

Topic 4

Risk and Information (Chapter 15)

Tools for Describing Risky Outcomes

Definition: A **lottery** is any event with an uncertain outcome.

Examples: Investment, Roulette, Football Game.

Definition: A **probability** of an outcome (*of a lottery*) is the likelihood that this outcome occurs.

Example: The probability often is estimated by the historical frequency of the outcome.

Tools for Describing Risky Outcomes

Definition: The **probability distribution** of the lottery depicts all possible payoffs in the lottery and their associated probabilities.

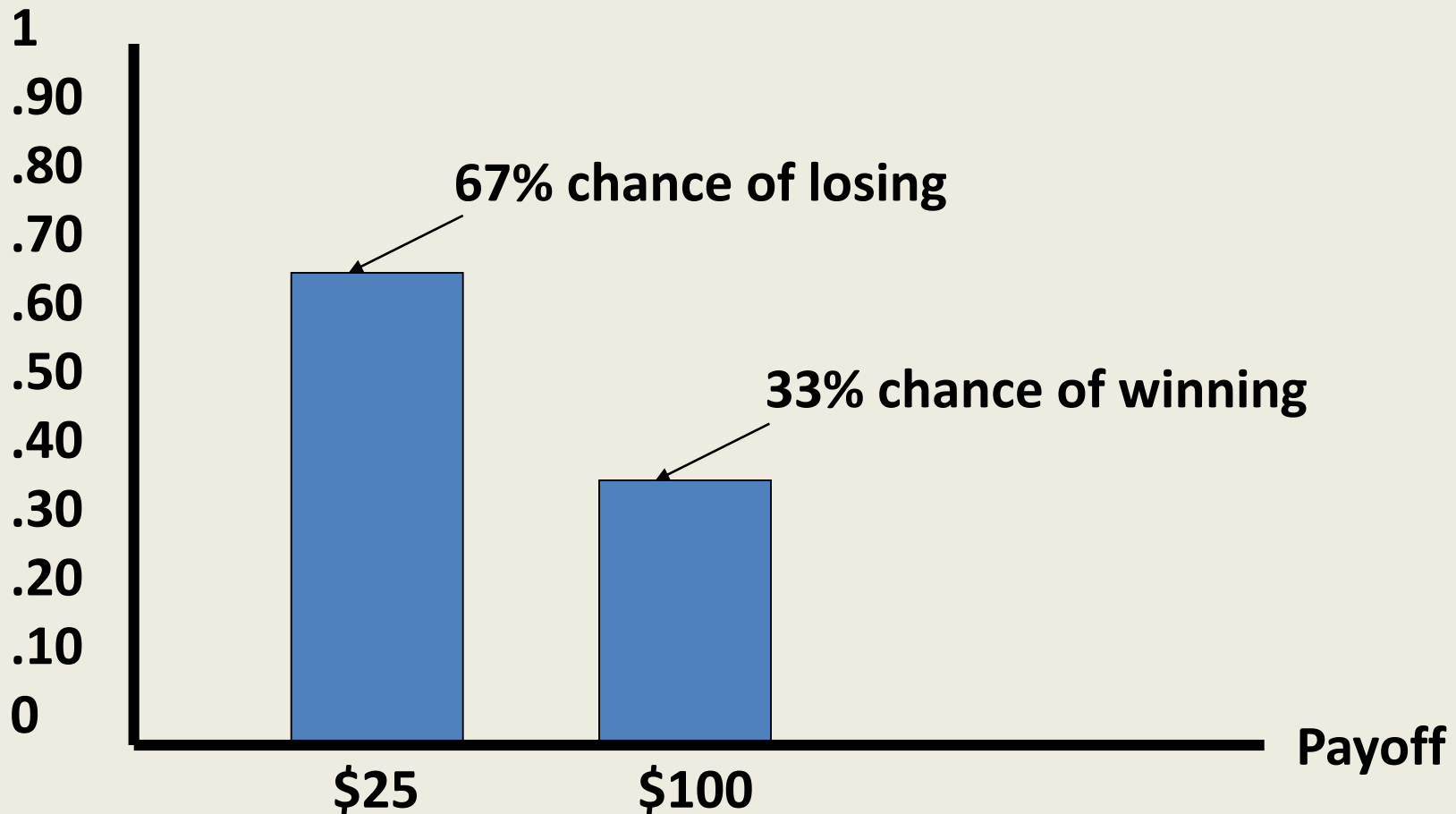
Property:

- The probability of any particular outcome is between 0 and 1.
 - The sum of the probabilities of all possible outcomes equals 1.
-

Definition: Probabilities that reflect subjective beliefs about risky events are called **subjective probabilities**.

Probability Distribution

Probability



Expected Value and Expected Utility

Definition: The **expected value** of a lottery is a measure of the average payoff that the lottery will generate.

Let X be a lottery with three possible outcomes A , B , and C .

