

Assignment 6

1. Find all *vertical*, *horizontal*, *slant* asymptotes (if any) of the following relations.

(a)

$$r = \left\{ (x, y) \in X \times Y \mid yx^3 + y = -2x^3 - 3x + 5 \right\}.$$

(b)

$$s = \left\{ (x, y) \in X \times Y \mid 2x^2 + x - xy - y = 5 \right\}.$$

(c)

$$f = \left\{ (x, y) \in X \times Y \mid y = \frac{6x^3 - 1}{18 - 2x^2} \right\}.$$

2. [**CORRECTED**] Consider the relation f defined from X to Y , $X, Y \subseteq \mathbb{R}$,

$$f = \left\{ (x, y) \in X \times Y \mid y = \frac{x^3}{x^2 - 4} \right\}.$$

Hint: For $f(x) = \frac{x^3}{x^2 - 4}$, the first and the second derivatives are given, respectively, by

$$f'(x) = \frac{x^4 - 12x^2}{(x^2 - 4)^2}, \quad \text{and} \quad f''(x) = \frac{8x^3 + 96x}{(x^2 - 4)^3}.$$

- (a) Find the domain of f .
- (b) Find x -intercepts and y -intercepts (if any).
- (c) Determine the symmetry of f .
- (d) Find the horizontal and vertical asymptotes for f (if any).
- (e) Find the critical number of f . Determine the intervals on which f is increasing and decreasing. Determine the extrema (maximum and minimum) of f .
- (f) Determine the intervals on which f is concave up and concave down. Find the points of inflection (if any).
- (g) Sketch the curve of f .

Optional Problems

1. Find the critical numbers of the given functions on $(-\infty, \infty)$.

(a) $f(x) = x^3 - 3x^2 + 3x - 1$	(b) $f(x) = \frac{x^2}{x^2 + 2}$	(c) $f(x) = e^{-x} + 2x$
(d) $f(x) = -x + \sin(x)$	(e) $f(x) = x^2 - 8 \ln(x)$	
2. Determine the intervals on which the given function f is increasing and the interval on which f is decreasing.

(a) $f(x) = x^2 + 6x - 1$	(b) $f(x) = x^4 - 4x^3 + 9$	(c) $f(x) = x^2 e^{-x}$	(d) $f(x) = x^2 - 2 x $
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3. Use the *First Derivative Test* to find the relative extrema of the given function.

(a) $f(x) = x^3 - 3x$	(b) $f(x) = x^3 + x - 3$	(c) $\frac{x^2 + 3}{x + 1}$
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4. For each given function,
- (i) use the Second Derivative Test, when applicable, to find the relative extrema;
 - (ii) find intercepts, vertical/horizontal asymptotes, and points of inflection, (if any);
 - (iii) find the intervals on which it is increasing and the interval on which it is decreasing;
 - (iv) find the intervals on which it is concave up and the intervals on which it is concave down;
 - and (v) sketch the graph.

(a) $f(x) = x^3 + 3x^2 + 3x + 1$ (b) $f(x) = 6x^5 - 10x^3$ (c) $f(x) = \frac{x}{x^2+2}$
(d) $f(x) = 2x - x \ln(x)$ (e) $\frac{x^2+3}{x+1}$

5. Let $X, Y_1, Y_2 \subseteq \mathbb{R}$. Consider the relations f_1, f_2 defined by

$$f_1 = \left\{ (x, y) \in X \times Y_1 \mid y = \sqrt{\frac{x+1}{2x-3}} \right\}, \quad f_2 = \left\{ (x, y) \in X \times Y_2 \mid (x, -y) \in f_1 \right\}.$$

Sketch the curve of the relation $f = f_1 \cup f_2$.

6. Find the vertical and horizontal asymptotes (if any) of $f(x) = \frac{x^2-1}{2x}$ and show that $y = \frac{x}{2}$ is a slant asymptote of f .