Long-Run
Production and Cost

Consumption versus Production

Consumption

- Utility Function
- Two Goods: X and Y
- Marginal Utility
- Diminishing MU
- Indifference Curve
- Marginal Rate of Substitution (MRS)
- Budget Line
- Utility-Max Problem

Production

- Production Function
- Two Inputs: L and K
- Marginal Product
- Diminishing MP
- Iso-quant Curve
- Marginal Rate of Technical Substitution (MRTS)
- Iso-cost Line
- Cost-Min Problem

Long-Run Production

Production Function:

$$Q = f(L, K)$$

- Q = output
- K = Capital
- L = Labor

We will now consider the **long-run** production where are factors of production, K and L, are variable.

Isoquants (for 2-input production)

Definition: An **isoquant** traces out all the combinations of inputs (labor and capital) that allow that firm to produce the same quantity of output

Example: $Q = K^{1/2}L^{1/2}$

*What is the **equation** of the **isoquant for Q = 20**?*

$\boldsymbol{20 = K^{1/2}L^{1/2}}$

=> $400 = KL$

=> $\boldsymbol{K = 400/L}$

$\boxed{\text{Slope} = dK/dL < 0}$

Isoquants (for 2-input production)

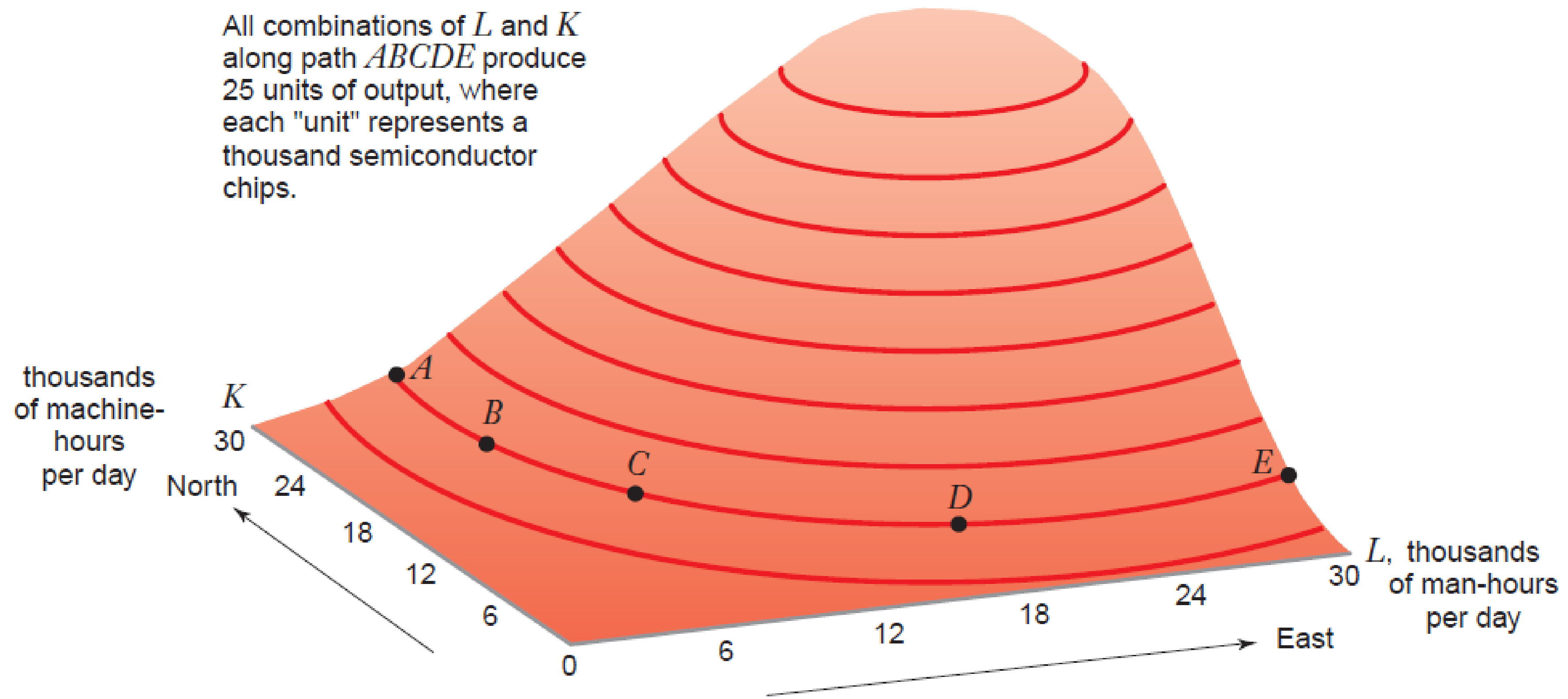
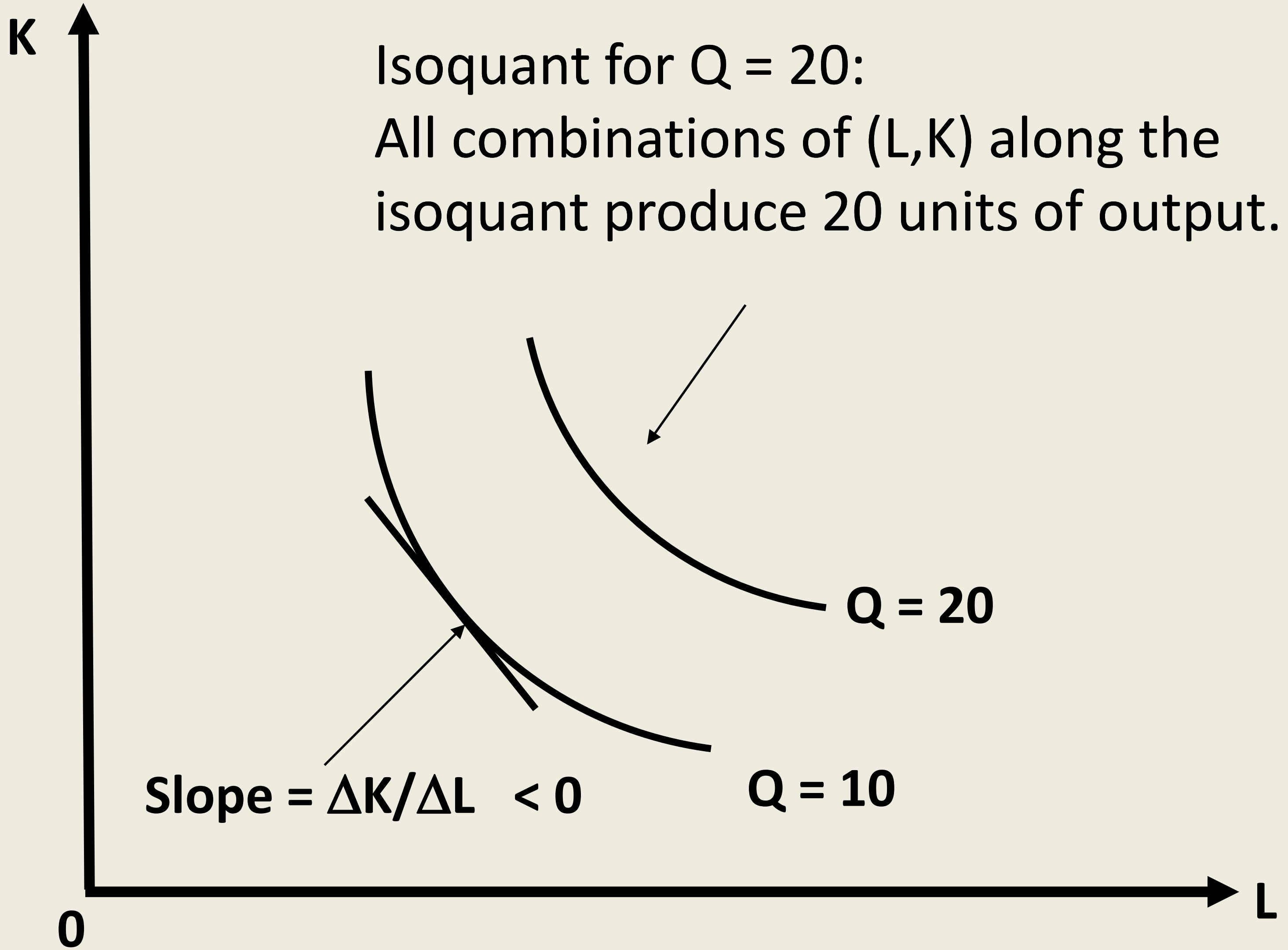


FIGURE 6.6 **Isoquants and the Total Product Hill**
If we start at point *A* and walk along the hill so that our elevation remains unchanged at 25 units of output, then we will trace out the path *ABCDE*. This is the 25-unit isoquant for this production function.

Isoquants (for 2-input production)

Isoquant for Q = 20:
All combinations of (L,K) along the isoquant produce 20 units of output.



Marginal Rate of Technical Substitution

Definition: The **marginal rate of technical substitution (MRTS)** is the rate at which capital can be reduced for every one-unit increase in labor, holding the quantity of output constant.

$MRTS_{L,K}$ $= MP_L/MP_K$
$= - \Delta K/\Delta L$ (for a given isoquant or output)
$=$ "negative" slope of an isoquant

Example Suppose MRTS = 4.
The firm can reduce K by 4 units if it wants to employ one more L. In doing so, the output will not change.

Marginal Rate of Technical Substitution

- If both marginal products are positive, the slope of the isoquant is negative, but the MRTS is positive.

- If we have diminishing marginal returns, we also have a diminishing marginal rate of technical substitution.