$$E[X] = \Pr(A) \cdot A + \Pr(B) \cdot B + \Pr(C) \cdot C$$

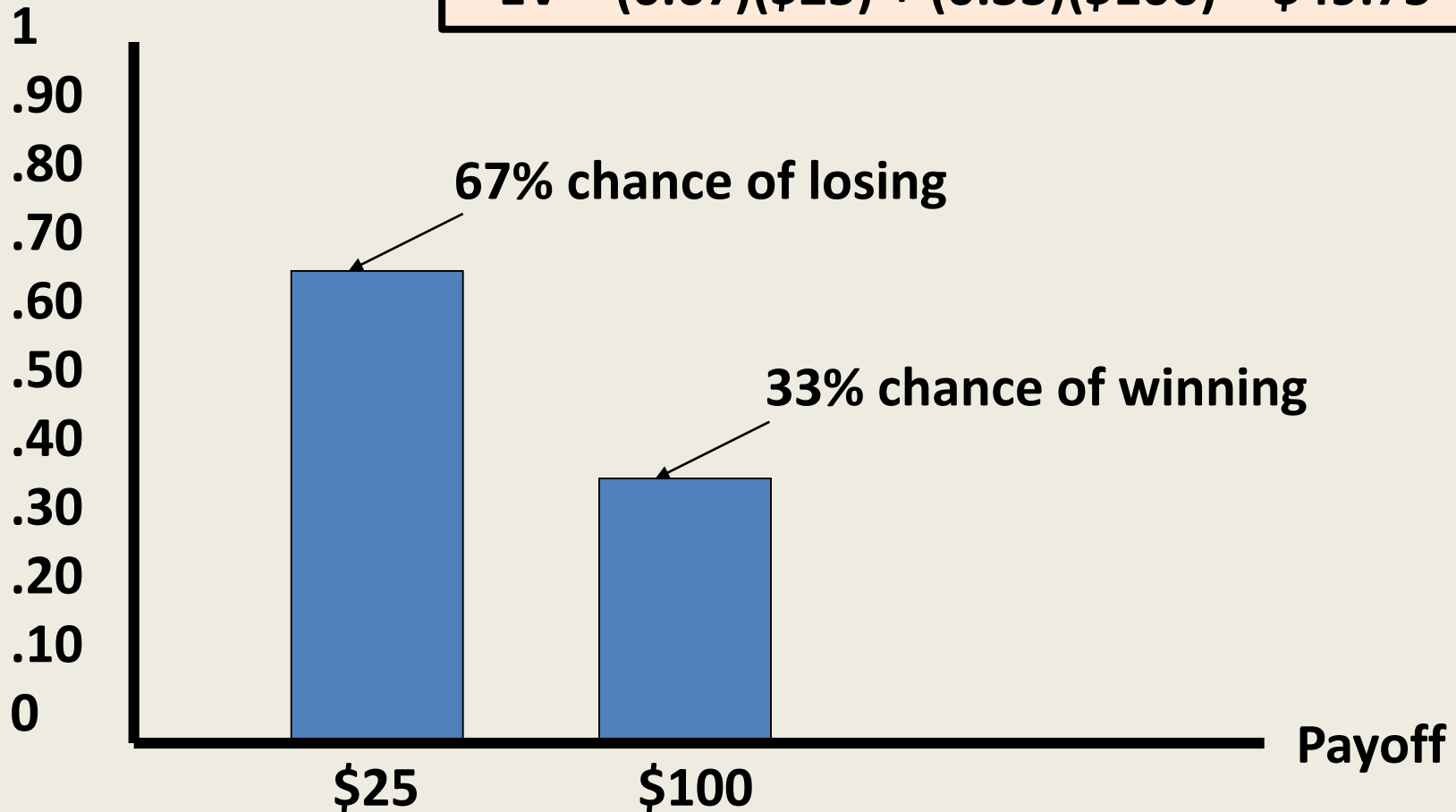
Definition: The **expected utility** of a lottery is the average value of the utility levels that the lottery will generate.

$$E[U(X)] = \Pr(A) \cdot U(A) + \Pr(B) \cdot U(B) + \Pr(C) \cdot U(C)$$

Expected Value

Probability

$$EV = (0.67)(\$25) + (0.33)(\$100) = \$49.75$$



Variance & Standard Deviation

Definition: The **variance** of a lottery is a measure of the lottery's riskiness.

$$\text{Var}[X] = E[X^2] - (E[X])^2$$

$$\text{Var}[X] = (x - E[X])^2 \cdot \Pr(X = x)$$

Definition: The **standard deviation** of a lottery is the square root of the variance. It is an alternative measure of risk.

Example

$X = x$	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$\Pr(X = x)$	0.1	0.2	0.4	0.3

$$E[X] = 0(0.1) + 1(0.2) + 2(0.4) + 3(0.3) = 1.9$$

$$\begin{aligned} \text{Var}[X] &= (x - E[X])^2 \cdot \Pr(X = x) \\ &= (0.1)(0-1.9)^2 + (0.2)(1-1.9)^2 + (0.4)(2-1.9)^2 + (0.3)(3-1.9)^2 \\ &= 0.89 \end{aligned}$$

$$\text{Var}[X] = E[X^2] - (E[X])^2$$

$$E[X^2] = (0^2)(0.1) + (1^2)(0.2) + (2^2)(0.4) + (3^2)(0.3) = 4.5$$

$$\text{Var}[X] = 4.5 - 1.9^2 = 0.89$$

Risk Preferences

Notes:

