

CHAPTER 3

Static and Comparative Static Equilibrium Analysis

Topics: Static and Comparative Static Equilibrium Analysis, PART1

Outline:

- Linear models in economics
- Simultaneous system of equations
- * A partial-equilibrium market model: A model of price determination in an isolated market
- * Excise Tax and Market Equilibrium
- What is elasticity ? How can we derive elasticity?
- * Tax incidence and Elasticity

“Equilibrium” means

→ a situation that selected interrelated variables so adjusted to one another that no tendency to change

→ All variables in the model must be simultaneously endogenous in a state of rest

“Static” means

statics = equilibrium

Nongoal type of equilibrium: → Ch3, Ch4, Ch6, 8

use matrix to solve equilibrium, optimization, goal-type equilibrium

→ The equilibrium that is not a result of any particular objective, but is derived from an impersonal process of interaction

⇒ we could have underemployment equilibrium level of national income

Linear economic model vs. Nonlinear economic model

$$\begin{aligned} \rightarrow Q^D &= Q^S \\ \rightarrow Q^D &= a - bp \\ \rightarrow Q^S &= -c + dp \end{aligned} \quad \left. \begin{array}{l} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{array} \right\}$$

$$\begin{aligned} Q^D &= Q^S \\ Q^D &= a - bp^2 \\ Q^S &= -c + dp^2 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

“Simultaneous system of equations” means

system of equations in which each equation is considered to be related

→ e.g. finding equilibrium = finding solution to the system of equations
The solution to system of equations must make every equations true simultaneously

3.1

A partial-equilibrium market model: A model of price determination in an isolated market

Suppose we are interested in one commodity, energy drink. Since only one commodity is being considered, the economic model for this market is comprised of:

1. Q^D
2. Q^S
3. P

After having chosen the variables, we next make certain assumptions regarding the working of the market. First, we must specify an equilibrium condition- something indispensable in an equilibrium model. The standard assumption is that equilibrium occurs in the market if and only if the excess demand is zero, that is, if and only if the market is cleared. The Clearing Market Condition can be written as:

$$Q^D = Q^S$$

to note: $P^D = P^S$ (1)

Equation (1.) is conditional equation of the market. The model requires that we specify behavioral equations to explain how exactly the demand and supply are each determined.

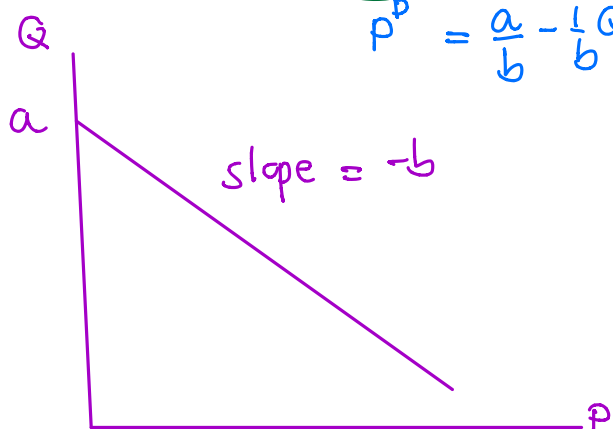
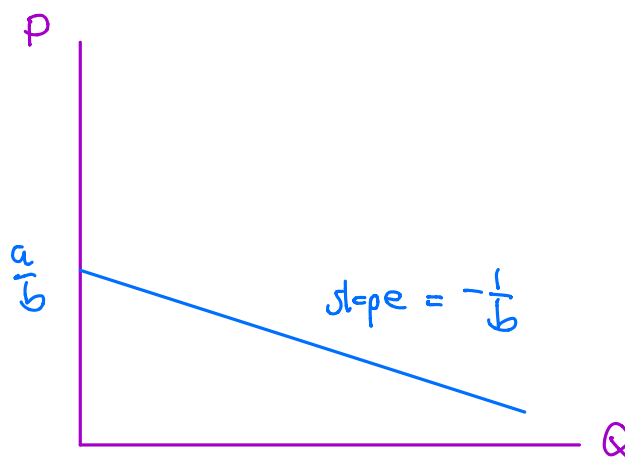
use (*) (*) to solve for

Demand for energy drinkWe assume that Q_d is a decreasing linear function of P .

