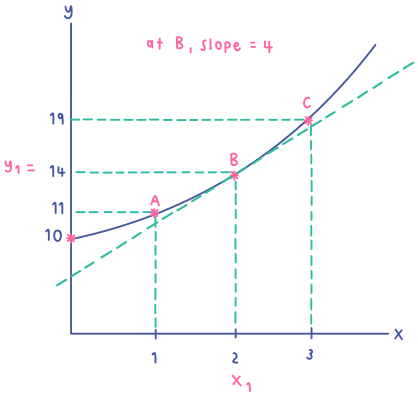
Approximation of change of y as a result of change of x

$y = f(x) = 10 + x^2$

- When $x_1 = 2, y_1 = 14$. If $\Delta x = 0.1$, we can approximate



$f'(x) = 2x$

$$\Delta y \approx f'(x_1).\Delta x$$
$$= f'(2).0.1$$
$$= 2(2) \cdot 0.1 = 0.4$$

- What is the real Δy? $x_1 = 2$, $\Delta x = 0.1$, $x_2 = 2.1$

$$y_2 = f(2.1) = 10 + (2.1)^2 = 14.41$$
$$\Delta y = y_2 - y_1 = 14.41 - 14 = 0.41$$

- We underestimate the real change of y.
- What if $\Delta x = -0.2$? Approximate the change of y.

$\Delta y \approx f'(x_1)\Delta x$

When x + y have linear relationship

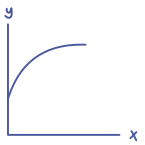
$y = 10 + 2x \rightarrow \frac{dy}{dx} = 2$
$y = 10 - 3x \rightarrow \frac{dy}{dx} = -3$
Constant

HW Given $y = 10 + \sqrt{x}$,

a) Find the derivative $f'(x)$.
b) Fill in the table

Point	X	Y	$f'(x)$
	0	10	
A	1	11	1/2
B	2	11.414	0.354
C	3	11.732	0.289

$\frac{1}{2\sqrt{x}}$



c) Does the slope increase as x increases? No, the slope decrease
d) Approximate the change in Y when $\Delta x = 0.2$ at $x_1 = 3$. Is the approximation under- or over-estimate? over estimate

Note: If the function $f(x)$ is linear, the approximation is exact.

$y = 10 + 2x$ — slope $= \frac{dy}{dx} = f'(x) = 2$

at $x_1 = 2$ $\Delta x = 0.1$
$\Delta y = f'(x) \cdot \Delta x$
$= 2 \cdot 0.1 = 0.2$