- Utility as a function of income only

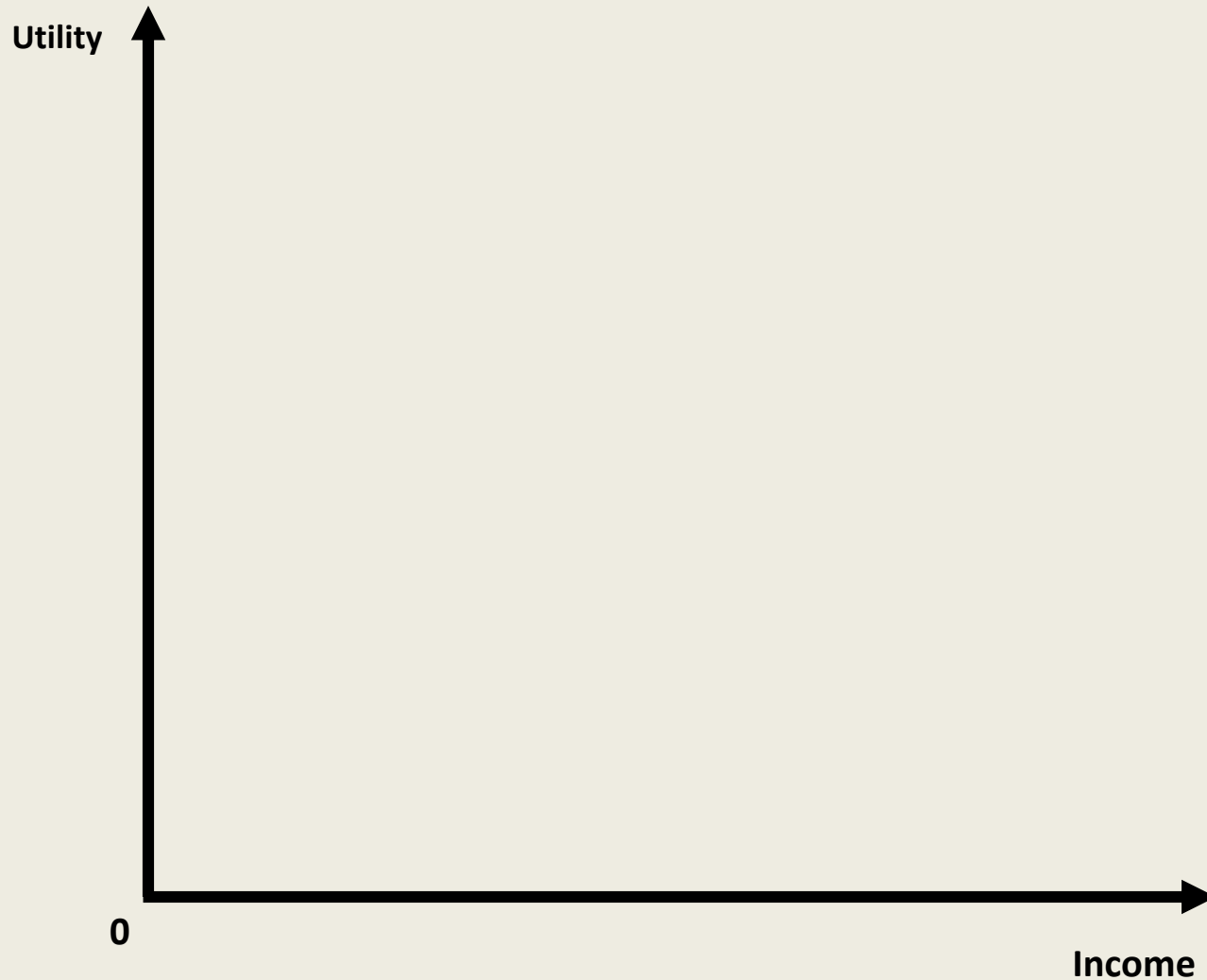
Definition: The risk preferences can be classified as follows:

An individual who prefers a sure thing to a lottery with the same expected value is **risk-averse**.

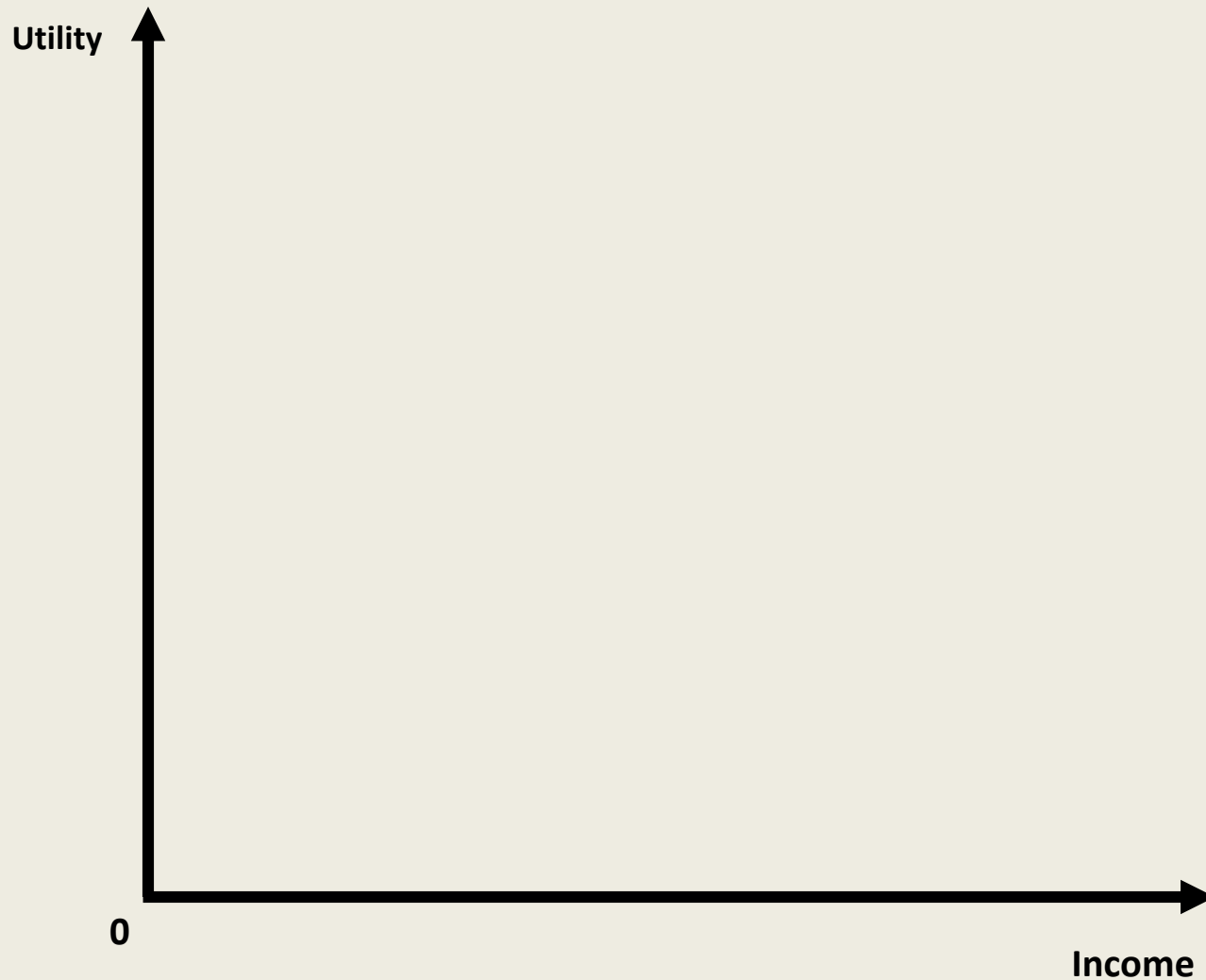
An individual who is indifferent about a sure thing or a lottery with the same expected value is **risk-neutral**.

