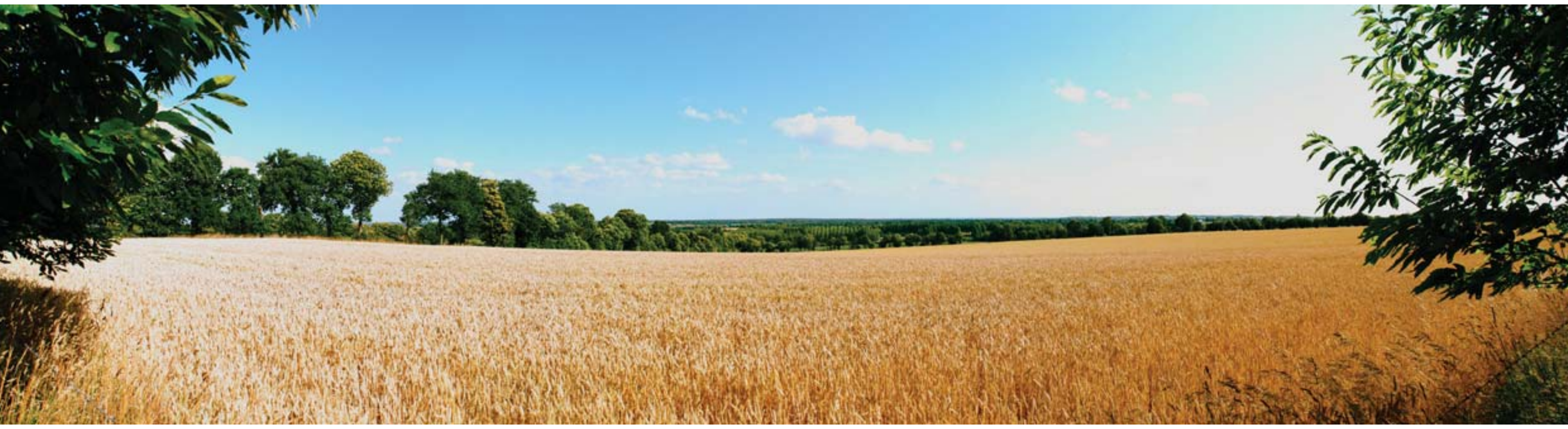


2. TWO-VARIABLE REGRESSION ANALYSIS

In order to understand two-variable regression, consider the data given in Table 2.1.
The data in the below table refer to a total **Population** of 42 families with their weekly income (X) and weekly consumption expenditure (Y).

Table 2.1: Weekly family Expenditure (Y), Baht and Income (X), Baht

	X=Weekly family Income, Baht					
	500	600	700	800	900	1000
	360	376	458	610	600	700
	313	475	422	468	531	679
	322	380	498	575	670	730
Y= Weeky	310	382	560	542	630	591
Family Expenditure	390	390	442	588	544	550
	315	425	440	466	565	620
	390	442	-	461	-	695
	400	-	-	-	-	635
Total	2800	2870	2820	3710	3540	5200
Conditional means of Y, $E(Y\|X)$	350	410	470	530	590	650

Notes -

Table 2.2: Conditional Probabilities $p(Y|X_i)$ for the Weekly Family Income (X) and Expenditure (Y)

			X=Weekly family Income, Baht			
	500	600	700	800	900	1000
	1/8	1/7	1/6	1/7	1/6	1/8
	1/8	1/7	1/6	1/7	1/6	1/8
	1/8	1/7	1/6	1/7	1/6	1/8
Y= Weekly	1/8	1/7	1/6	1/7	1/6	1/8
Family Expenditure	1/8	1/7	1/6	1/7	1/6	1/8
	1/8	1/7	1/6	1/7	1/6	1/8
	1/8	1/7	-	1/7	-	1/8
	1/8	-	-	-	-	1/8
Conditional means of Y, $E(Y|X)$	350	410	470	530	590	650

Notes -

Conditional expected value of weekly consumption expenditure given the income level =X , $E(Y|X)$