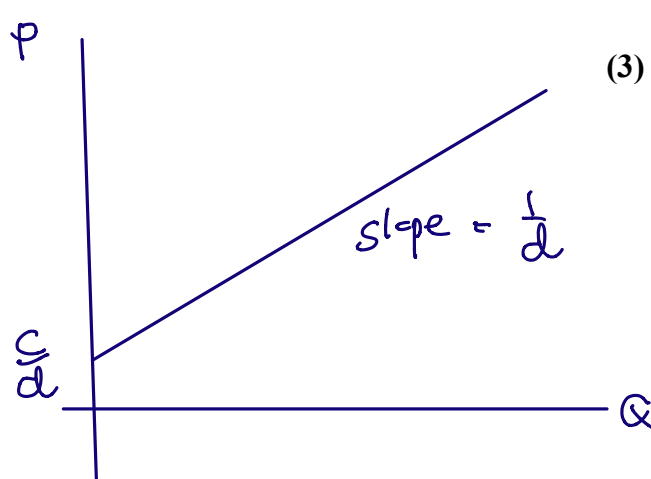
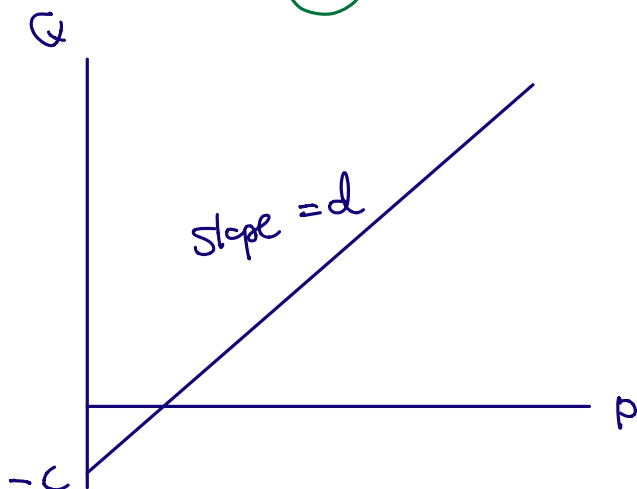
$$Q^D = a - bP, \quad b > 0 \quad (*)$$

$$P^D = \frac{a}{b} - \frac{1}{b}Q$$

to solve for Q^*, P^*
 $Q^D = Q^S$
 indicate
 (2) restrictions on a, b, c, d
 such that Q^*, P^* can
 happen

**Supply for energy drink**We assume that Q_s is an increasing linear function of P .

$$Q^S = -c + dP, \quad d > 0, \quad c > 0 \quad (*)$$



In all, the model will contain one equilibrium condition plus two behavioral equations which govern the demand and supply sides of the market.

$$\begin{aligned} (1) & \quad Q_d = Q_s \Rightarrow 1 \text{ Conditional equation} \\ (2) & \quad Q_d = a - bP \quad (a, b > 0) \\ (3) & \quad Q_s = -c + dP \end{aligned} \quad \left. \vphantom{\begin{aligned} (1) & \quad Q_d = Q_s \Rightarrow 1 \text{ Conditional equation} \\ (2) & \quad Q_d = a - bP \quad (a, b > 0) \\ (3) & \quad Q_s = -c + dP \end{aligned}} \right\} 2 \text{ Behavioral equations} \quad (4)$$

The next step is to obtain the solution values of the three endogenous variables, Q_d, Q_s, P .

The solution values are those values that satisfy the three equations in (4) simultaneously.

We usually denote the solution value of an endogenous variable with an asterisk. Thus, the solution values of Q_d, Q_s, P are denoted by Q_d^*, Q_s^*, P^* .