An individual who prefers a lottery to a sure thing that equals the expected value of the lottery is **risk-loving**.

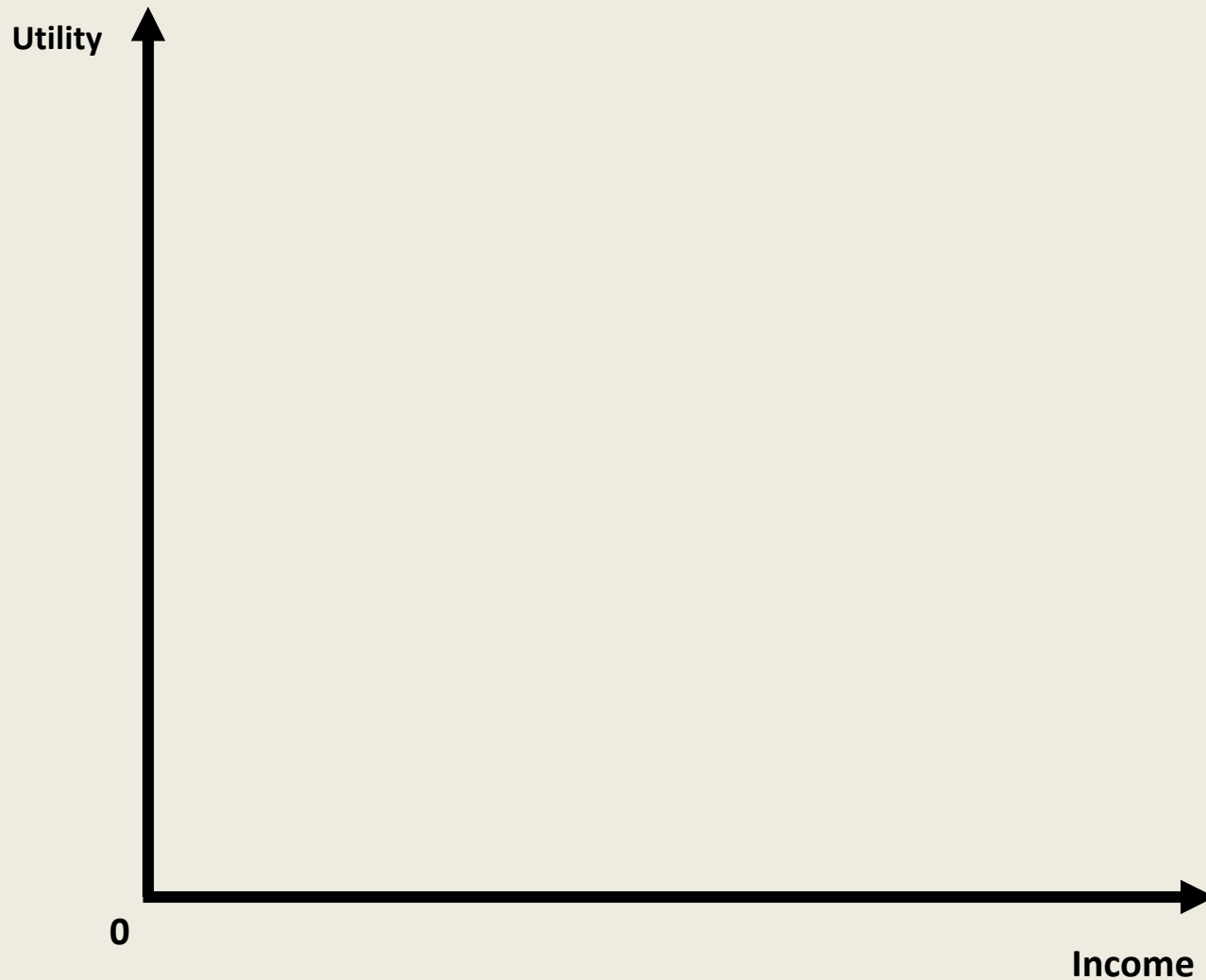
Risk-Averse Individuals



Risk-Neutral Individuals



Risk-Loving Individuals



Risk Preferences

Let X be a lottery.