Unconditional expected value ,$E(Y)$

Figure 2.1: Conditional Distribution of Expenditure for Various Levels of Income

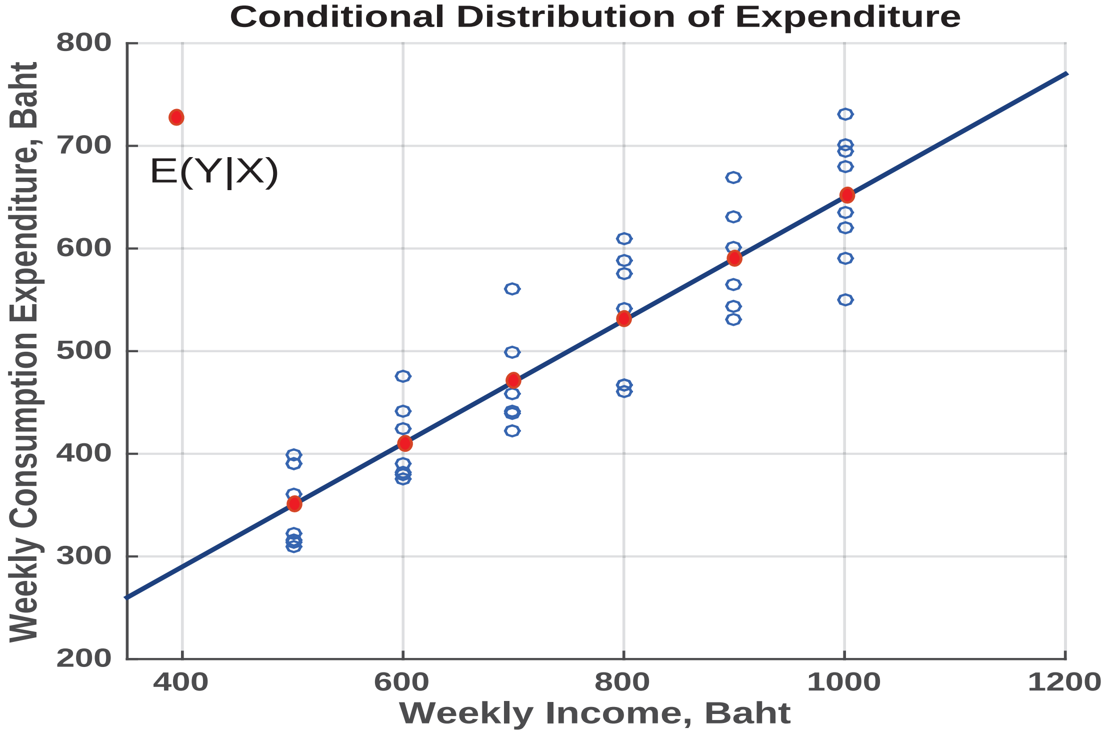
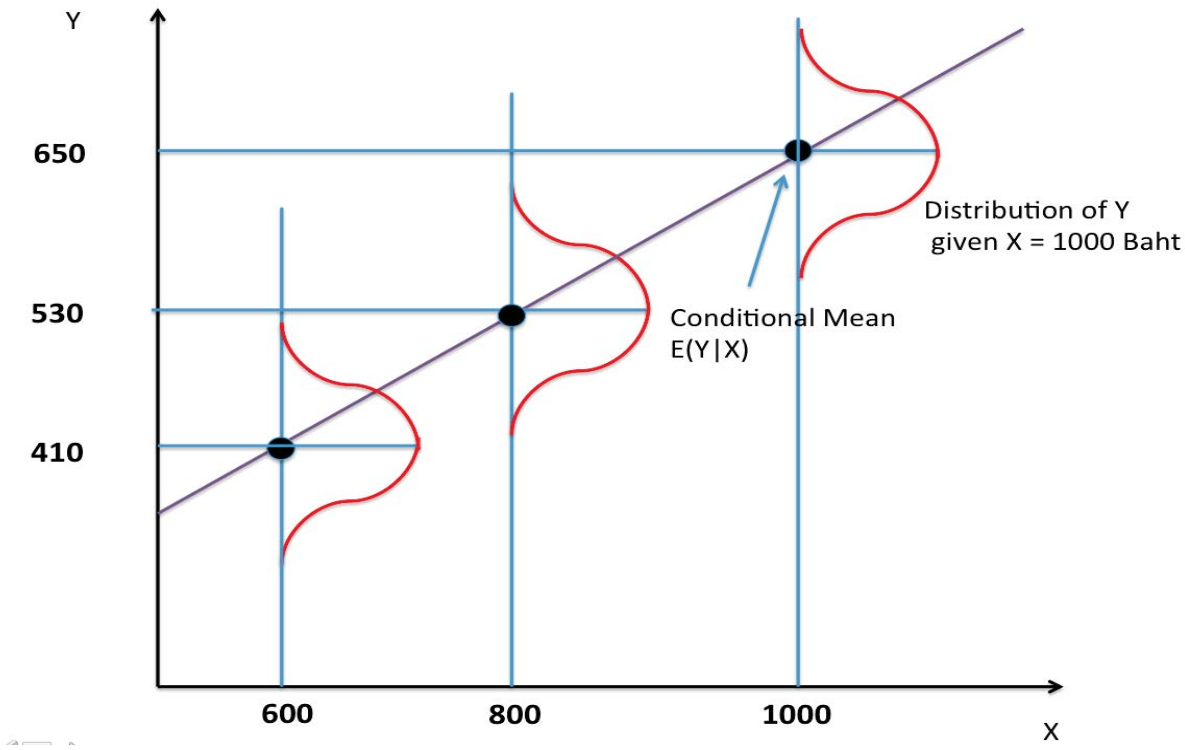