Since $Q_d^* = Q_s^*$ they can be replaced by a single symbol Q^* . $Q_d^* = Q_s^* = Q^*$

Hence, an equilibrium solution of the model may simply be denoted by an ordered pair (P^*, Q^*) .

★ That is, we equate quantity demanded with quantity supplied & solve for market price ★
at equilibrium

Solution by Elimination of Variables: From equilibrium mkt. price, we can further solve for related market quantity at equilibrium

From mkt. equilibrium condition:

$$Q^D = Q^S$$

$$a - bP = -c + dP$$

$$a + c = (b + d)P$$

$$P^* = \frac{a + c}{b + d}$$

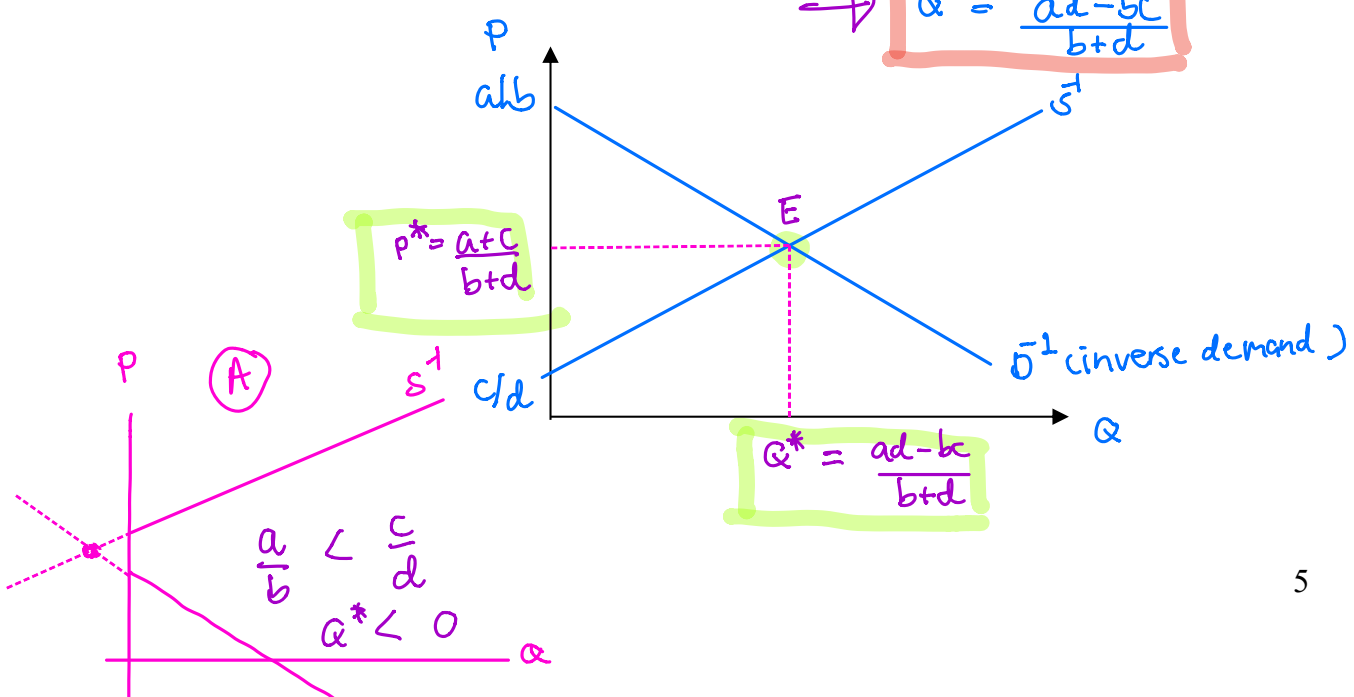
the solution that we are looking for
⇒ endo Var = f(parameters, exo Var.)