$U(E[X])$ denotes **Utility of the EV (sure thing) of the lottery X .**

$E[U(X)]$ denotes **Expected Utility of the lottery X .**

risk-averse: $U(E[X]) > E[U(X)]$

happiness from sure thing > expected happiness from lottery

risk-neutral: $U(E[X]) = E[U(X)]$

happiness from sure thing = expected happiness from lottery

risk-loving: $U(E[X]) < E[U(X)]$

happiness from sure thing < expected happiness from lottery

Computing the Expected Utility for Two Lotteries

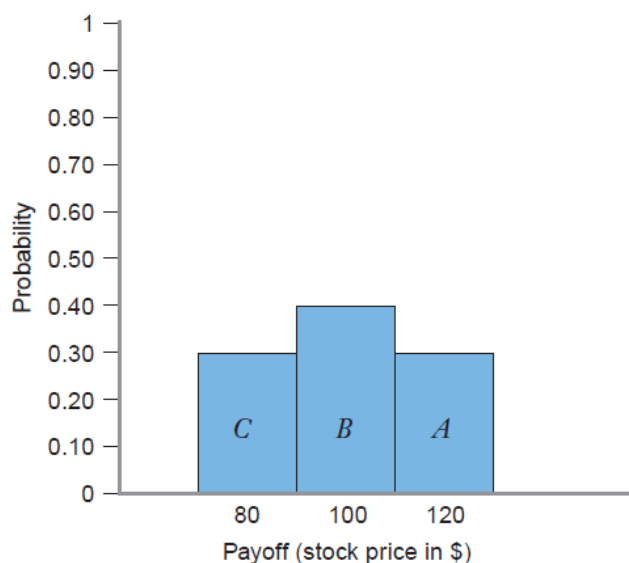
Consider the two lotteries depicted in Figure 15.3. They have the same expected value, but the first (investing in the Internet company's stock) has a larger variance than the second (investing in the public utility company's stock). This tells us that the first lottery is riskier than the second lottery. Suppose that a risk-averse decision maker has the utility function $U(I) = \sqrt{100I}$, where I denotes the payoff of the lottery.

Problem Which lottery does the decision maker prefer—that is, which one has the bigger expected utility?

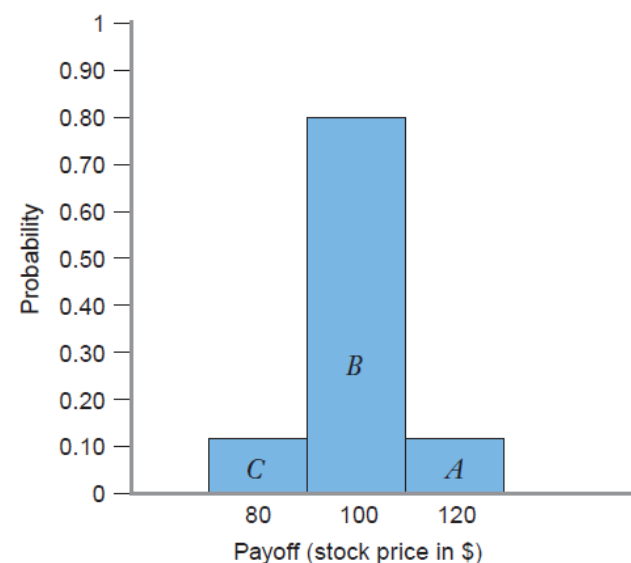
One decision maker is risk neutral, with the utility function $U(I) = 100I$, while the other is risk loving, with the utility function $U(I) = 100I^2$, where I denotes the payoff of the lottery.

Problem

- Which lottery does the risk-neutral decision maker prefer?
- Which lottery does the risk-loving decision maker prefer?



(a) Internet company



(b) Public utility company

Risk Premium

Most people are risk-averse, but some are willing to take risks if they are fairly compensated. The money needed as the compensation is called the **risk premium**.

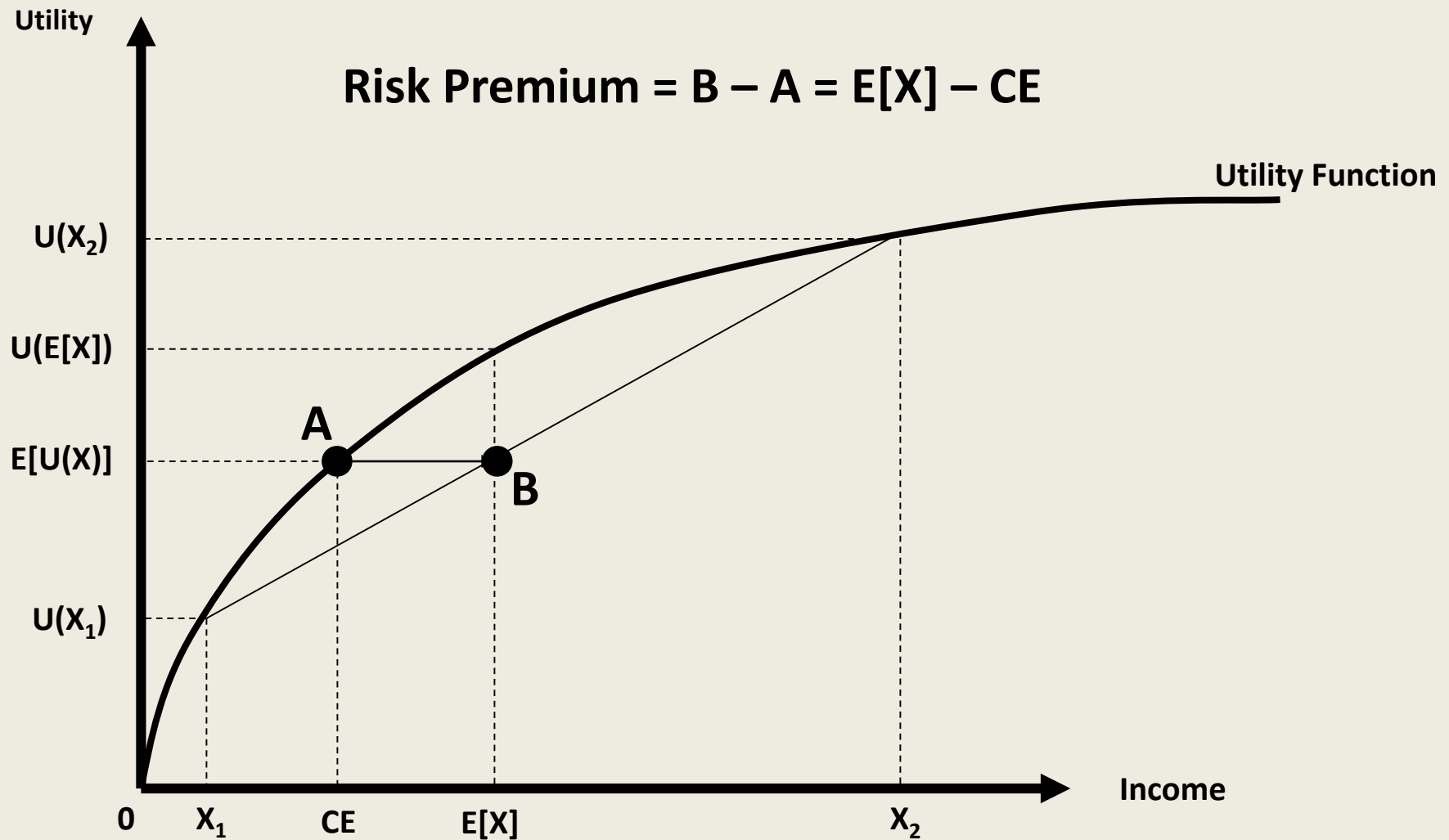
Definition: The **certainty equivalent (CE)** is the guaranteed amount of money (sure thing) such that an individual is indifferent between the lottery and the said money.

