ln \left(\frac{Y}{L} \right) = a + \frac{\alpha}{1-\alpha} \ln(s) - \frac{\alpha}{1-\alpha} \ln(n + g + \delta) \quad -- (*)$$

Data and Samples

- Data are from the Real National Accounts constructed by Summers and Heston (1988).
- Data are annual and cover the period 1960-1985.
- n measures the average growth rate of the working-age (15-64) population.
- s is the average share of real investment in real GDP.
- Y/L is real GDP in 1985 divided by the working-age pop.
- There are three samples:
 - 98 non-oil producing countries
 - 75 countries – exclude those with <1m pop or those with data problems
 - 22 OECD countries with pop > 1m

TABLE I
ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48 (1.59)	5.36 (1.55)	7.97 (2.48)
$\ln(I/GDP)$ $\alpha/(1 - \alpha)$	1.42 (0.14)	1.31 (0.17)	0.50 (0.43)
$\ln(n + g + \delta)$ $-\alpha/(1 - \alpha)$	-1.97 (0.56)	-2.01 (0.53)	-0.76 (0.84)
$\bar{R}^2$	0.59	0.59	0.01
s.e.e.	0.69	0.61	0.38
Restricted regression:			
CONSTANT	6.87 (0.12)	7.10 (0.15)	8.62 (0.53)
$\ln(I/GDP) - \ln(n + g + \delta)$	1.48 (0.12)	1.43 (0.14)	0.56 (0.36)
$\bar{R}^2$	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.37
Test of restriction:			
<i>p</i> -value	0.38	0.26	0.79
Implied α	0.60 (0.02)	0.59 (0.02)	0.36 (0.15)
A priori value of $\alpha = 1/3$			

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

Augmented Solow Model

- Let the production function be:

$$Y(t) = K(t)^\alpha H(t)^\beta (A(t)L(t))^{1-\alpha-\beta}, 0 < \alpha + \beta < 1,$$

Where H = stock of human capital.

- Let s_k and s_h be the fractions of income invested in physical capital and human capital, respectively.
- The evolution of the economy is determined by:

$$\dot{k} = s_k y(t) - (n + g + \delta)k(t)$$

$$\dot{h} = s_h y(t) - (n + g + \delta)h(t)$$

Where $k = K/AL$, $h = H/AL$, and $y = Y/AL$.

Augmented Solow Model (cont'd)

- Hence, the steady-state levels of K and H are:

$$k^* = \left[\frac{s_k^{1-\beta} s_h^\beta}{n+g+\delta} \right]^{1/(1-\alpha-\beta)} \quad \text{and} \quad h^* = \left[\frac{s_k^\alpha s_h^{1-\alpha}}{n+g+\delta} \right]^{1/(1-\alpha-\beta)}$$

- Substituting k^* and h^* in the production function yields:

$$\ln\left(\frac{Y}{L}\right) = \ln A(0) + gt - \frac{\alpha+\beta}{1-\alpha-\beta} \ln(n+g+\delta) + \frac{\alpha}{1-\alpha-\beta} \ln(s_k) + \frac{\beta}{1-\alpha-\beta} \ln(s_h)$$

Where s_h = secondary school enrollment ratio x proportion of labor force of secondary school age.

- Alternatively, the above equation can be rewritten as:

$$\ln\left(\frac{Y}{L}\right) = \ln A(0) + gt + \frac{\alpha}{1-\alpha} \ln(s_k) - \frac{\alpha}{1-\alpha} \ln(n+g+\delta) + \frac{\beta}{1-\alpha} \ln(h^*)$$

TABLE II
ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89 (1.17)	7.81 (1.19)	8.63 (2.19)
$\ln(I/GDP)$	$\frac{\alpha}{1 - \alpha - \beta}$ 0.69 (0.13)	0.70 (0.15)	0.28 (0.39)
$\ln(n + g + \delta)$	$-\frac{\alpha + \beta}{1 - \alpha - \beta}$ -1.73 (0.41)	-1.50 (0.40)	-1.07 (0.75)
$\ln(SCHOOL)$	$\frac{\beta}{1 - \alpha - \beta}$ 0.66 (0.07)	0.73 (0.10)	0.76 (0.29)
$\bar{R}^2$	0.78	0.77	0.24
<i>s.e.e.</i>	0.51	0.45	0.33