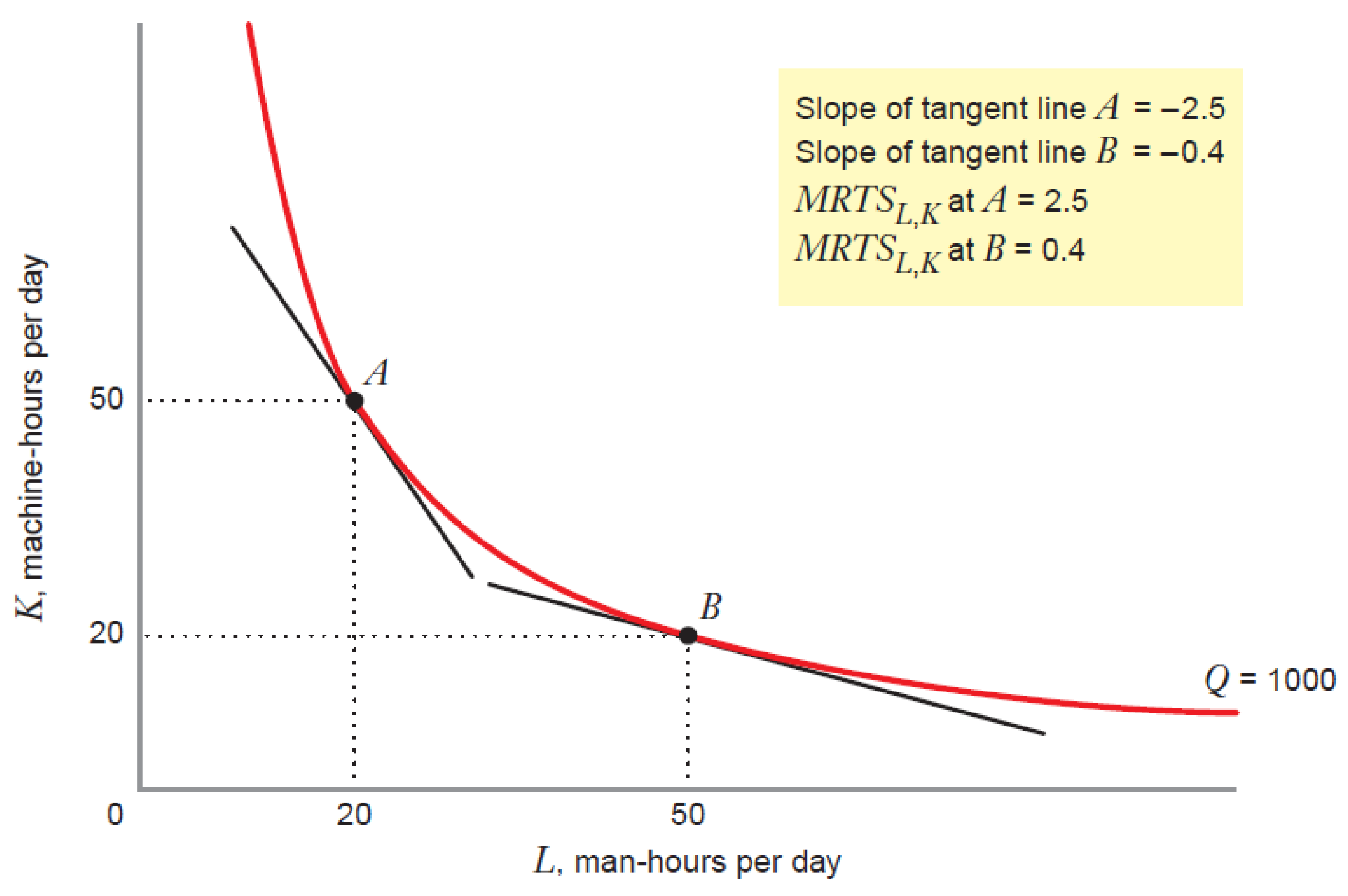
- Diminishing returns:
  - For L, $dMPL/dL < 0$. For K, $dMPK/dK < 0$.

- Diminishing MRTS: $dMRTS_{L,K}/dL < 0$.
  - **As more L is added, the isoquant becomes flatter.**

Marginal Rate of Technical Substitution

Diminishing MRTS

FIGURE 6.10
Marginal Rate of Technical Substitution of Labor for Capital ($MRTS_{L,K}$) along an Isoquant
At point *A*, the $MRTS_{L,K}$ is 2.5. Thus, the firm can hold output constant by replacing 2.5 machine-hours of capital services with an additional man-hour of labor. At point *B*, the $MRTS_{L,K}$ is 0.4. Here, the firm can hold output constant by replacing 0.4 machine-hours of capital with an additional man-hour of labor.



Elasticity of Substitution

Definition: The **elasticity of substitution**, σ, measures how the capital-labor ratio, K/L, changes relative to the change in the $MRTS_{L,K}$. **It measures of how easy is it for a firm to substitute labor for capital.**

$$\sigma = \frac{Percentage change \text{ in capital - labor ratio}}{Percentage change \text{ in } MRTS_{L,K}}$$

$$= \frac{\%\Delta\left(\frac{K}{L}\right)}{\%\Delta MRTS_{L,K}}$$

Inputs are Perfect Substitutes σ = ∞

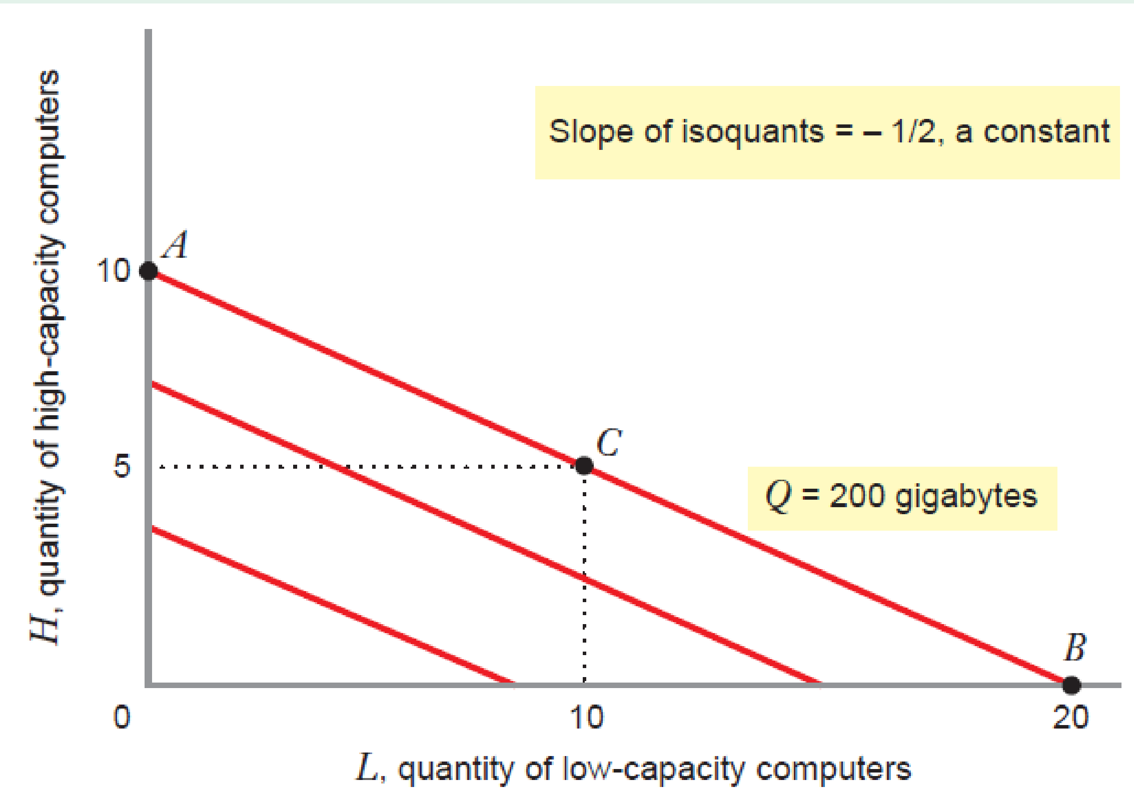


FIGURE 6.14 **Isoquants for a Linear Production Function**
The isoquants for a linear production function are straight lines. The $MRTS_{L,H}$ at any point on an isoquant is thus a constant.

Inputs are Perfect Complements σ = 0

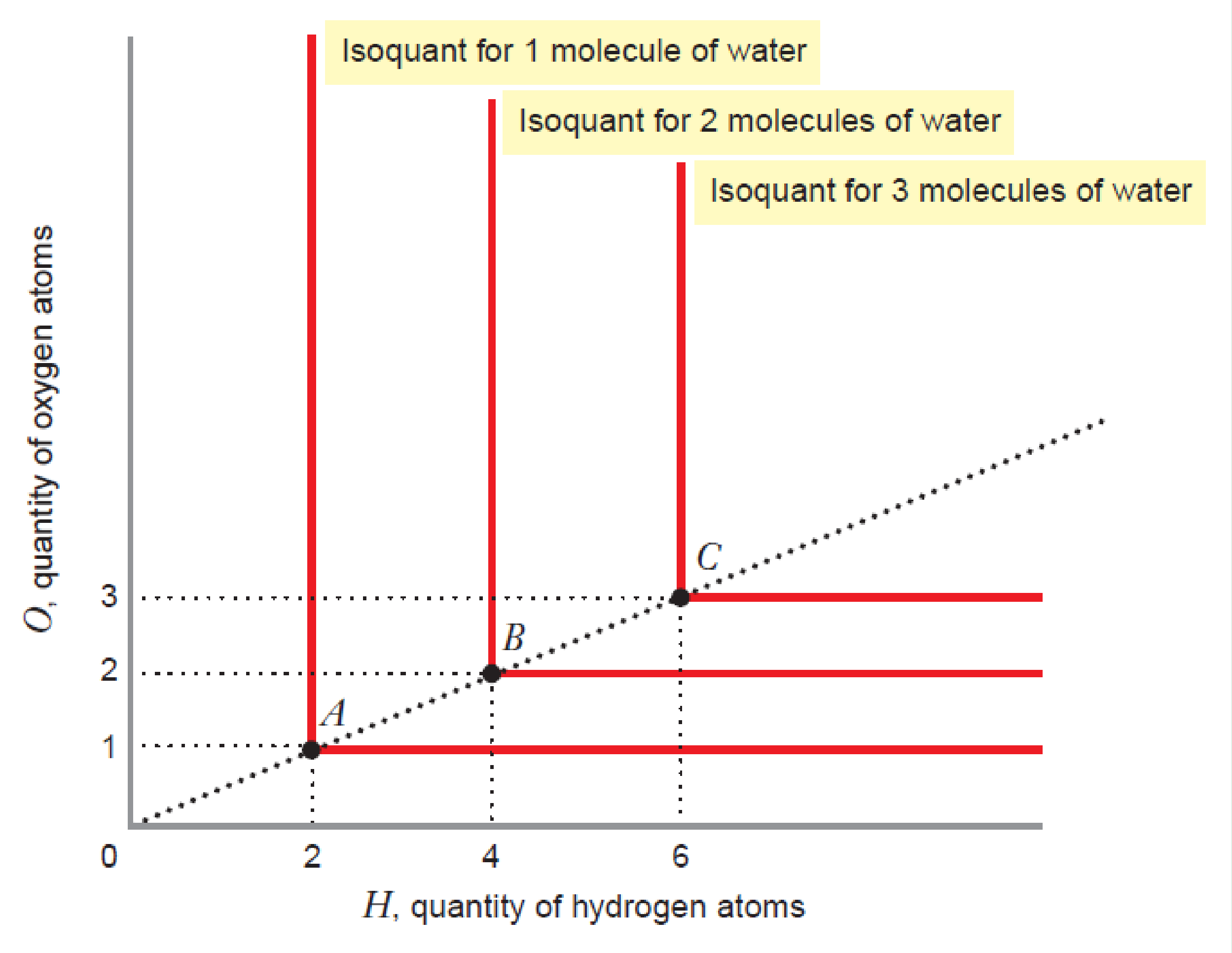


FIGURE 6.15 Isoquants for a Fixed-Proportions Production Function Two atoms of hydrogen (*H*) and one atom of oxygen (*O*) are needed to make one molecule of water. The isoquants for this production function are L-shaped, which indicates that each additional atom of oxygen produces no additional water unless two additional atoms of hydrogen are also added.