$$U(CE) = E[U(X)]$$

Definition: The **risk premium (RP)** of a lottery is the difference between the EV of the lottery and its CE.

$$RP = E[X] - CE$$

Risk Premium



Risk Premium

Risk-Averse: $CE < E[X]$

Risk Premium is positive

They are willing to accept a smaller sure thing rather than the lottery.

Risk-Loving: $CE > E[X]$

Risk Premium is negative

They require a larger sure thing because they love the lottery.

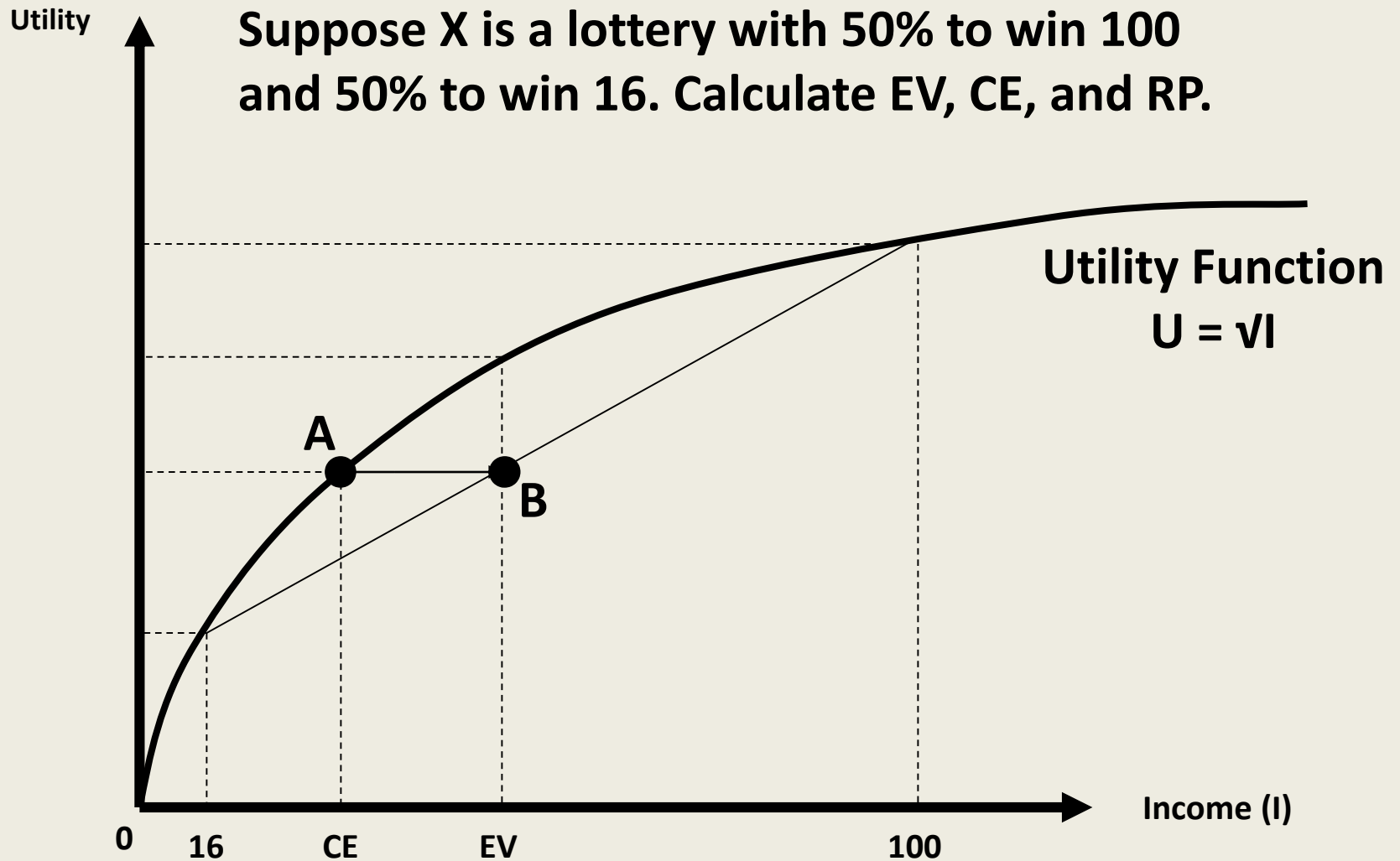
Risk-Neutral: $CE = E[X]$

Risk Premium is zero

They are indifferent between the sure thing and the lottery.