$$Q^D = Q^S = Q^*$$

$$a - b\left(\frac{a + c}{b + d}\right) = -c + d\left(\frac{a + c}{b + d}\right) = Q^*$$

$$Q^* = \frac{a(b + d) + d(a + c) - ab - bc}{b + d}$$

$$Q^* = \frac{ad - bc}{b + d}$$



Additional restriction for an economically meaningful solution:

If we would like to have $Q^* > 0$

$$\frac{ad-bc}{b+d} > 0$$

$$ad - bc > 0$$

$$ad > bc$$

$$\frac{a}{b} > \frac{c}{d}$$

Note: Compare between partial equilibrium and general equilibrium model

two mkt. : x, y

① Partial equilibrium

$$Q_d^x = a - bP_x + cP_y$$

$$Q_s^x = d + eP_x$$

$$Q_d^x = Q_s^x$$

take as given

$$P_x^*, Q_x^* = Q_d^* = Q_s^*$$

Solving for 2 solutions (P_x^*, Q_x^*)

P_y is exogenous var

P_x, Q_d^x, Q_s^x endo var

②

General equilibrium

$$Q_d^x = a - bP_x + cP_y$$

$$Q_s^x = d + eP_x$$

$$Q_d^y = f - gP_y + hP_x$$

$$Q_s^y = j + kP_y$$

$$Q_d^x = Q_s^x$$

$$Q_d^y = Q_s^y$$

3.2

$Q_d^x, Q_s^x, Q_d^y, Q_s^y, P_x, P_y$ are endogenous variables

Excise Tax and Market Equilibrium

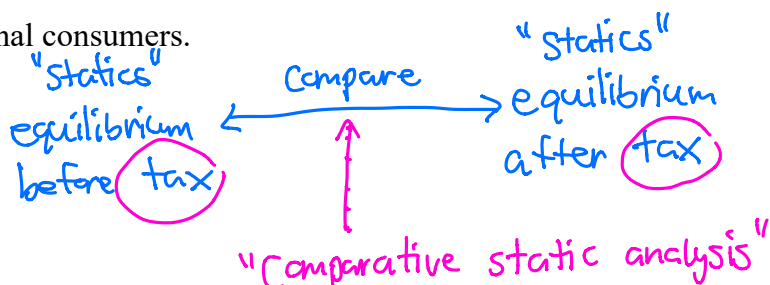
Solving for 4 solutions

$(P_x^*, Q_x^*) (P_y^*, Q_y^*)$

We will use a partial-equilibrium market model from previous section in an analysis of the impact of excise tax.

Definition of an excise tax:

Excise taxes are narrowly based taxes on consumption, levied on specific goods, services, and activities. They can be either a per unit tax (such as the per gallon tax on gasoline) or a percentage of price (such as the airline ticket tax). Generally, excise taxes are collected from producers or wholesalers, and are embedded in the price paid by final consumers.



(a.) **Specific Tax** or per unit tax or unit tax: is a tax that is defined as a fixed amount for each unit of a good or service sold, such as cents per kilogram.

..... t baht per unit e.g. 3 baht per gallon of gasoline

(b.) **Ad Valorem Tax** is a tax whose amount is based on the value of a transaction or of property. It is a charge based on a fixed percentage of the product value. It is typically imposed at the time of a transaction, as in the case of a sale or value-added tax (VAT).

..... $t\%$ of price levied on the price of a product
 $7\% \times 10 \text{ baht}$, $7\% \times 20 \text{ baht}$