Cobb-Douglas Production Function

- $Q = AK^{\alpha}L^{\beta}$ where A, α, and β are positive.

- Cobb-Douglas function has the following properties:
  – MPK and MPL > 0.
  – MPK and MPL are diminishing w.r.t. their inputs.
  That is, $dMPK/dK < 0$ and $dMPL/dL < 0$.
  – Elasticity of Substitution = σ = 1

- The function implies that the two inputs are both substitutes and complements for each other.

CES Production Function

- CES stands for Constant Elasticity of Substitution.

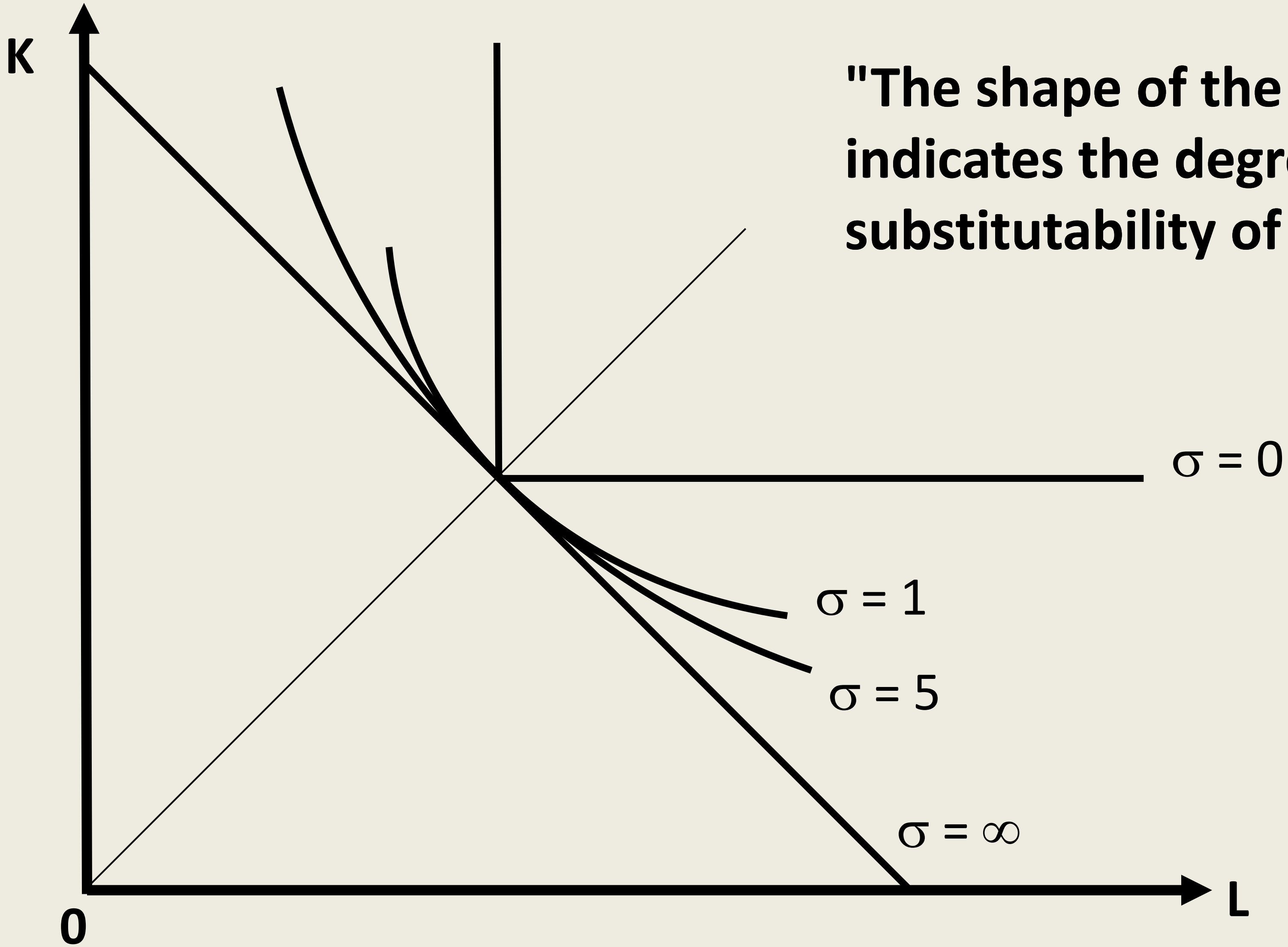
$$Q = \left[aL^{\frac{\sigma-1}{\sigma}} + bK^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

where a, b, and σ are positive.

- σ is the elasticity of substitution, σ ϵ [0, ∞).

- It can represent all three types of production functions:
  - Cobb-Douglas
  - Perfect Substitutes
  - Perfect Complements

Elasticity of Substitution



"The shape of the isoquant indicates the degree of substitutability of the input."

Returns to Scale

Definition: **Returns to scale (RTS)** tells us how output will change when ALL inputs are changed.

For example, when we double inputs (K and L double), will the output double too?

$$\text{Returns to Scale} = \frac{\%\Delta(\text{quantity of output})}{\%\Delta(\text{quantity of } \textit{all} \text{ inputs})}$$

Returns to Scale vs Marginal Returns

- **Returns to scale**: simultaneous increase in all inputs
- **Marginal Returns**: increase in a single input, holding all others constant.

- The marginal product of a single factor may diminish while the returns to scale do not.
- Returns to scale need not be the same at different levels of production.

Returns to Scale

Let $\lambda > 1$ represent the proportion by which both inputs, labor and capital, increase.

- **Increasing RTS** when $F(\lambda K, \lambda L) > \lambda F(K, L)$:

A proportional increase in all inputs results in a greater than proportional increase in output.

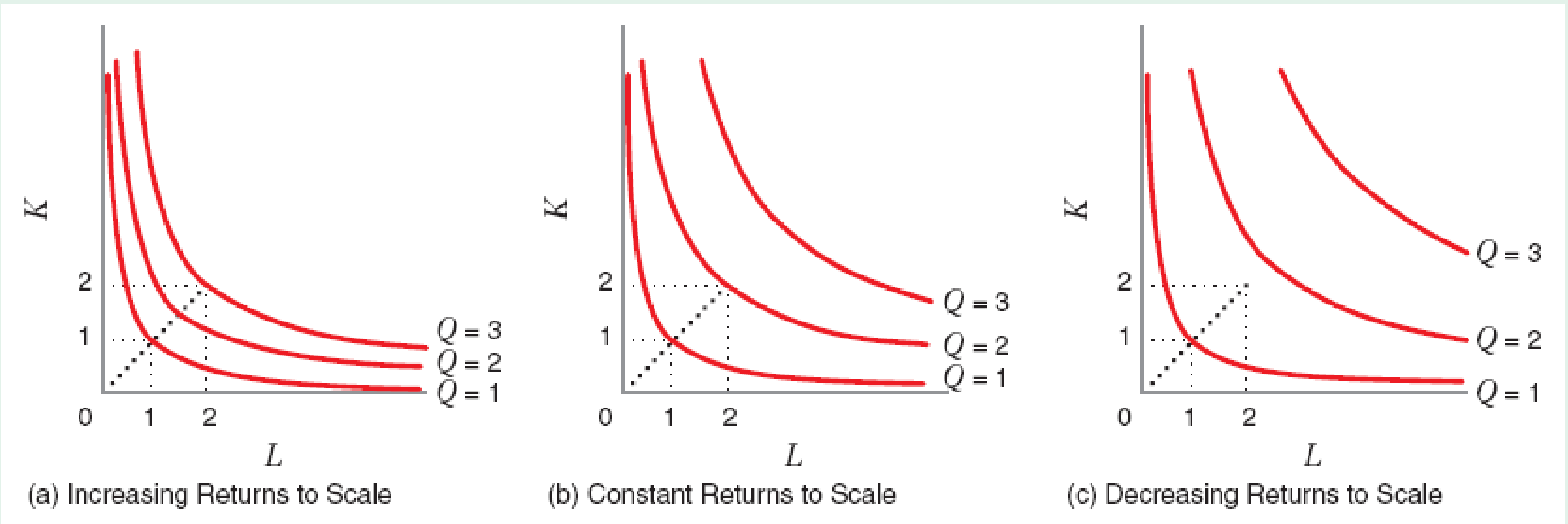
- **Decreasing RTS** when $F(\lambda K, \lambda L) < \lambda F(K, L)$:

A proportional increase in all inputs results in a less than proportional increase in output.

- **Constant RTS** when $F(\lambda K, \lambda L) = \lambda F(K, L)$:

A proportional increase in all inputs results in the same proportional increase in output.

Returns to Scale



Long-Run Cost Minimization

In SR, a firm chooses ONE input, L, with K being fixed.
In LR, a firm chooses TWO inputs, K and L, to minimize its costs, subject to the firm producing a given output.

Cost to the Firm: $$TC = wL + rK$$

TC:	Total Cost
w:	Price of Labor (wage rate)
L:	Quantity of Labor
r:	Price of Capital (interest rate)
K:	Quantity of Capital

Long-Run Cost Minimization

Suppose that a firm wishes to minimize costs, subject to a given output. Let such output be Q_0.

Production Function: $Q = f(L,K)$

Cost Minimization Problem:

$$\min_{(K,L)} \quad TC = rK + wL \quad \text{subject to } Q_0 = f(L,K)$$

That is, we are looking for K and L on the lowest isocost, such that Q_0 can be produced.