The larger the variance of the lottery, the riskier the lottery, and the larger the risk premium

Risk Premium – Example



Measuring Risk Aversion

- (Arrow-Pratt) coefficient of **absolute risk aversion**:

$$ARA = -\frac{U''(W)}{U'(W)}$$

- (Arrow-Pratt) coefficient of **relative risk aversion**:

$$RRA = -W \frac{U''(W)}{U'(W)}$$

Measuring Risk Aversion

The larger the coefficient, the more risk-averse the agent.

RRA is better than ARA when the agent's preference to risk varies as his wealth changes, e.g. he is risk-averse when poor and risk-loving when rich..

Example:

$$U = \sqrt{W}$$

$$ARA = (1/2)W$$

$$RRA = 1/2$$

Insurance

Risk Premium is the money that risk-averse people accept to take risks.

Insurance Premium is the “price” of the insurance that risk-averse people buy in order to eliminate the risks.

Fairly-Priced Insurance Policy is a policy where the insurance premium is equal to the expected value of the insurance payment.

Fairly-Priced Insurance – Example

Example

Insurance Premium (price) = \$500

It promises to pay \$10,000 if Bad Thing happens.
It promises to pay \$0 if Nothing happens.

$\text{Pr}(\text{BadThings}) = 5\%$

$\text{Pr}(\text{Nothing}) = 95\%$

$$500 = .05(10,000) + .95(0)$$

That is, insurance premium = expected payment.
Thus, this insurance is fairly priced.



LEARNING-BY-DOING EXERCISE 15.4

The Willingness to Pay for Insurance

Your current disposable income is \$90,000.

Suppose that there is a 1 percent chance that your house may burn down, and if it does, the cost of repairing it will be \$80,000, reducing your disposable income to \$10,000.

Suppose, too, that your utility function is $U = \sqrt{I}$.

Problem

- (a) Would you be willing to spend \$500 to purchase an insurance policy that fully insures you against your loss?
- (b) What is the highest price that you would be willing to pay for an insurance policy that fully insures you in the event that your house burns down?

Adverse Selection & Moral Hazard

Definition: **Asymmetric Information** is a situation in which one party knows more about its own actions or characteristics than another party.

Definition: **Adverse Selection** is *opportunism* characterized by an informed person's taking advantage of a less informed person who does not know about an *unobserved characteristic* of the informed person.

Definition: **Moral Hazard** is *opportunism* characterized by an informed person's taking advantage of a less informed person through an *unobserved action*.

Adverse Selection & Moral Hazard

Adverse Selection / Hidden Information:

- Good and Bad cars in second-hand garages.
- Healthy and Unhealthy people in health insurance markets.

Moral Hazard / Hidden Action:

- People drive carelessly after buying car insurances.
- Employers fails to observe lazing employees.
- Doctors over-prescribe treatments for patients with health insurances.

Decision Trees

Definition: A **decision tree** is a diagram that describes the options available to a decision maker, as well as the risky events that can occur at each point in time.

Decision Tree has the following elements:

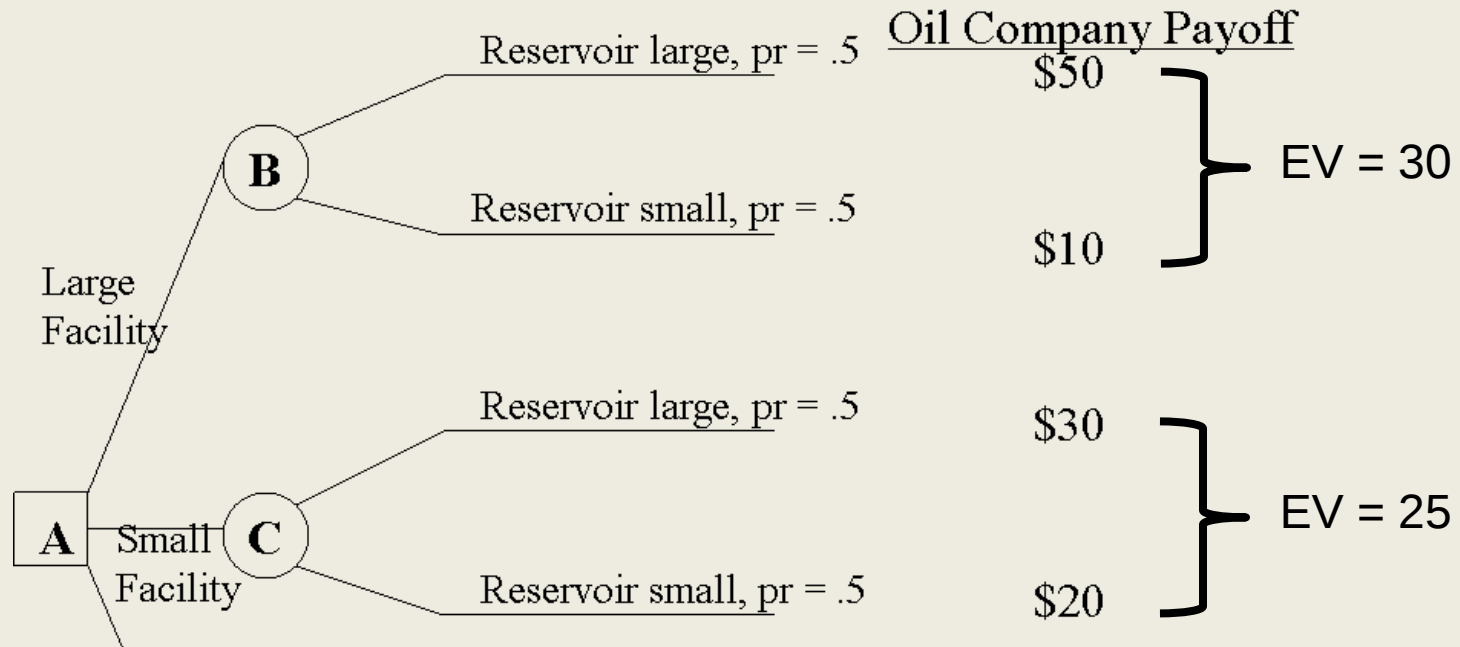
1. Decision Nodes
2. Chance Nodes
3. Probabilities
4. Payoffs

