Student ID.................................

EE431 Economics of Financial Markets and Institutions
Problem Set 2 : Debt Market and Structure of Interest Rate

Solution

1. Suppose you expect the interest rate to go down in the future. You are considering to buy one of these bonds, hold it for a year and then sell it out. Which bond you will buy in order to get the highest returns? Explain.

   - Bond A, 10 years bond with 1,000 Baht face value, 10% annual coupon, 10% yield to maturity.
   - Bond B, 3 years bond 100 Baht face value, 10% annual coupon, 10% yield to maturity.

   Answer. If we are going to buy the bond for one year and sell it out. Then, the actual rate of return we get is not equal to yield to maturity of the bond.

   Since interest rate is expected to go down, bond prices are expected to increase in the future. Bond A and Bond B have the same coupon rate and the same yield to maturity but Bond A has a longer time to maturity than Bond B. Therefore, Bond A is more price sensitive than Bond B. For a given decrease in interest rate, the percentage increase in price for Bond A is larger than that for Bond B. The rate of capital gain on Bond A is higher than that on Bond B.

   Initial current yields from investing in Bond A and Bond B are equal but Bond A has a higher rate of capital gain than bond B. Thus, the actual rate of return from one-year investment in Bond A is greater than that from Bond B. For this reason, we should choose to invest in Bond A to get the highest return.

   Note : You may answer this question by giving a numerical example together with an explanation. For example, you may assume that the interest rate is believed to decrease by 1 percent. Then, you compare the actual rates of return from investing in the two bonds. Your answer must include both the definition of actual rate of return and an explanation on bond price sensitivity. The objective of this question is to test your understanding about the two concepts mentioned previously. Make sure you make a clear explanation on both concepts. Plus, you must provide sufficient details.

2. Consider bond XYZ with a modified duration of 6.33. Suppose that the market value of this bond is $3 million. The approximate dollar price change for a 1 percentage change in yield to maturity is ...6.33%.....

   Reason : .....Modified duration is the percentage change in price from a 1% change in yield. The modified duration is 6.33. We can say that a 1% change in yield to maturity leads to a $1\% \times 6.33 = 6.33\%$ change in price in the opposite direction. If the yield to maturity increases by 1%, the percentage decrease in the price of bond XYZ is approximately 6.33%. The current value of the bond is $3 million. Hence, as the yield to maturity increases by 1%, the value of the bond will decrease approximately by $6.33\% \times 3 = 0.1899$ million dollars....

3. Consider a 5% coupon bond with the par value of 2,000 Baht, selling for 2,228.46372 Baht. The bond will mature in 4 years. Find the modified duration of the coupon bond and explain its meaning.

$D = \sum_t \frac{\text{Present Value of } CF_t}{\text{Bond Market Price}} \times t$, where Bond Market Price $= \sum_t \frac{CF_t}{(1+k)^t}$.

First, find the yield to maturity of the bond. Yield to maturity is defined as the discount rate that equates present value of cashflows from bonds to its market price.

$$\begin{aligned} \text{Bond Price} &= \sum_t \frac{CF_t}{(1+k)^t}, \\ 2228.46372 &= \frac{100}{(1+YTM)} + \frac{100}{(1+YTM)^2} + \frac{100}{(1+YTM)^3} + \frac{100}{(1+YTM)^4} + \frac{2,000}{(1+YTM)^4}, \\ &= 100 \times PVIFA_{YTM\%,4} + 2,000 \times PVIF_{YTM\%,4}. \end{aligned}$$

The yield to maturity usually cannot be solved directly. It is determined by using trial and error or iterative method. The method consists of a sequence of iterations, involving repeated calculations.

The bond in the question is sold at premium. Hence, the coupon rate is greater than the discount rate.

Therefore, guess the initial value of yield to maturity as 4%. Then,

$$\begin{aligned} \text{Bond Price} &= 100 \times PVIFA_{4\%,4} + 2,000 \times PVIF_{4\%,4}, \\ &= (100 \times 3.62990) + (2,000 \times 0.85480), \\ &= 362.99 + 1709.6, \\ &= 2072.59. \end{aligned}$$

2,072.59$<$ 2,228.46372. Therefore, we decrease the trial discount rate to 3% and recalculate.

$$\begin{aligned} \text{Bond Price} &= 100 \times PVIFA_{3\%,4} + 2,000 \times PVIF_{4\%,4}, \\ &= (100 \times 3.71710) + (2,000 \times 0.88849), \\ &= 371.71 + 1,776.98, \\ &= 2,148.69. \end{aligned}$$

2,148.69 $<$ 2,228.46372. Therefore, we decrease the trial discount rate to 2% and recalculate.

$$\begin{aligned} \text{Bond Price} &= 100 \times PVIFA_{2\%,4} + 2,000 \times PVIF_{2\%,4}, \\ &= (100 \times 3.80773) + (2,000 \times 0.92385), \\ &= 380.773 + 1,847.7, \\ &= 2,228.473. \end{aligned}$$

2,228.473 $\approx$ 2,228.46372. (Note that the two numbers are not exactly equal. This is because the calculation is based on the present value table in which the numbers are rounded off to five decimal places. The difference between the two numbers is lower than 0.01. The two numbers are close enough. In this case, we conclude that yield to maturity is approximately equal to 2%. If we try 1% discount rate, we would get the bond price equal to 2,312.157, which is even further from 2,228.46372. Of all the numbers we can obtain from the calculation by using the present value table, 2,228.473 is the closest number to 2228.46372. If we use a financial calculator or a computer program, we would find that the yield to maturity for this bond is equal to 2%. A MS Excel file for this question is provided on the BE moodle, you may have a look.)

