(a) **(5 Marks)** Use the above information to compute OLS estimates of the intercept coefficient β_1 and that of the slope coefficient β_2.

From the above information:

$$\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{20.58}{211.00} = 0.0975$$

On the average, when the SAT score increases (decreases) 1 score, the grade point average will goes up (goes down) by 0.0975.

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = \frac{50.90}{16} - 0.0975\left(\frac{388}{16}\right) = 0.8169$$

when SAT score is zero, the grade point average is 0.8169 on average.

(b) **(5 Marks)** Interpret the slope coefficient estimate you calculated in part(a)–i.e.,explain in words what the numeric value you calculated for $\hat{\beta}_2$ means.

(see 4.a).

(c) **(10 Marks)** Calculate an estimate of σ^2.

We know that

$$TSS = ESS + RSS$$

$$\Sigma y_i^2 = \Sigma \hat{y}_i^2 + \Sigma \hat{u}_i^2$$

$$\therefore \quad \Sigma \hat{u}_i^2 = 2.5844 - 2.0063 = 0.5781$$

$\therefore$ the estimated σ^2 is $\hat{\sigma}^2$

$$\hat{\sigma}^2 = \frac{\Sigma \hat{u}_i^2}{n-2} = \frac{0.5781}{16-2} = 0.0413$$

(d) **(5 Marks)** Compute the value of r^2. Briefly explain what the calculated value of r^2 means.

By the definition

$$r^2 = \frac{\Sigma \hat{y}_i^2}{\Sigma y_i^2} = \frac{2.0063}{2.5844} = 0.7763$$

The variation of the grade point average (Y_i) can be explained by the variation of the SAT scores (X_i) by 77.63 %.

(e) **(5 Marks)** Compute the estimated variance of $\hat{\beta}_1$ and the estimated variance of $\hat{\beta}_2$.

we know that

$$Var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\Sigma x_i^2} = \frac{0.0413}{211.00} = 0.000195$$

$$var(\hat{\beta}_1) = \frac{\Sigma X^2 \hat{\sigma}^2}{n \Sigma x_i^2} = \frac{0.0413 \times (9620)}{16 \times (211.00)} = 0.1177$$

(f)**(5 Marks)** Compute the two-sided 90% confidence interval for the intercept coefficient β_1. Briefly explain what the two-sided 90% confidence interval means.

A 90% confidence interval for β_1 is

$$\hat{\beta}_1 \pm t_{\frac{0.1}{2}, 16-2} \cdot se(\hat{\beta}_1)$$

$$0.8169 \pm t_{0.05, 14} \sqrt{0.1177}$$

$$0.8169 \pm 1.761 \times \sqrt{0.1177}$$

$$0.8169 \pm 0.6042 \quad - \text{ (1)}$$

In the long-run, in 90 out of 100 cases intervals like (1) will contain the true β_1.

(g)(**5 Marks**) Perform a test of the null hypothesis $H_0 : \beta_1 = 0$ against the alternative hypothesis $H_1 : \beta_1 \neq 0$ at the 1 % significance level (i.e.,for significance level $\alpha = 0.01$). Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. Briefly explain what the test outcome means.

step 1

$H_0: B_1 = 0$

$H_1: B_1 \neq 0$

step 2

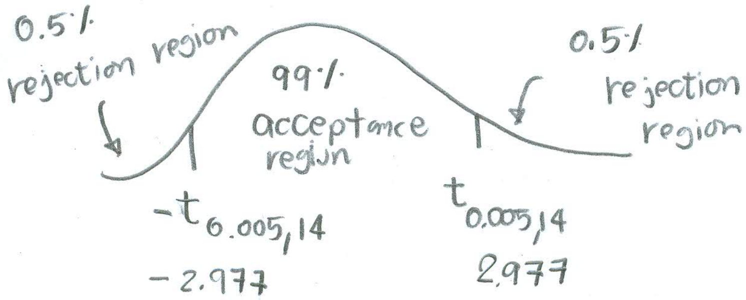
$\alpha = 0.01$

step 3

$$t_{cal} = \frac{0.8169}{0.3431} = 2.3809$$

step 4

$$t_{critical} = t_{\frac{0.01}{2}, 14} = 2.977$$



step 5 compase b/w t_{cal} and $t_{critical}$ value

$$|t_{cal}| < t_{0.005, 14}$$

We cannot reject the null hypothesis that $H_0: B_1 = 0$ with 99% confidence interval.

(h) **(5 Marks)** Perform a test of the null hypothesis $H_0 : \sigma^2 \geq 0.025$ against the alternative hypothesis $H_1 : \sigma^2 < 0.025$ at the 5 % significance level (i.e.,for significance level $\alpha = 0.05$). Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. Briefly explain what the test outcome means.

step 1:

H_0: $\sigma^2 \geqslant 0.025$

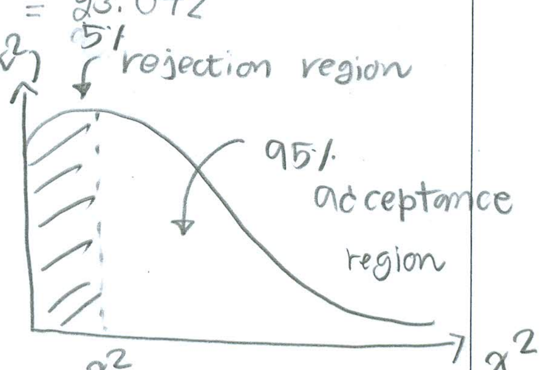
H_1 $\sigma^2 < 0.025$

step 2: $\alpha = 0.05$.

Step 3:

$$\chi^2_{cal} = (n-2).\frac{\hat{\sigma}^2}{\sigma^2}$$

$$= (16-2).\frac{0.0413}{0.025} = 23.072$$



step4: $\chi^2_{critical} = \chi^2_{1-0.05,14}$

$\chi^2_{0.95,14} = 6.571$

step 5: Compare χ^2_{cal} and $\chi^2_{critical}$

since $\chi^2_{cal} > \chi^2_{critical}$

$\therefore$ With $\alpha = 0.05$ we cannot reject the null hypothesis that $\sigma^2 \geqslant 0.025$ #.