Isocost Lines

Isocost Line: The set of combinations of labor and capital that yield the same total cost for the firm.

Isocost Equation:

$TC = wL + rK$

$rK = TC – wL$

$K = TC/r – (w/r)L$

Isocost Slope

$= – (w/r)$

Example

$w = \$10/\text{hour}$

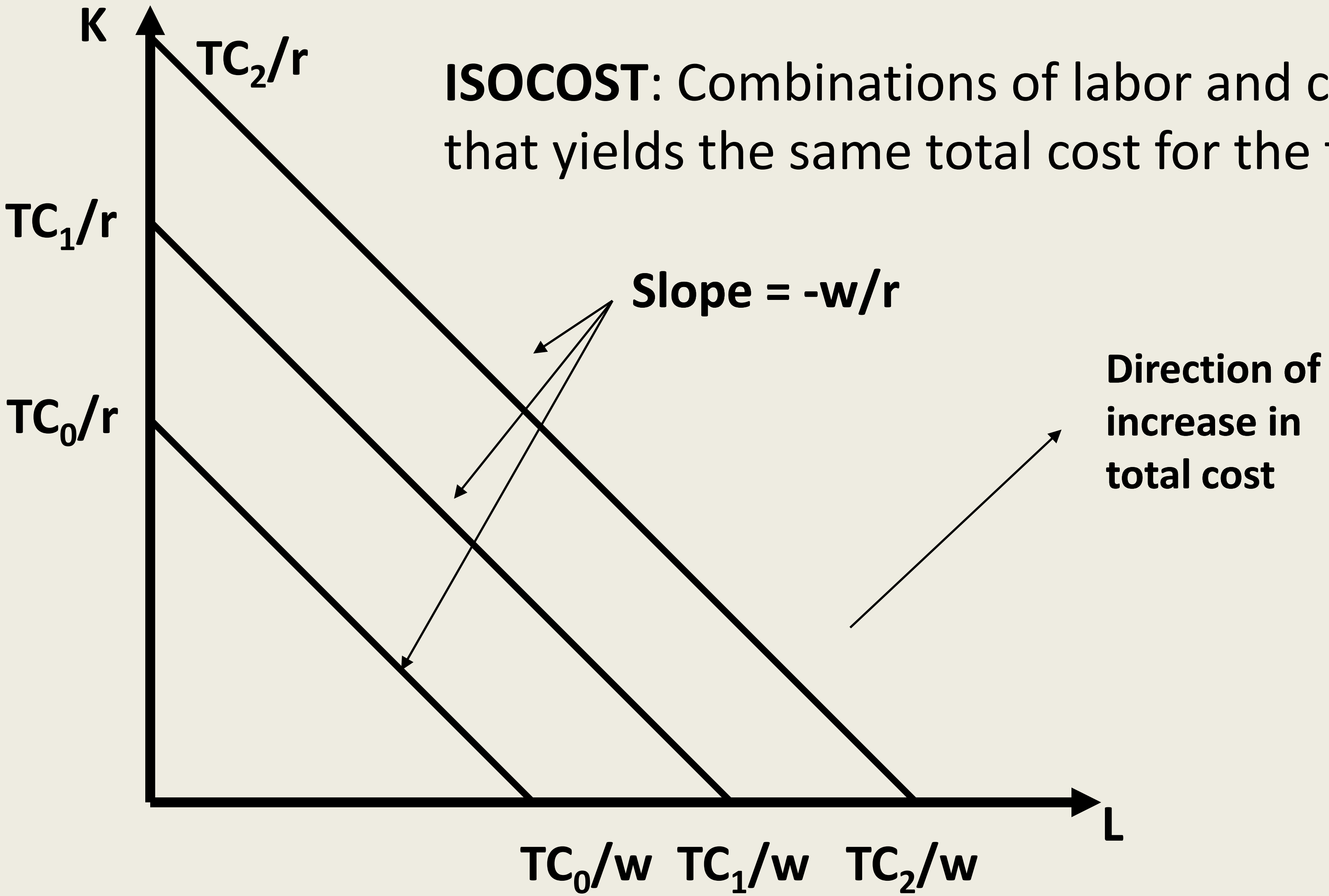
$r = \$20/\text{hour}$

$TC = \$1000$

$\Rightarrow 1000 = 10L + 20K$

$\Rightarrow K = 1000/20 – (10/20)L$

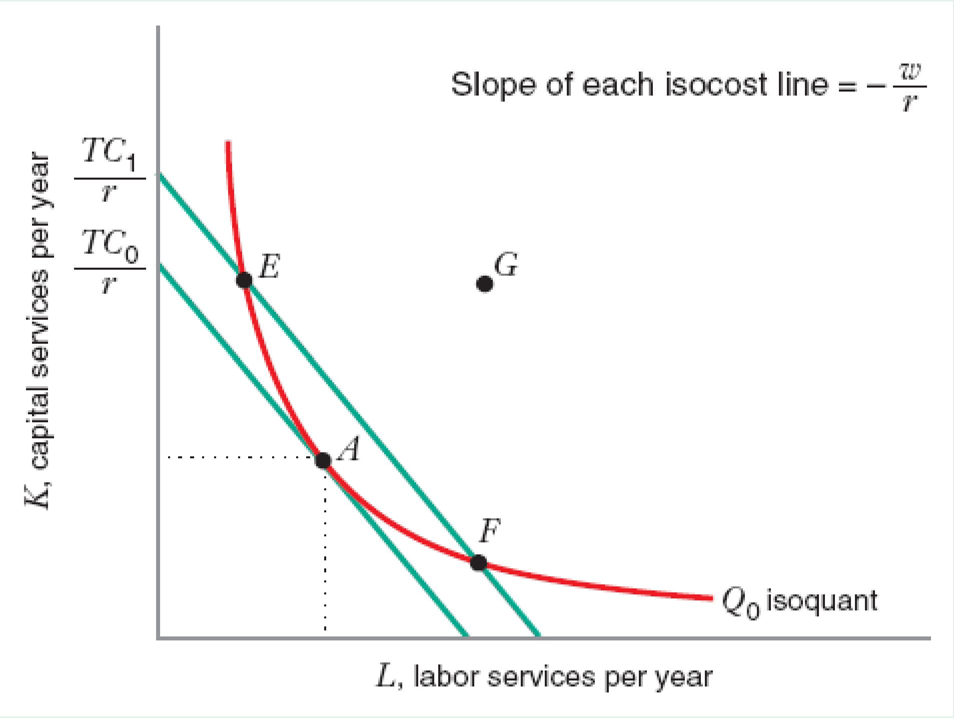
Isocost Lines



ISOCOST: Combinations of labor and capital that yields the same total cost for the firm.

Long-Run Cost Minimization

Cost Minimization: looking for K and L on the lowest isocost, such that Q_0 can be produced.

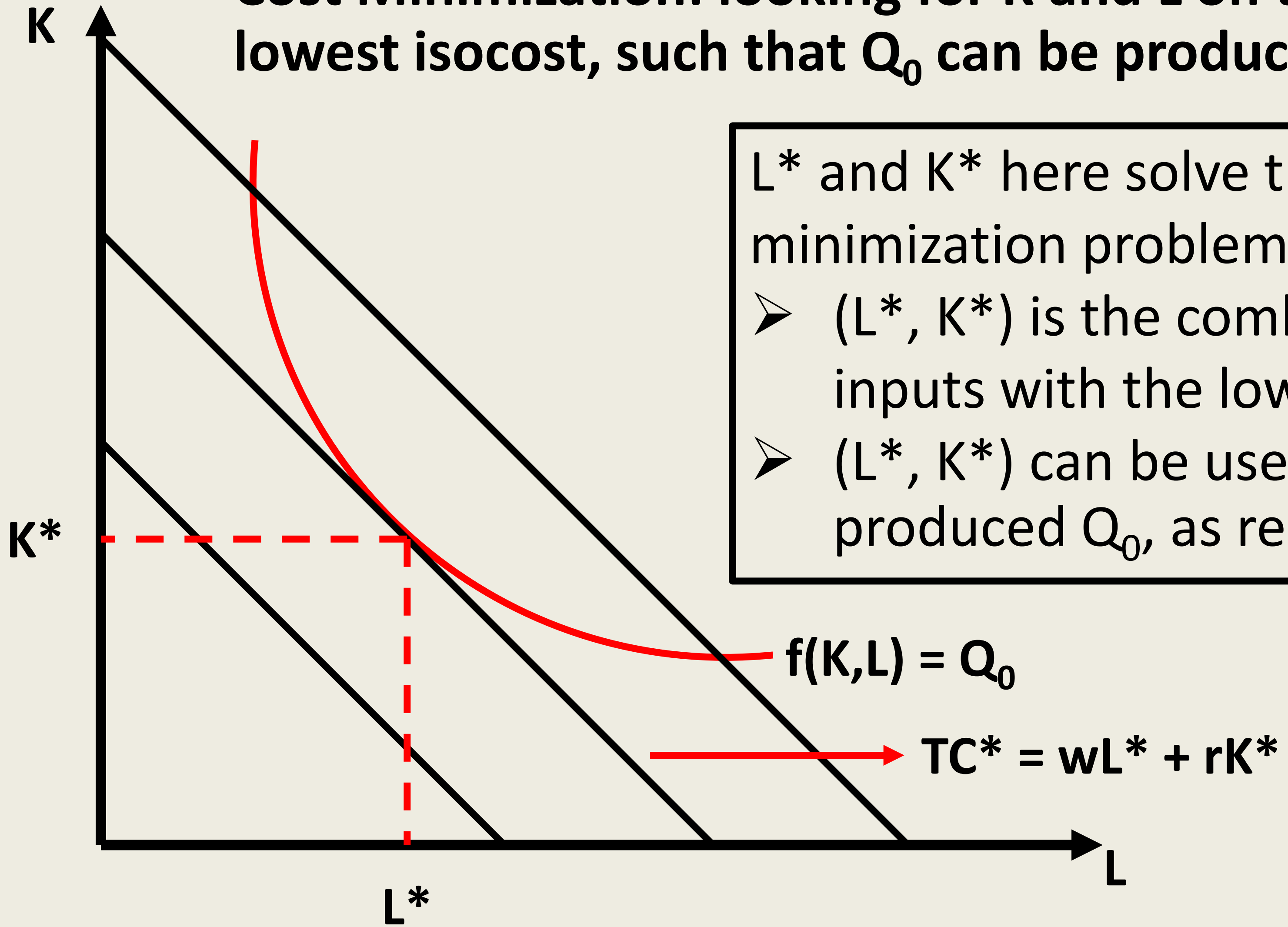


G is not the solution because less inputs can be used to produce Q_0.