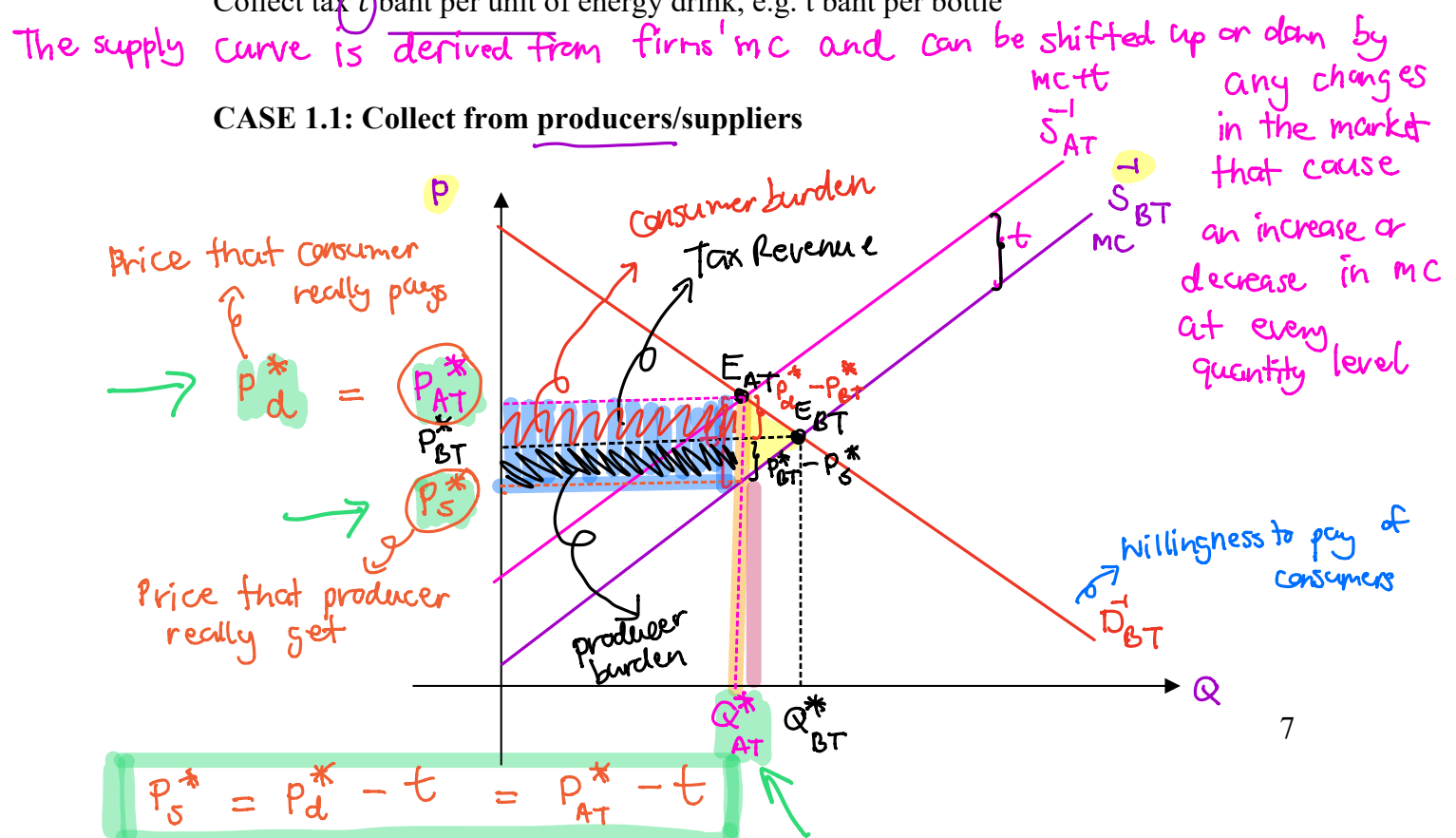
Tax can be collected from buyers or producers, depending on convenience and efficacy of tax collection. Either way, market equilibrium will change. We can compare *market equilibrium before tax* to *market equilibrium after tax*, and analyze how the change is. This is called “Comparative Static Analysis”.

Comparative statics, as the name suggested, is concerned with the comparison of different equilibrium states that are associated with different sets of values of parameters and exogenous variables.