TABLE II (Cont'd)
ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
Restricted regression:			
CONSTANT	7.86 (0.14)	7.97 (0.15)	8.71 (0.47)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.73 (0.12)	0.71 (0.14)	0.29 (0.33)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.67 (0.07)	0.74 (0.09)	0.76 (0.28)
$\bar{R}^2$	0.78	0.77	0.28
<i>s.e.e.</i>	0.51	0.45	0.32
Test of restriction:			
<i>p</i> -value	0.41	0.89	0.97
Implied α	0.31 (0.04)	0.29 (0.05)	0.14 (0.15)
Implied β	0.28 (0.03)	0.30 (0.04)	0.37 (0.12)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Endogenous Growth and Convergence

- Goal: reexamine the evidence on convergence of Y/L .
- Previous model assumes that countries in 1985 were in their steady states (or that the deviations from steady state were random). $\rightarrow$ relax this assumption!
- Let y^* be the steady-state income per capita and $y(t)$ be the actual value at time t .
- The **speed of convergence** is given by:

$$\frac{d\ln(y(t))}{dt} = \lambda[\ln(y^*) - \ln(y(t))],$$

Where $\lambda = (n + g + \delta)(1 - \alpha - \beta)$ is the convergence rate.

- Example: If $\alpha = \beta = \frac{1}{3}$ and $n + g + \delta = 0.06$, then $\lambda = 0.02$.

Convergence Rate

- The above equation implies that:

$$\ln(y(t)) = (1 - e^{-\lambda t})\ln(y^*) + e^{-\lambda t}\ln(y(0)),$$

Where $y(0)$ is income per effective worker at an initial date.

- Subtracting $\ln(y(0))$ and substituting y^* gives:

$$\begin{aligned} & \ln(y(t)) - \ln(y(0)) \\ &= (1 - e^{-\lambda t}) \frac{\alpha}{1 - \alpha - \beta} \ln(s_k) + (1 - e^{-\lambda t}) \frac{\beta}{1 - \alpha - \beta} \ln(s_h) \\ & \quad - (1 - e^{-\lambda t}) \frac{\alpha + \beta}{1 - \alpha - \beta} \ln(n + g + \delta) - (1 - e^{-\lambda t}) \ln(y(0)) \end{aligned}$$

TABLE III
TESTS FOR UNCONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	−0.266 (0.380)	0.587 (0.433)	3.69 (0.68)
$\ln(Y60) - (1 - e^{-\lambda t})$	0.0943 (0.0496)	−0.00423 (0.05484)	−0.341 (0.079)
$\bar{R}^2$	0.03	−0.01	0.46
<i>s.e.e.</i>	0.44	0.41	0.18
Implied λ	−0.00360 (0.00219)	0.00017 (0.00218)	0.0167 (0.0023)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960.