E & F are not the solution. While they can be used to produce Q_0, **they do not minimize cost.**

Long-Run Cost Minimization

Cost Minimization: looking for K and L on the lowest isocost, such that Q_0 can be produced.



L* and K* here solve the cost minimization problem.

- (L*, K*) is the combination of inputs with the lowest cost.
- (L*, K*) can be used to produced Q_0, as required.

Long-Run Cost Minimization

Solution to Cost Minimization

- Slope of isoquant = Slope of isocost line

i.e. Ratio of marginal products = Ratio of input prices

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

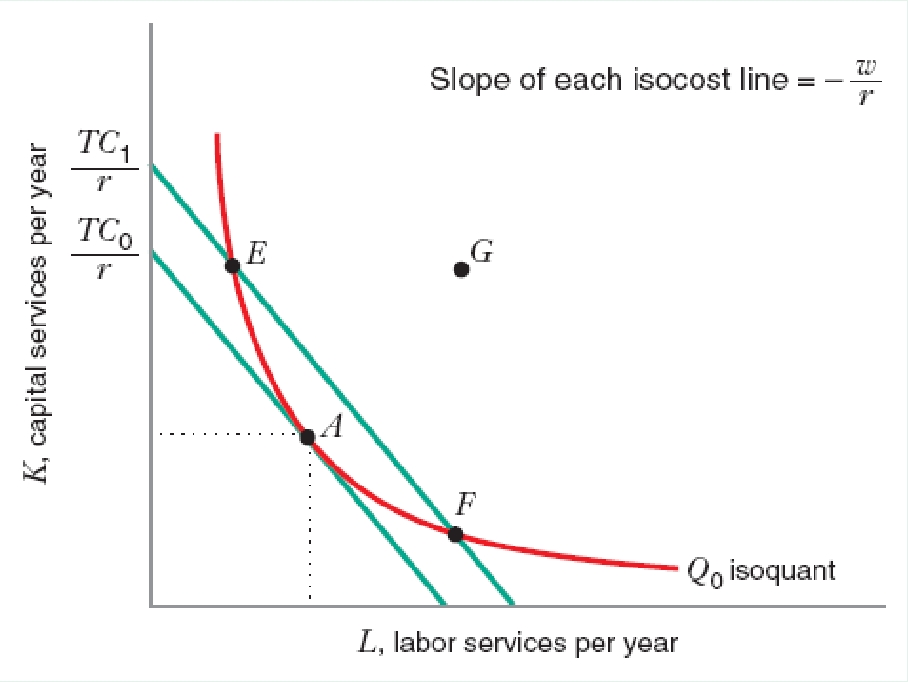
$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

- This means **"one \$ spent on K gives the same number of marginal products as one \$ spent on L"**.

Long-Run Cost Minimization

- At point E, Isoquant is **steeper** than Isocost:

$$\frac{MP_L}{MP_K} > \frac{w}{r} \ (or) \ \frac{MP_L}{w} > \frac{MP_K}{r}$$



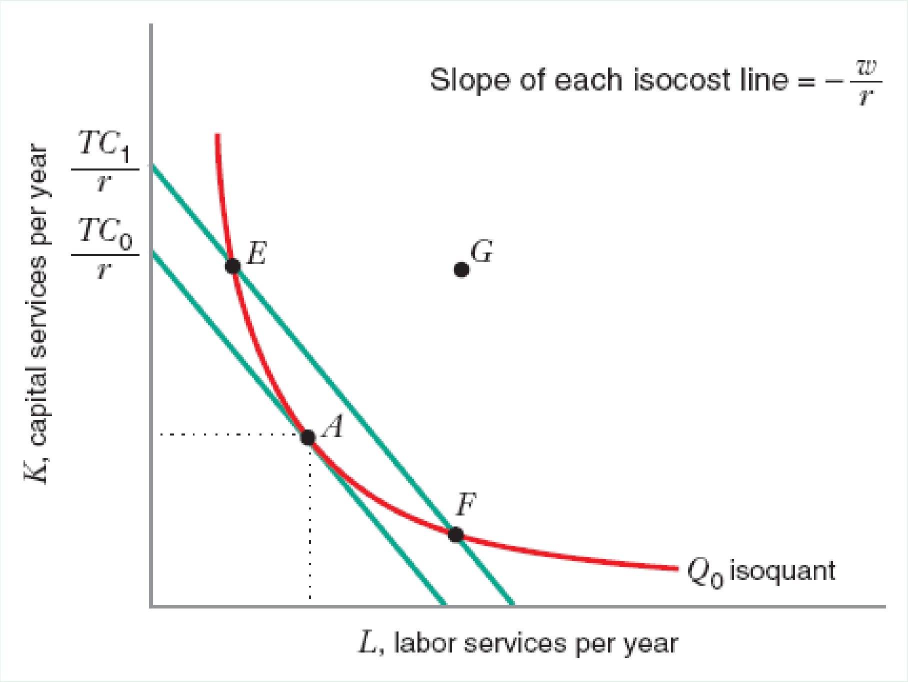
One $ on L gives greater output than one $ on K.

The firm can save more by employing **more L and less K.**

Long-Run Cost Minimization

- At point F, Isoquant is **flatter** than Isocost:

$$\frac{MP_L}{MP_K} < \frac{w}{r} \quad (or) \frac{MP_L}{w} < \frac{MP_K}{r}$$



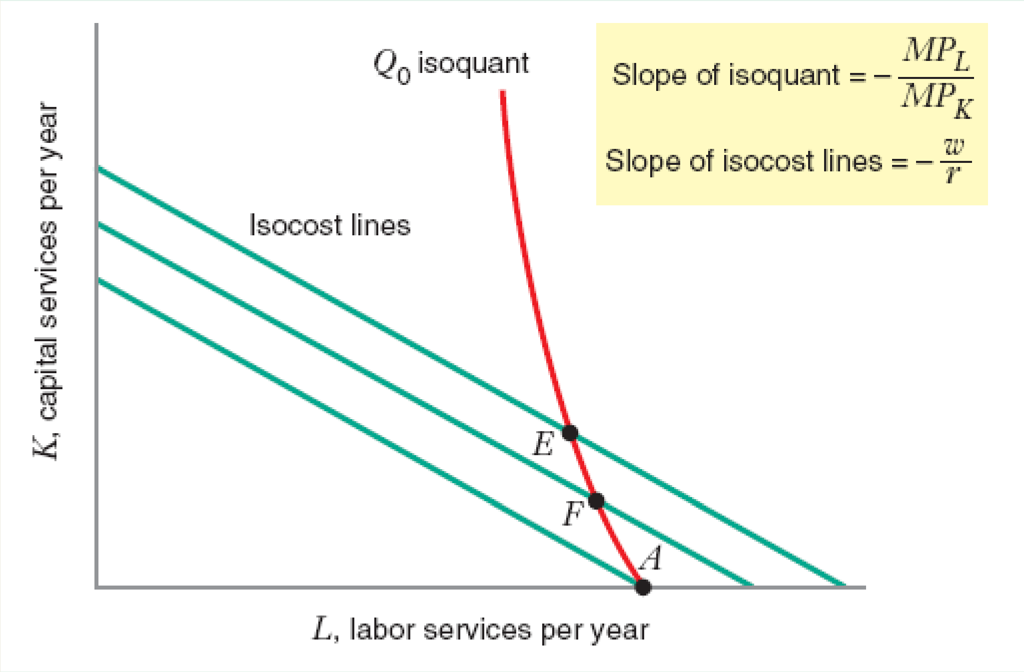
One \$ on K gives greater output than one \$ on L.

The firm can save more by employing **more K and less L.**

Corner Solution

In some cases, using one input is cheaper than using both inputs, and doing so can still produce the required output.

In such cases, we will have a corner solution, e.g. point A.



$$-(\frac{MP_L}{MP_K}) < -(\frac{w}{r})$$

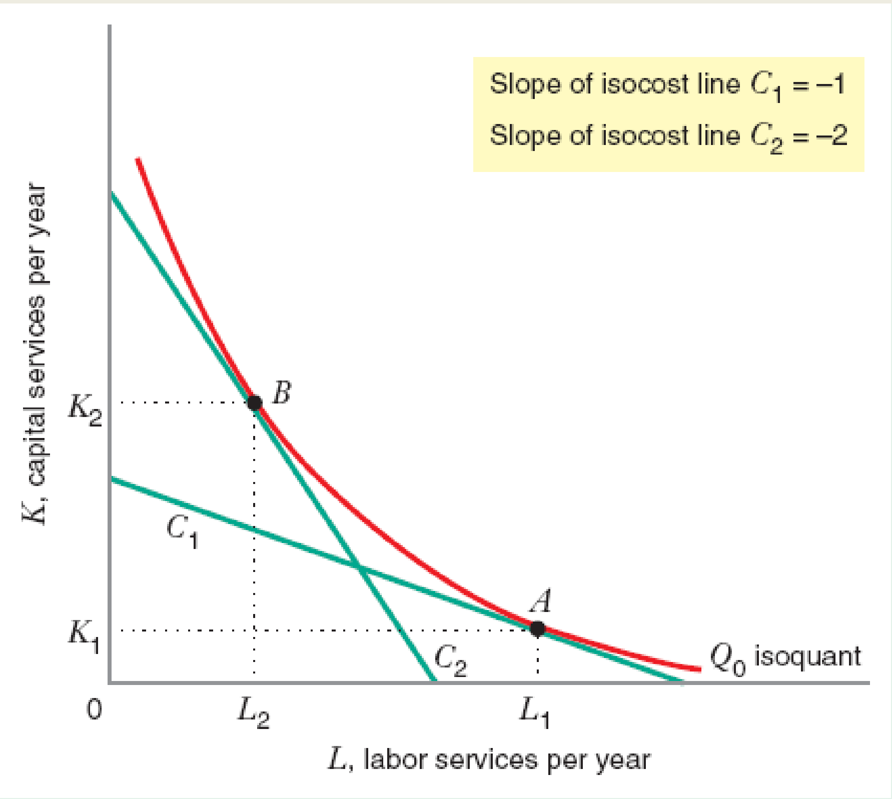
$$\Rightarrow \quad \frac{MP_L}{w} > \frac{MP_K}{r}$$

ANALYSIS: Change in Relative Prices of Inputs

A change in the relative price of inputs changes the slope of the isocost line.