CASE 1: Specific tax

Collect tax t baht per unit of energy drink, e.g. t baht per bottle

CASE 1.1: Collect from producers/suppliers



Market equilibrium before tax

Short version

$$Q^d = Q^s$$

$$Q^d = a - bP, a, b > 0$$

$$Q^s = -c + dP, c, d > 0$$

$$a - bP = -c + dP$$

$$P^* = \frac{a+c}{b+d}, Q^* = \frac{ad-bc}{b+d}$$

Market equilibrium after tax

long version

$$① Q^d = Q^s$$

$$② Q^d = a - bP_d : \text{demand is a fcn of the price really paid by consumers}$$

$$③ Q^s = -c + dP_s : \text{supply is a fcn of the price really received by producers}$$

At mkt equilibrium with no tax

$$④ P_d = P_s = P \text{ mkt price that we are going to solve for}$$

$$\text{To solve, } Q^d = Q^s$$

$$a - bP_d = -c + dP_s$$

$$a - bP = -c + dP$$

$$\rightarrow ① Q^d = Q^s$$

$$\rightarrow ② Q^d = a - bP_d$$

$$\rightarrow ③ Q^s = -c + dP_s$$

$$\rightarrow ④ P_s = P_d - t$$

$P_d = P$ market price to solve for
 $P_s = P - t$ price received by producers is the price paid by consumer minus tax

To solve :

$$Q^d = Q^s$$

$$a - bP_d = -c + dP_s$$

$$a - bP = -c + d(P - t)$$

$$a + c + dt = (b + d)P$$

$$P_{AT}^* = \frac{a+c+dt}{b+d} = P_d^*$$

since $d > 0$, $P_{AT}^* > P_{BT}^*$

$$Q_{AT}^* = a - bP_d^*$$

$$= a - b\left(\frac{a+c+dt}{b+d}\right)$$

$$Q_{AT}^* = \frac{ad - bc - bdt}{b+d}$$

since $b, d > 0$
 $Q_{AT}^* < Q_{BT}^*$

$$p_s^* = p_d^* - t = \frac{a+c+dt}{b+d} - t = \frac{a+c-bt}{b+d}$$

$$p_s^* < p_{BT}^*$$

Intro Math Econ, sec 046402, sem 2/2020, chapter 3, part 1

Tax revenue is equal to:

$$\text{Tax Revenue} = t Q_{AT}^*$$

$$= t \left[\frac{ad - bc - bdt}{b+d} \right]$$

Tax burden on consumers/buyers

$$\text{CB per unit} = p_d^* - p_{BT}^* = \frac{dt}{b+d} : \text{tax burden per unit on consumer}$$

$$\text{CB} = (p_d^* - p_{BT}^*) Q_{AT}^* = \frac{dt}{b+d} \left[\frac{ad - bc - bdt}{b+d} \right]$$