We analyze decision problems by working backward along the decision tree to decide the optimal decision.

Decision Trees

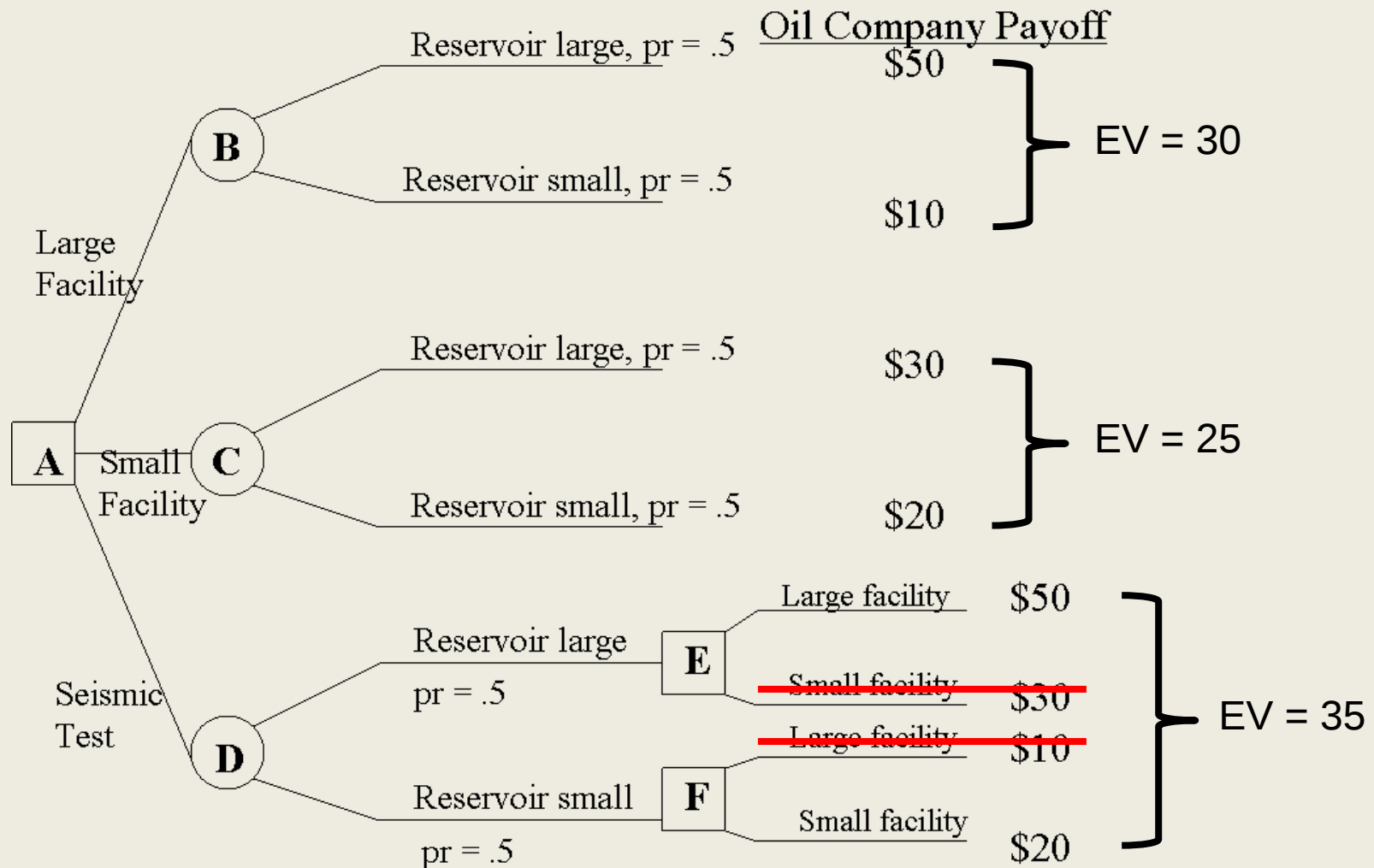
- An oil company is considering whether it should build a large facility (good if the oil reservoir is large) or a small facility (good if the oil reservoir is small).
- $\Pr(\text{Small Reservoir}) = 0.5$ and $\Pr(\text{Large Reservoir}) = 0.5$.
- However, it can implement “free seismic test” before building a facility to determine if the reservoir is large or small.

Decision Trees



Without Seismic Test, the company would build a large facility, and its expected payoff is 30.

Decision Trees



With Seismic Test, the expected payoff is 35.

Perfect Information

Definition: The **value of perfect information** is the increase in the decision maker's expected payoff when the decision maker can -- at no cost -- obtain information that reveals the outcome of the risky event.

Example:

- Expected payoff to conducting test: \$35M
- Expected payoff to not conducting test: \$30M

The value of information: \$5M