- Given a diminishing $MRTS_{L,K}$, an increase in the price of one input will cause the cost-minimizing quantity of that input to go down.
- Given that both inputs are perfect complements, an increase in the price of one input will not change the cost-minimizing quantity of that input.

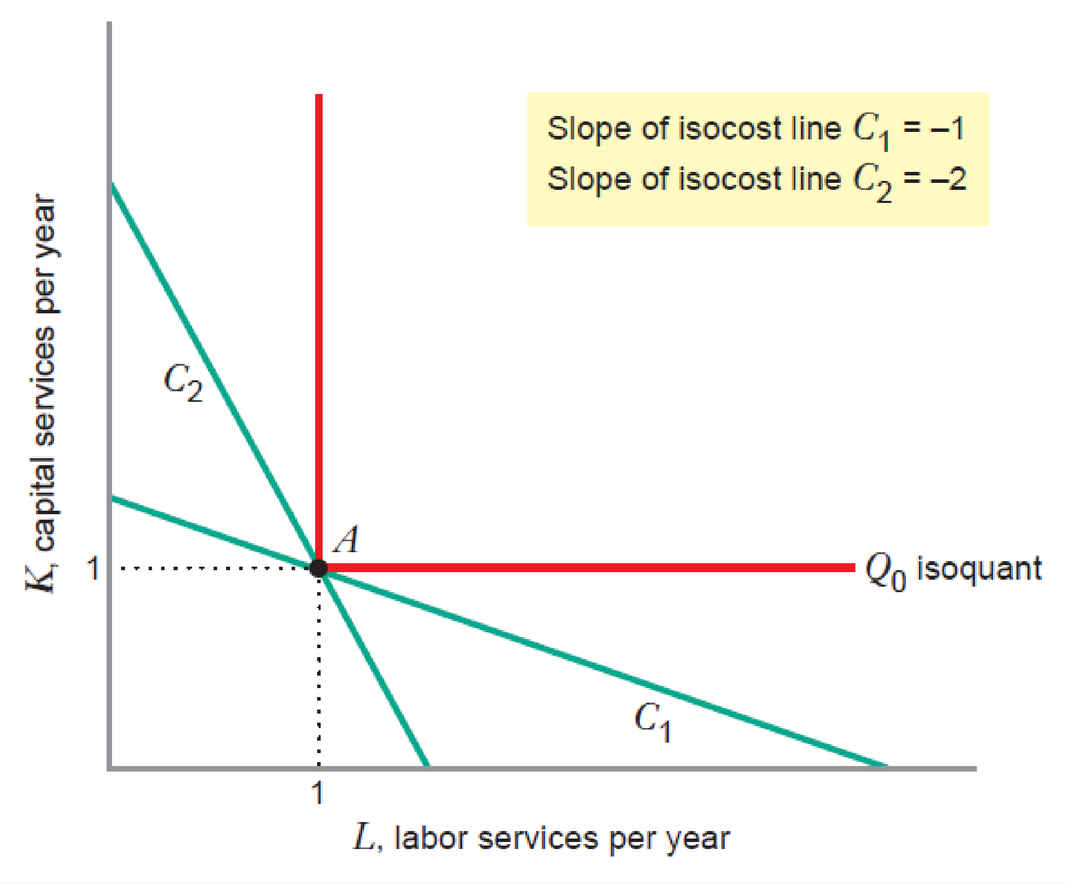
ANALYSIS: Change in Relative Prices of Inputs



- Price of capital r = 1.
- Quantity of output Q_0 is constant.
- When price of labor w = 1, the isocost line is C1. >>> optimal point A.
- When price of labor w = 2, isocost line is C2. >>> optimal point B.

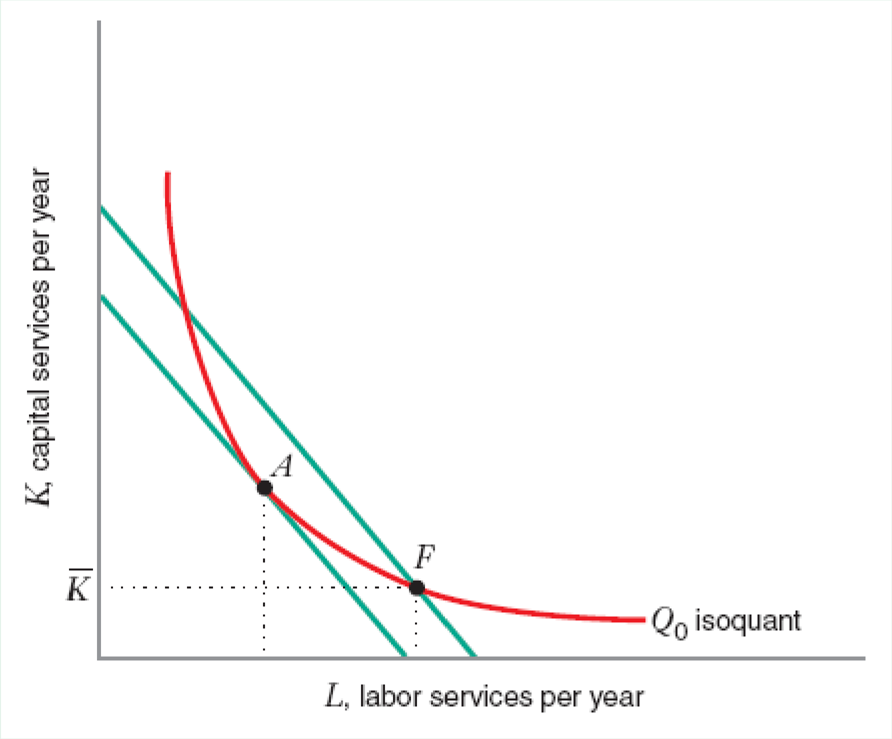
ANALYSIS: Change in Relative Prices of Inputs

FIGURE 7.6 **Comparative Statics Analysis of the Cost-Minimization Problem with Respect to the Price of Labor for a Fixed-Proportions Production Function**
The price of capital $r = 1$, and the quantity of output Q_0 are held constant. When the price of labor $w = 1$, the isocost line is C_1 and the ideal input combination is at point A ($L = 1$, $K = 1$). When the price of labor $w = 2$, the isocost line is C_2 and the ideal input combination is still at point A. Increasing the price of labor does not cause the firm to substitute capital for labor.



Short-Run Cost Minimization

In SR, capital is fixed at $\overline{K}$.



- Short-run combination is at Point F.
- If the firm were free to adjust all of its inputs, the cost-minimizing combination is at Point A.

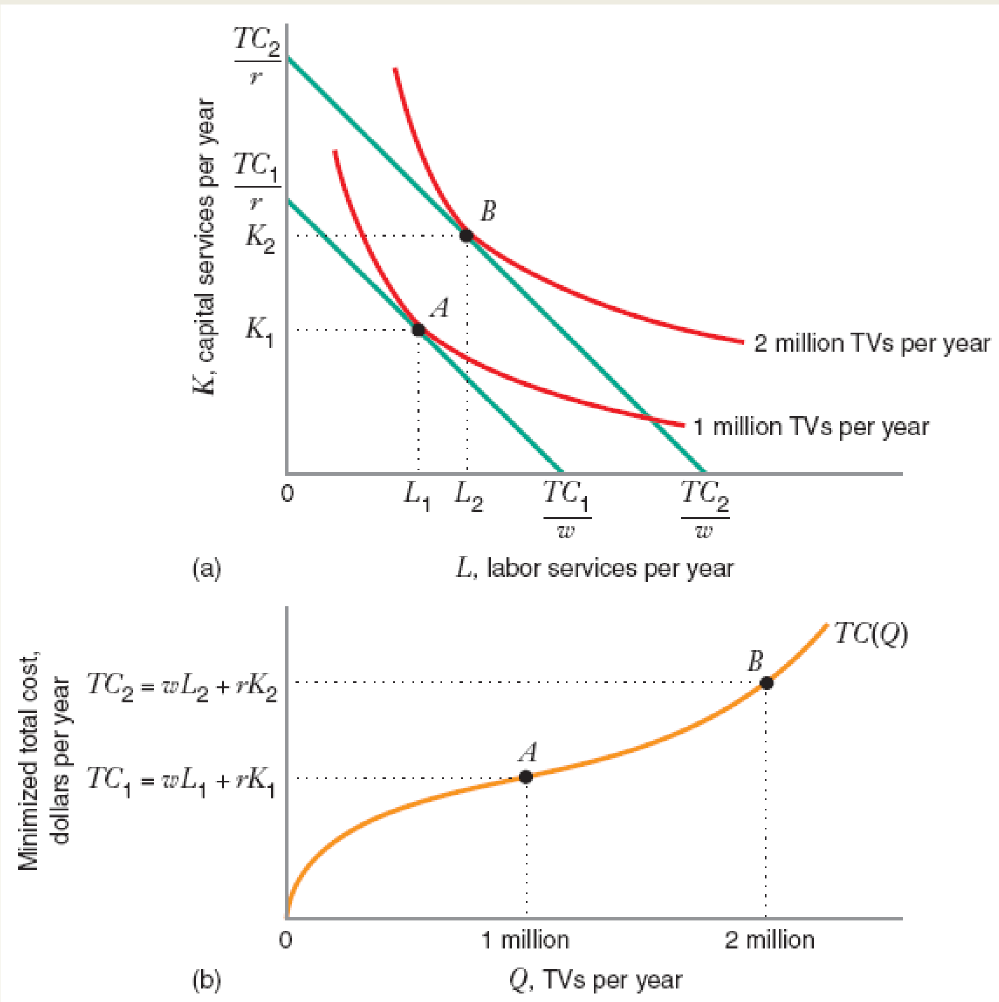
Long Run Cost Functions

Definition: The **long-run total cost (LRTC) function** relates minimized total cost to output, Q, and to the factor prices (w and r).

$TC(Q,w,r) = wL^*(Q,w,r) + rK^*(Q,w,r)$

Where: L^* and K^* are the solution to the cost-minimization problem.

Long Run Cost Functions



Suppose w and r are constant.

As output increases from 1 million to 2 million, …

- cost-minimizing input combination moves from (L_1, K_1) to (L_2, K_2)
- total cost rises from TC_1 to TC_2, which gives the TC(Q) curve.