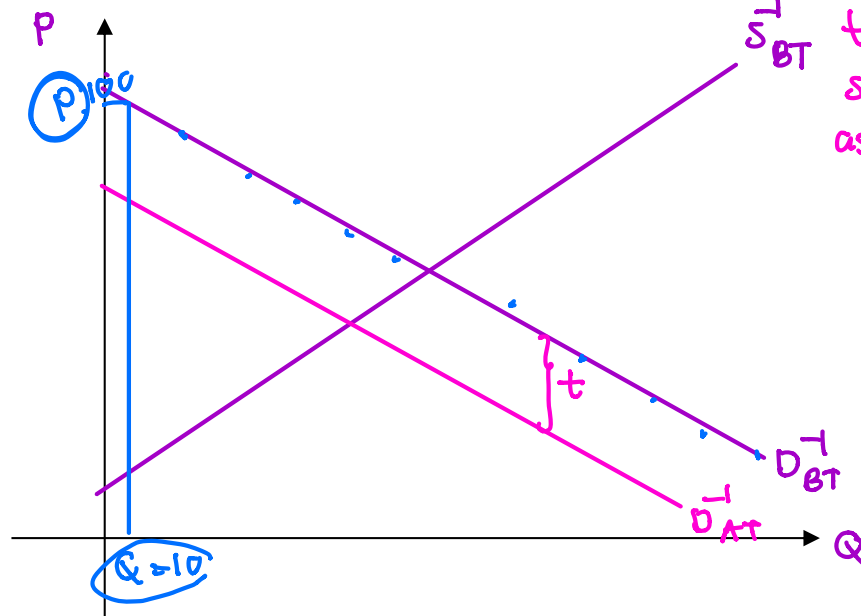
Tax burden on producers/sellers

$$\text{PB per unit} = p_{BT}^* - p_s^* = \frac{bt}{b+d} : \text{tax burden per unit on producer}$$

$$\text{PB} = (p_{BT}^* - p_s^*) Q_{AT}^* = \frac{bt}{b+d} \left[\frac{ad - bc - bdt}{b+d} \right]$$

CASE 1.2: Collect from consumers/buyers

Consumer will have lower WTP so that after paying tax, they pay the same amount as before tax



$$p_d^* = p_s^* + t$$

Whether p_{AT}^* , Q_{AT}^* , Tax burden, Tax revenue are the same as the case of p_s^* , p_{BT}^*

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Tax revenue is equal to:

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Tax burden on consumers/buyers

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Tax burden on producers/sellers

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OBSERVATION:

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CASE 2: Ad Valorem tax

Collect tax $t\%$ of price, e.g. 7% of price per bottle

CASE 2.1: Collect from producers/suppliers



Market equilibrium before tax

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Market equilibrium after tax

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Tax revenue is equal to:

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Tax burden on consumers/buyers

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Tax burden on producers/sellers

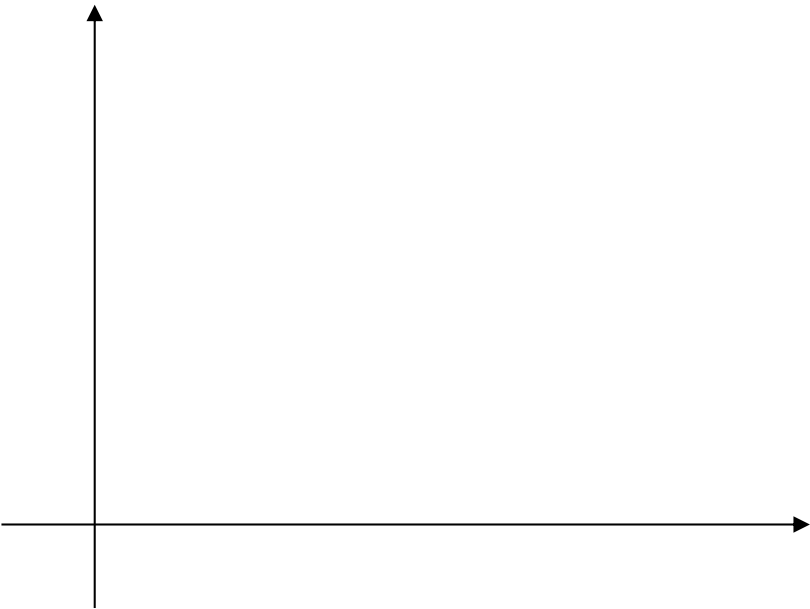
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CASE 2.2: Collect from consumers/buyers



Market equilibrium before tax

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Market equilibrium after tax

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Tax revenue is equal to:

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Tax burden on consumers/buyers

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Tax burden on producers/sellers

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OBSERVATION:

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Tax incidence (or incidence of tax) is an economic term for understanding the division of a tax burden between buyers and sellers or producers and consumers.

Tax incidence is related to the price elasticity of supply and demand. When supply is more elastic than demand, the tax burden falls on the buyers. If demand is more elastic than supply, producers will bear the cost of the tax.

Digression

What is elasticity ? How can we derive elasticity?

From linear demand function in (2):

.....

With P on Y-axis and Q on x-axis, the inverse demand function is:

.....

From linear supply function in (3):

.....

The inverse supply function is:

.....