Second, find the duration of the bond.

$D = \sum_t \frac{\text{Present Value of } CF_t}{\text{Bond Market Price}} \times t$, where Bond Market Price $= \sum_t \frac{CF_t}{(1+k)^t}$.

(1) t	(2) Present Value of CF_t	(3) $\frac{\text{PV of } CF_t}{\text{Bond Market Price}}$	(4) $\frac{\text{PV of } CF_t}{\text{Bond Market Price}} \times t$ (3) × (1)
1	$100 \times PVIF_{2\%,1}$ $= 100 \times 0.98039$ $= 98.039$	$\frac{98.039}{2,228.473}$ $= 0.0439938$	0.0439938×1 $= 0.0439938$
2	$100 \times PVIF_{2\%,2}$ $= 100 \times 0.96117$ $= 96.117$	$\frac{96.117}{2,228.473}$ 0.0431313	0.0431313×2 $= 0.0862626$
3	$100 \times PVIF_{2\%,3}$ $=100 \times 0.94232$ 94.232	$\frac{94.232}{2,228.473}$ $= 0.0422855$	0.0422855×3 $= 0.1268565$
4	$2{,}100 \times PVIF_{2\%,4}$ $= 2,100 \times 0.92385$ $1,940.085$	$\frac{1,940.085}{2,228.473}$ $= 0.8705894$	0.8705894×4 3.4823576
Total	$2,228.473$ = Bond Market Price	1	3.7394705

The bond's duration is equal to 3.7394705.

Third, find the modified duration of the bond.

$$\begin{aligned}\text{Modified Duration} &= \frac{\text{Duration}}{1+\text{YTM}}, \\ &= \frac{3.7394705}{1+2\%}, \\ &= \frac{3.7394705,}{1.02} \\ &= 3.66615.\end{aligned}$$

Fourth, explain the meaning of the bond's modified duration.
If the interest rate (yield to maturity of the bond) changes by 1%, the bond price will change by about 3.6615%, in an opposite direction.
In other words, if the interest rate (yield to maturity of the bond) increases by 1%, the bond price will decrease by about 3.6615%.

*Note that A student may get slightly different numbers due to round-offs. The difference should be small enough; otherwise it is unacceptable.

4. When interest rates fall, the prices of outstanding bonds ...rise.... (rise or fall).
5. The market price of longer maturity bonds fluctuates ..more...... (more or less) compared with shorter maturity bonds as interest rates change.
6. Write the Fisher Equation, relating the nominal interest rate i, the real interest rate r, and expected (or anticipated) inflation π^e :

Answer : $$\begin{aligned}(1+i) &= (1+r)(1+\pi^e) \\ i &\approx r+\pi^e\end{aligned}$$

7. Suppose your earns 7% nominal interest from your deposit account. If inflation is 5%, what is the real rate of return?

   Answer : From the Fisher Equation, $(1+i) = (1+r)(1+\pi)$, or $(1+i) = (1+r)(1+\pi^e)$. * SEE the note at the end of this solution to problem set.

$$\begin{aligned}(1+i) &= (1+r)(1+\pi^e)\\ (1+0.07) &= (1+r)(1+0.05)\\ (1+r) &= \frac{(1+0.07)}{(1+0.05)}\\ &= 1.01905\end{aligned}$$

   Thus, the real rate of return is 2.857%

   Alternatively,

$$\begin{aligned}i &\approx r+\pi^e\\ r &= i-\pi^e\\ &= 0.07-0.05\\ &= 0.02\end{aligned}$$

   Thus, the real rate of return is 3%

8. How does real interest rate in the financial market relate to marginal product of capital?

   [An approximate guideline.]

   The association of the equilibrium real rate of interest with the marginal product of capital is a principle of modern mainstream economics. Marginal productivity of capital refers to the incremental increase in output if we employ an additional unit of capital. The return to capital is the MPK-depreciation rate. The return to capital should be equal the real risk-free rate to ensure that firms maximize profits.

Note: Question from students and the answer:

Question : I have question about question 7 which is about fisher effect. Since for the Fisher equation, we need to use expected inflation not the current inflation so the answer in my opinion of this question should be "the question does not give enough information" [many thanks for asking.]

Answer :

“Regarding question 7 in homework 2, we can use fisher equation in two ways :

(1) before actual inflation rate is realised (we use expected inflation), real interest rate =nominal interest rate - expected inflation rate, and

(2) after actual inflation rate is realised, real interest rate = nominal interest rate - actual inflation rate.

For example, at the beginning go the year, we make a decision on whether or not to deposit our money into a bank for one-year and receive 3% interest rate. We expect inflation rate at the end of the year will be 2%. Then, from the fisher equation, we **expect to receive** real interest rate of 3%-2% = 1%. [nominal interest rate - expected inflation rate].

At the end of the year, suppose inflation rate is 0%. Then, from fisher equation, we **receive** real interest rate of 3%-0% = 3%. [nominal interest rate - actual inflation rate].

When we study **bond demand curve**, we focus on **the first way** since people make their decision based on expectation. We make our decision to deposit the money at the beginning of the year, based on 1% real interest rate we **expect** to receive.

Specifically, the question gives information on "inflation rate" which may refer to "actual" inflation or “expected” inflation.

In **some** other context/theory, **it is possible** that only one method can be applied, i.e. it must be expected inflation or it must be actual inflation.