Midterm 2014 Q1

For a function $f(x, y, z) = 2xy + 2yz + xz$

subject to $xyz = C$ where $x, y, z > 0$ and C is a constant

(a) Given that $C = 108$, use the **substitution method** to calculate any possible critical point(s). In addition, use the second derivative test to classify the critical point(s).

(b) Use the **Lagrange Multiplier method** to determine any possible critical point(s) and optimum value of the function **in term of C only**.

(c) Form (b), determine the value of C, which gives the optimum value of the function $f^*(x^*, y^*, z^*) = 108$, and identify the corresponding critical point.

(d) From (b) and (c), if the constant C in the constraint equation decreases by 0.4, estimate the new optimum value of the function.

a) The critical point is (6,3,6) which is a relative maximum.

b) $(x, y, z) = \left(2\sqrt[3]{\frac{C}{4}}, \sqrt[3]{\frac{C}{4}}, 2\sqrt[3]{\frac{C}{4}}, \right)$

$f^*(2y,y,y)=12y^2 = 6\sqrt[3]{\frac{C^2}{2}}$

c) The critical point is (6,3,6)

d) $\lambda = \frac{2}{3}$

$f^*_{new}=107.73$

Midterm 2012 Q2

(a) For a 2-variable function $f(x, y) = x^2 + y^2$

subject to the constraint $x^2 + xy + y^2 = 3$,

use the **Lagrange Multiplier** method, to determine the absolute maximum. (8 marks)

(b) **Estimate** the maximum value(s) if the constraint changes to $x^2 + xy + y^2 = 2.97$ (2 marks)

a) The critical points are (1,1) and (-1,-1)

f(1,1) and f(-1,-1) are absolute minimum. $f(-\sqrt{3}, \sqrt{3})$ and . $f(\sqrt{3}, -\sqrt{3})$ are absolute maximum.

b) $\lambda = 2$, $f^*_{new}=5.94$