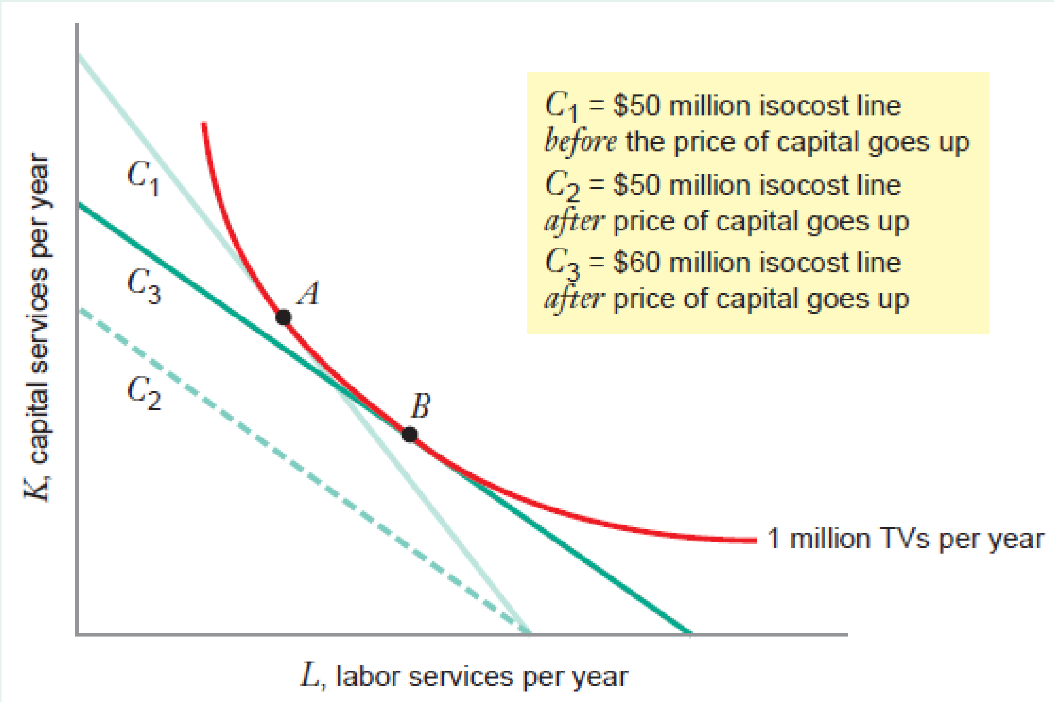
Long Run TC Curve

- A **movement** along the TC curve is a result from changing output.
- As we produce more, we face higher TC.

- What happens when input prices changes?
- **The TC curve will shift**.

- Let's consider two cases:
  - When r (price of K) rises.
  - When both r and w rises equally.

Long Run TC Curve – Higher r



Given higher r, we opt for a cheaper input, L.

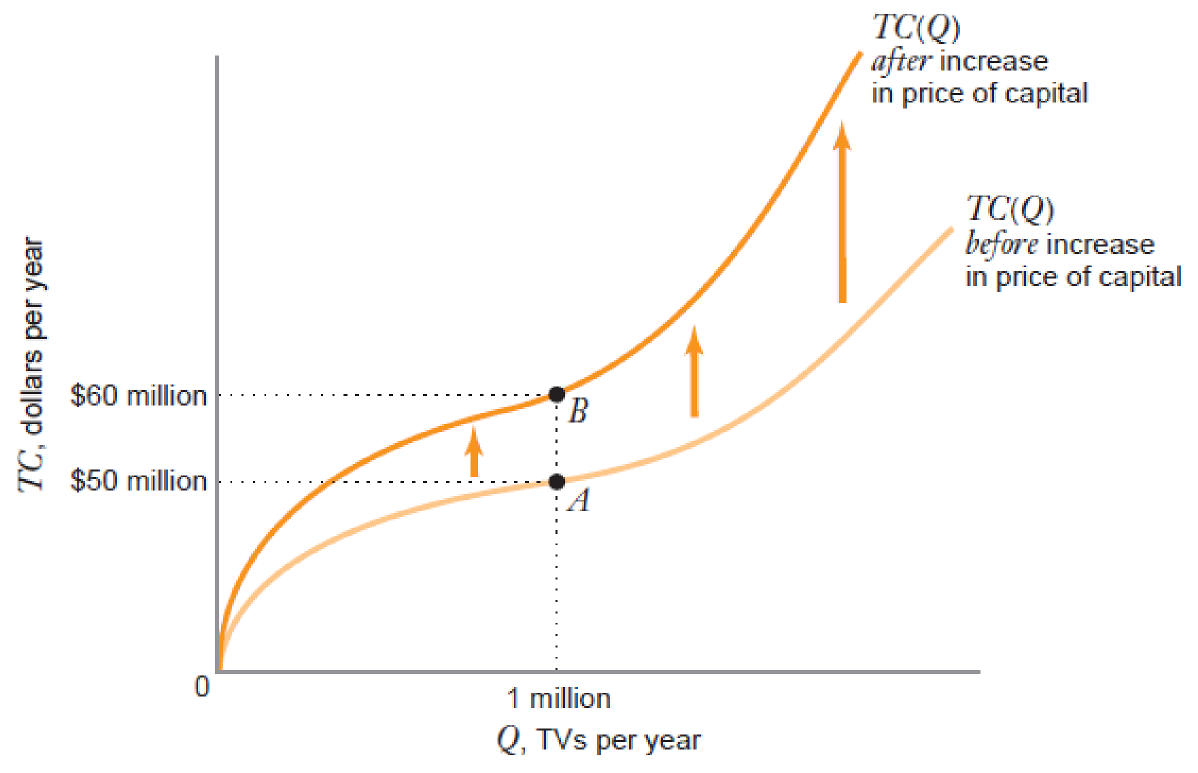
FIGURE 8.3 How a Change in the Price of Capital Affects the Optimal Input Combination and Long-Run Total Cost for a Producer of Television Sets
The firm's long-run total cost increases after the price of capital increases. The isocost line moves from C_1 to C_3 and the cost-minimizing input combination shifts from point *A* to point *B*.

C_2 = $50m BUT few K can be bought, so C_2 is not sufficient for producing 1m TVs.

C_3 is sufficient to produce the required output.

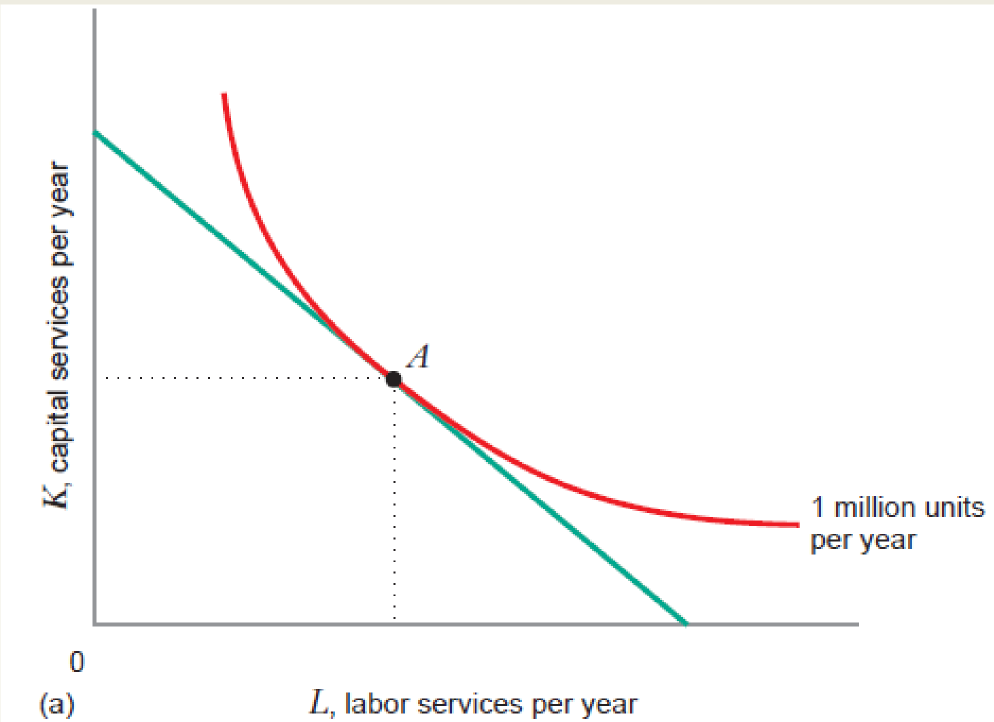
Long Run TC Curve – Higher r

FIGURE 8.4 How a Change in the Price of Capital Affects the Long-Run Total Cost Curve for a Producer of Television Sets An increase in the price of capital causes the long-run total cost curve *TC*(*Q*) to rotate upward. Points *A* and *B* correspond to the cost-minimizing input combinations in Figure 8.3.



With higher r, producing 1m TVs now requires $60m, instead of $50m

Long Run TC Curve – Higher r & w



When both r and w rise in the same proportion, the cost-minimizing K* and L* do not change.

We cannot opt for a cheaper input because both input prices rise.

Note that the TC will rise in the same proportion.

Long Run TC Curve – Higher r & w

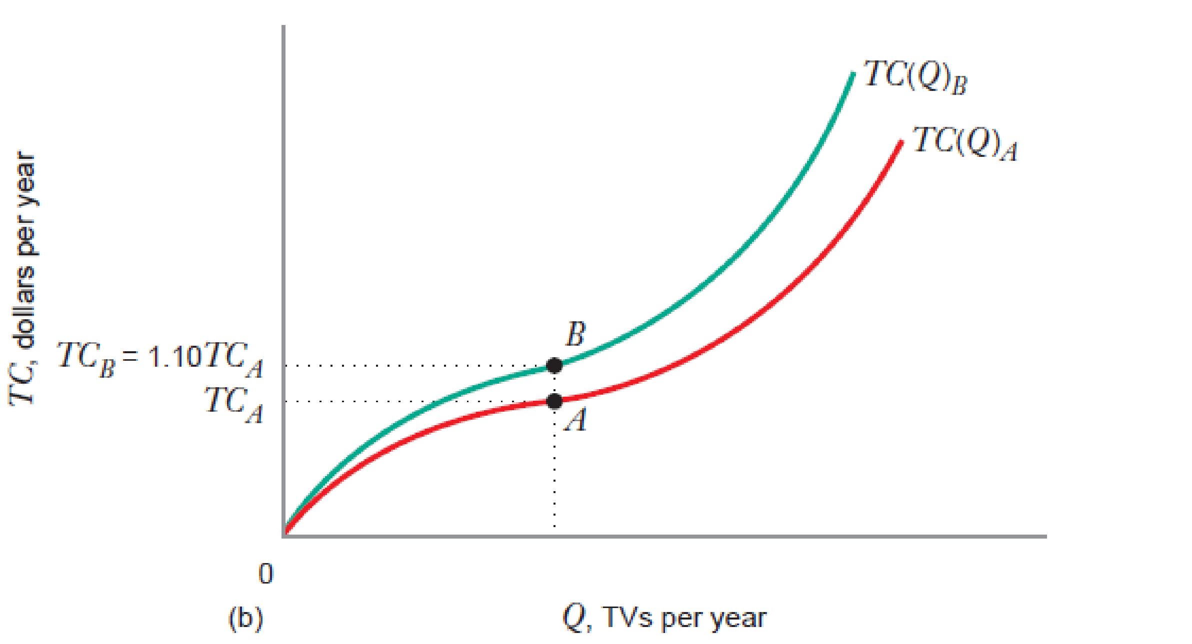


FIGURE 8.5 How a Proportionate Change in the Prices of All Inputs Affects the Cost-Minimizing Input Combination and the Total Cost Curve
The price of each input increases by 10 percent. Panel (a) shows that the cost-minimizing input combination remains the same (at point *A*), because the slope of the isocost line is unchanged. Panel (b) shows that the total cost curve shifts up by the same 10 percent.