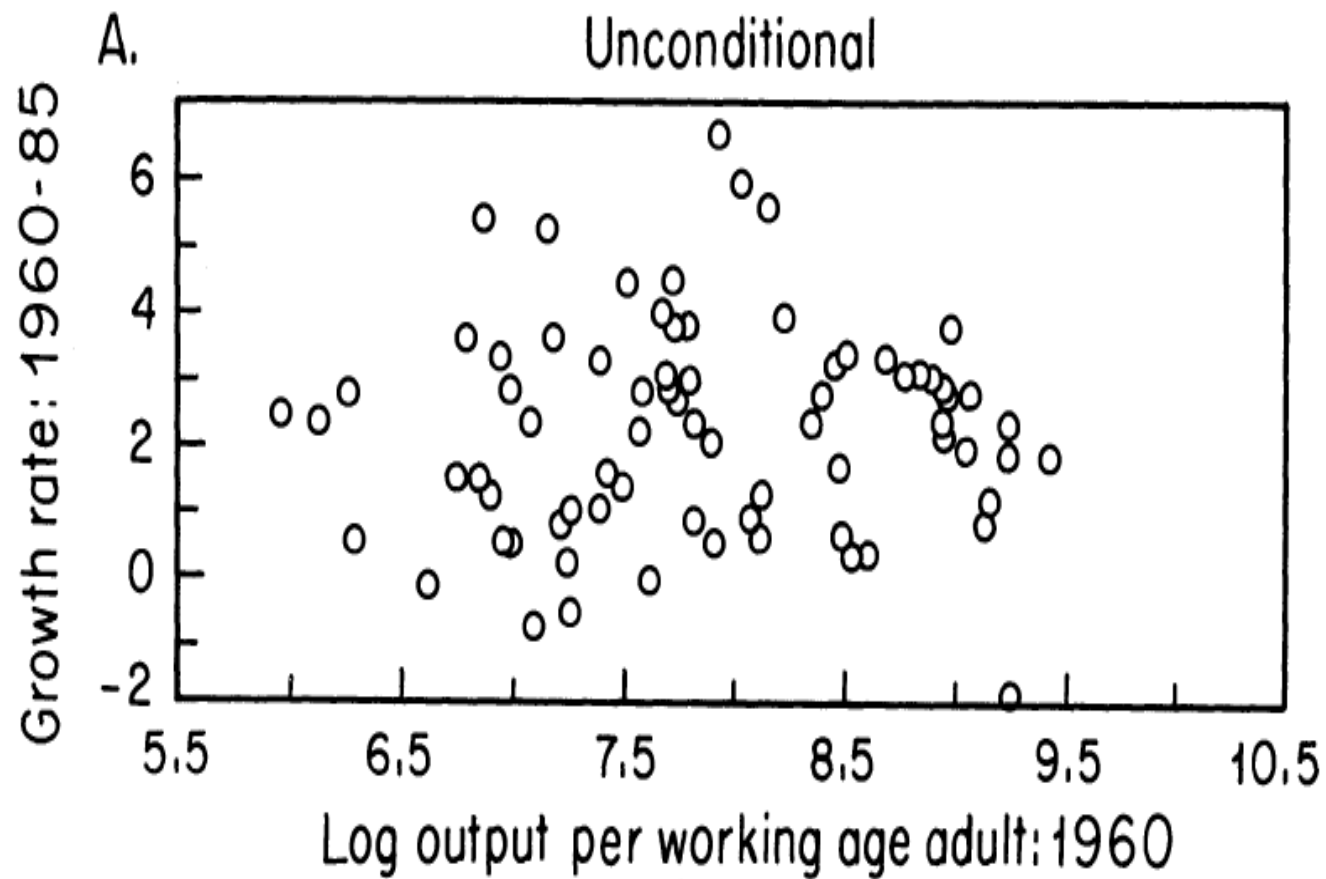


TABLE IV
TESTS FOR CONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	1.93 (0.83)	2.23 (0.86)	2.19 (1.17)
$\ln(Y60)$	−0.141 (0.052)	−0.228 (0.057)	−0.351 (0.066)
$\ln(I/GDP)$	0.647 (0.087)	0.644 (0.104)	0.392 (0.176)
$\ln(n + g + \delta)$	−0.299 (0.304)	−0.464 (0.307)	−0.753 (0.341)
$\bar{R}^2$	0.38	0.35	0.62
<i>s.e.e.</i>	0.35	0.33	0.15
Implied λ	0.00606 (0.00182)	0.0104 (0.0019)	0.0173 (0.0019)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

TABLE V
TESTS FOR CONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	3.04 (0.83)	3.69 (0.91)	2.81 (1.19)
$\ln(Y60) - (1 - e^{-\lambda t})$	-0.289 (0.062)	-0.366 (0.067)	-0.398 (0.070)
$\ln(I/GDP) - (1 - e^{-\lambda t}) \frac{\alpha}{1 - \alpha - \beta}$	0.524 (0.087)	0.538 (0.102)	0.335 (0.174)
$\ln(n + g + \delta) - (1 - e^{-\lambda t}) \frac{\beta}{1 - \alpha - \beta}$	-0.505 (0.288)	-0.551 (0.288)	-0.844 (0.334)
$\ln(SCHOOL) - (1 - e^{-\lambda t}) \frac{\alpha + \beta}{1 - \alpha - \beta}$	0.233 (0.060)	0.271 (0.081)	0.223 (0.144)
$\bar{R}^2$	0.46	0.43	0.65
s.e.e.	0.33	0.30	0.15
Implied λ	0.0137 (0.0019)	0.0182 (0.0020)	0.0203 (0.0020)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