$-b$ is

.....

.....

$-d$ is

To be able to compare how responsive of change in quantity to change in price across different commodities, the concept of “elasticity” comes in handy.

Elasticity is the measurement of the percentage change of one economic variable in response to one percentage change in another.

(4)

The price elasticity of demand is:

The price elasticity of supply is:

.....

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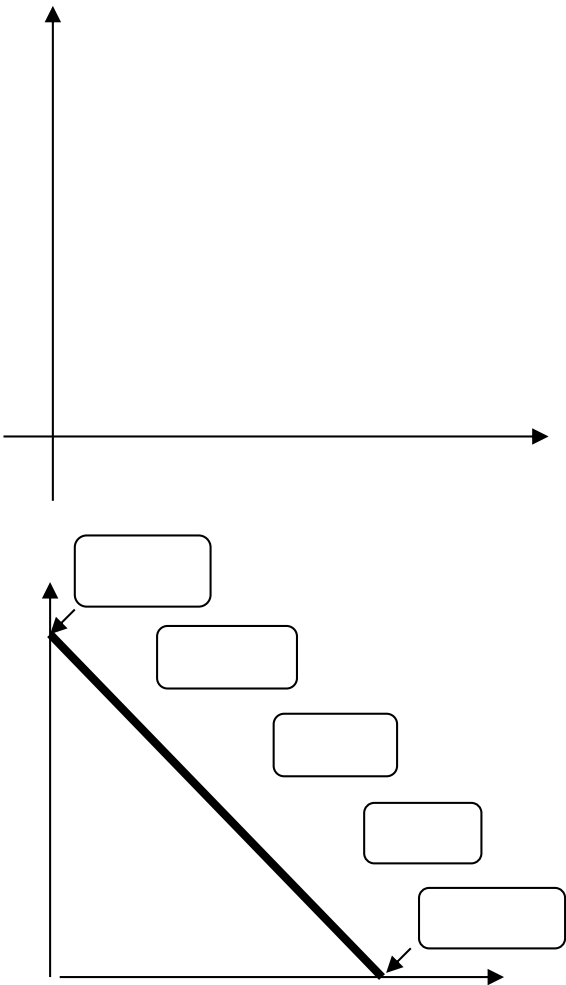
Inelastic vs. Elastic vs. Unit elastic

$E_p < 1$

$E_p > 1$

$E_p = 1$

Price Elasticity of Demand at different point on inverse demand function:

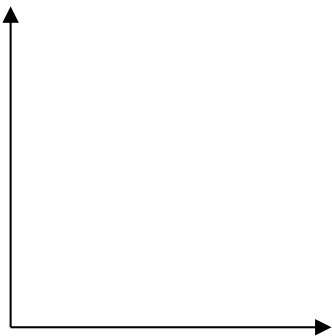
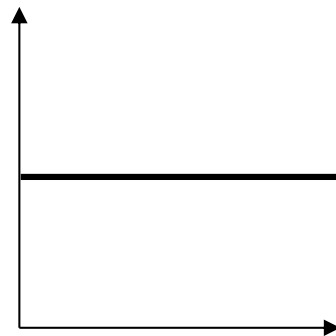
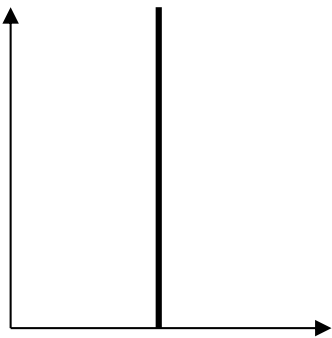


The price elasticity of demand at each point of linear demand function, with negative slope, is not equal to other point. This is because:

$$\frac{P}{Q}$$

$$\frac{\Delta Q}{\Delta P} = -b$$

In which case are the price elasticities of demand for different points equal?



Tax incidence and Elasticity

Whoever cannot adjust themselves instantaneously to change in price will have to bear a larger portion of the tax burden.