Here, r and w rise by 10%.

TC also rises by 10%.

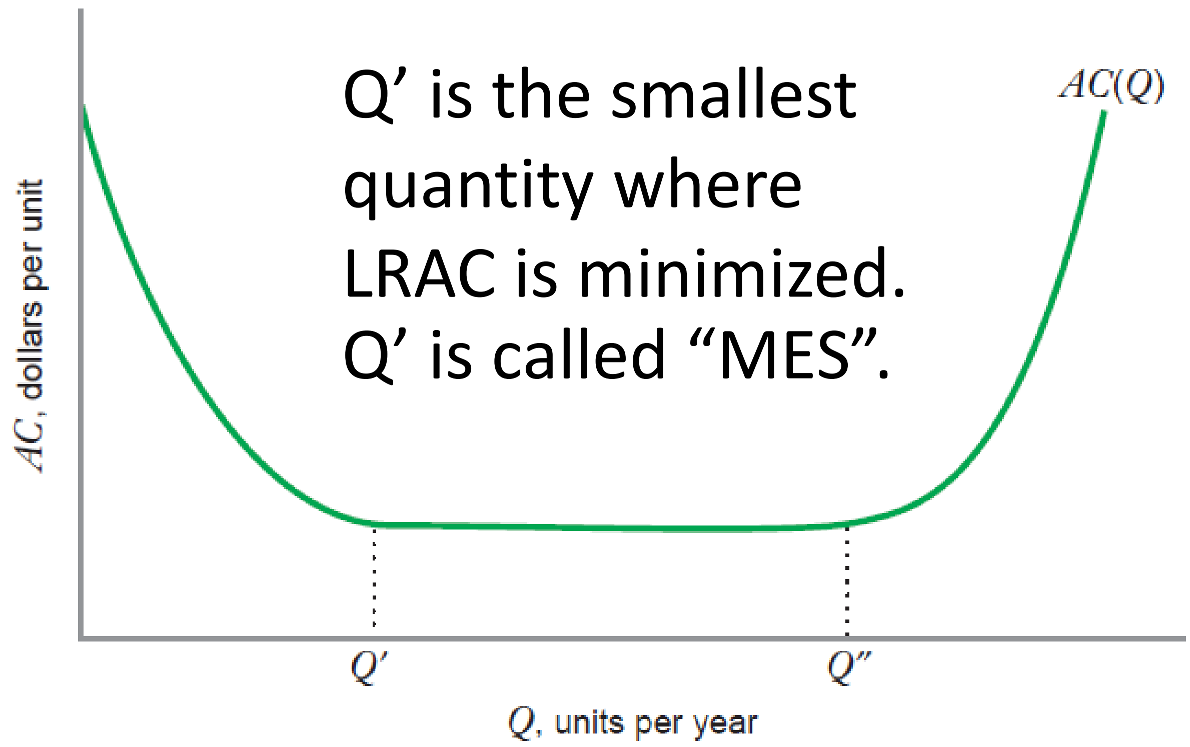
Economies & Diseconomies of Scale

Definition: If LRAC decreases as output rises, all else equal, the LRAC function exhibits economies of scale.

Definition: if LRAC increases as output rises, all else equal, the LRAC function exhibits diseconomies of scale.

Definition: The smallest quantity at which the LRAC curve attains its minimum point is called the minimum efficient scale (MES).

Economies & Diseconomies of Scale



Q’ is the smallest quantity where LRAC is minimized. Q’ is called “MES”.

FIGURE 8.11 **Economies and Diseconomies of Scale for a Typical Real-World Average Cost Curve** There are economies of scale for outputs less than *Q*′. Average costs are flat between and *Q*′ and *Q*″ and there are diseconomies of scale thereafter. The output level *Q*′ is called the minimum efficient scale.

From 0 to Q’, we have “**economies of scale**”.
From Q’’ onwards, we have “**diseconomies of scale**”.

Economies & Diseconomies of Scale

Reasons for EoS (lower AC when Q increases)

- Specialization & Division of Labor
- Bulk Buying & Quantity Discount
- Indivisible Inputs (e.g. large machine)
- Financial Economies

Reasons for DoS (higher AC when Q increases)

- Principal-Agent Problem
- Communication Problems
- Coordination Problems
- Managerial Diseconomies

Returns to Scale & Economies of Scale

- When the production function exhibits **increasing RTS**, the LRAC function exhibits **economies of scale** so that AC(Q) decreases with Q, all else equal.

- When the production function exhibits **decreasing RTS**, the LRAC function exhibits **diseconomies of scale** so that AC(Q) increases with Q, all else equal.

- When the production function exhibits **constant RTS, the LRAC function is flat**: it neither increases nor decreases with output.

Relationship between LR and SR Costs

The firm can minimize costs at least as well in the long run as in the short run because it is "**less constrained**".

Hence, **the short run total cost curve lies everywhere above the long run total cost curve.**

However, some inputs combination solves both SR cost-minimization and LR cost-minimization. At such inputs, SRTC = LRTC.

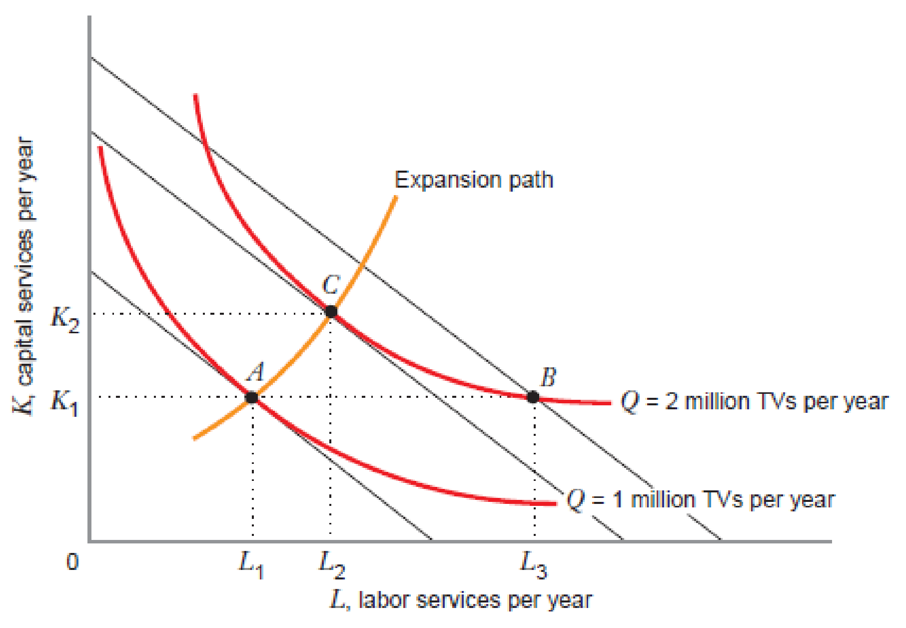


FIGURE 8.14 Total Costs Are Generally Higher in the Short Run than in the Long Run Initially, the firm produces 1 million TVs per year and operates at point *A*, which minimizes cost in both the long run and the short run, if the firm's usage of capital is fixed at K_1. If Q is increased to 2 million TVs per year, and capital remains fixed at K_1 in the short run, the firm operates at point *B*. But in the long run, the firm operates at point *C*, on a lower isocost line than point *B*.

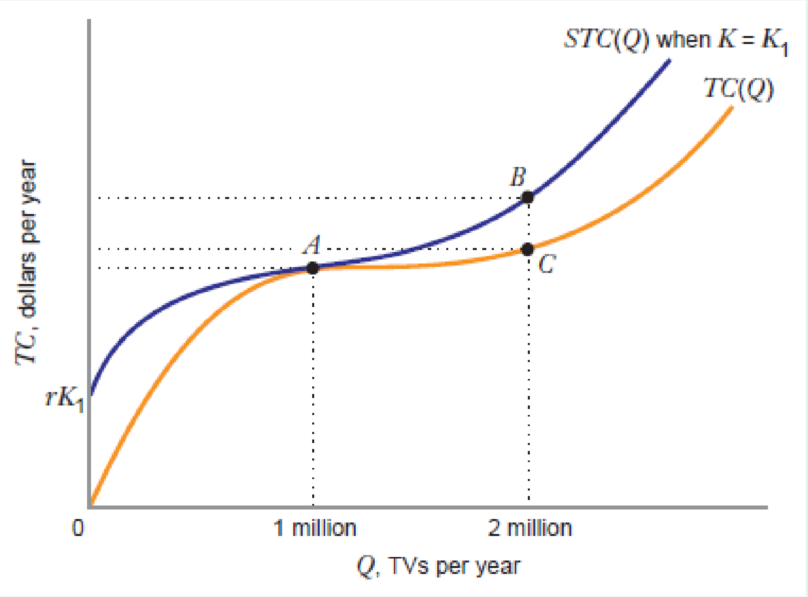


FIGURE 8.15 Relationship between Short-Run and Long-Run Total Cost Curves When the quantity of capital is fixed at K_1, $STC(Q)$ is always above $TC(Q)$, except at point *A*. Point *A* solves both the long-run and the short-run cost-minimization problem when the firm produces 1 million TVs per year.