Figure 2.2: Population Regression Line (PRL)



2.1 The Concept of Population Regression Function (PRF)

The population regression function (PRF) can be written as the function of X_i:

2.1.1 What form does the function f(X) assume?

If we assume the PRF $E(Y|X_i)$ is a linear function of X_i, we get

$$E(Y|X_i) = \beta_1 + \beta_2 X_i$$

2.1.2 What is the meaning of the term LINEAR?

LINEARITY in the variables

LINEARITY in the parameters

2.2 Stochastic Specification of PRF

We can write the **deviation** of an individual Y_i around it expected value as follows:

2.2.1 The roles of the stochastic disturbance term

1. Vagueness of theory

2. Unavailability of data

3. Core variables versus peripheral variables

4. Intrinsic randomness in human behavior

5. Poor proxy variable

6. Principle of parsimony

7.Wrong functional form

2.3 The Sample Regression Function (SRF)

As mentioned, in the real situation, we cannot find out all the population of Y values corresponding to the fixed X's. We only have a sample of Y values corresponding to some fixed X's.

Therefore, our goal in this section is to estimate the population regression line (PRF) on the basis of the **SAMPLE INFORMATION.**

As a result, for the fixed X's as given in table 2.1, we only have a randomly selected sample of Y values. For example, table 2.3 and table 2.4 show a random sample from the population of table 2.1

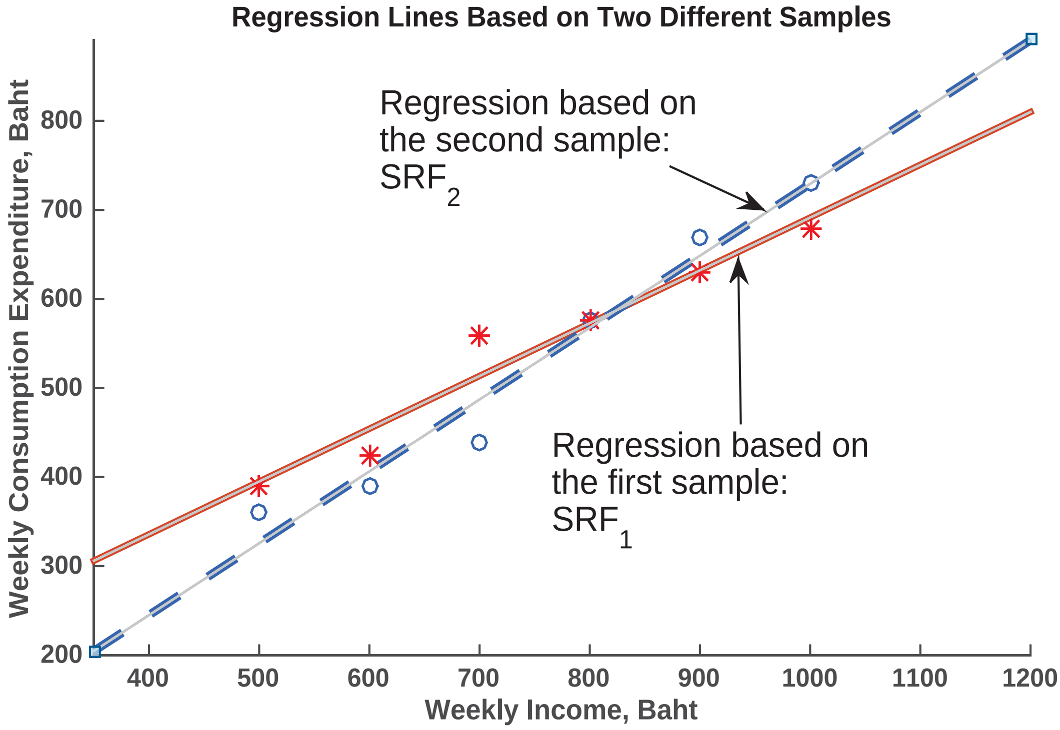
Table 2.3: A Random Sample From the Population

X	Y
500	390
600	425
700	560
800	575
900	630
1000	679

Table 2.4: Another Random Sample From the Population

X	Y
500	360
600	390
700	440
800	575
900	670
1000	730

Figure 2.3: Regression lines based on two different samples



The sample regression function (SRF) can be written as:

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

where $\hat{Y}$ is read as "Y-hat"

$\hat{Y}_i$ = estimator of $E(Y|X_i)$

$\hat{\beta}_1$=estimator of β_1

$\hat{\beta}_2$=estimator of β_2

We can express the SRF in its stochastic form as follows:

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{\mu}_i$$

In sum, our ultimate goal is to estimate
the PRF

on the basis of
the SRF

Figure 2.4: Sample and Population Regression Lines